

Signal Processing Toolbox

For Use with MATLAB®

Computation

Visualization

Programming

User's Guide

Version 4

How to Contact The MathWorks:



508-647-7000

Phone



508-647-7001

Fax



The MathWorks, Inc.
24 Prime Park Way
Natick, MA 01760-1500

Mail



<http://www.mathworks.com>
<ftp.mathworks.com>
<comp.soft-sys.matlab>

Web
Anonymous FTP server
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Signal Processing Toolbox User's Guide

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What Is the Signal Processing Toolbox?

This section describes how to begin using the Signal Processing Toolbox. It explains how to use this manual and points you to additional books for toolbox installation information.

The Signal Processing Toolbox is a collection of tools built on the MATLAB[®] numeric computing environment. The toolbox supports a wide range of signal processing operations, from waveform generation to filter design and implementation, parametric modeling, and spectral analysis. The toolbox provides two categories of tools:

- Signal processing functions
- Graphical, interactive tools

The first category of tools is made up of functions that you can call from the command line or from your own applications. Many of these functions are MATLAB M-files, series of MATLAB statements that implement specialized signal processing algorithms. You can view the MATLAB code for these functions using the statement

`type function_name`

or by opening the M-file in the MATLAB Editor/Debugger. You can change the way any toolbox function works by copying and renaming the M-file, then modifying your copy. You can also extend the toolbox by adding your own M-files.

Secondly, the toolbox provides a number of interactive tools that let you access many of the functions through a *graphical user interface* (GUI). The GUI-based tools provide an integrated environment for filter design, analysis, and implementation, as well as signal exploration and editing. For example, with the graphical user interface tools you can

- Use the mouse to graphically edit the magnitude response of a filter or measure the slope of a signal with onscreen rulers
- Play a signal on your system's audio hardware by selecting a menu item or pressing a corresponding keystroke combination
- Customize the parameters and method of computing the spectrum of a signal

How to Use This Manual

If you are a new user. Begin with Chapter 1, “Signal Processing Basics.” This chapter introduces the MATLAB signal processing environment through the toolbox functions. It describes the basic functions of the Signal Processing Toolbox, reviewing its use in basic waveform generation, filter implementation and analysis, impulse and frequency response, zero-pole analysis, linear system models, and the discrete Fourier transform.

When you feel comfortable with the basic functions, move on to Chapter 2 and Chapter 3 for a more in-depth introduction to using the Signal Processing Toolbox.

- Chapter 2, “Filter Design” for a detailed explanation of using the Signal Processing Toolbox in infinite impulse response (IIR) and finite impulse response (FIR) filter design and implementation, including special topics in IIR filter design.
- Chapter 3, “Statistical Signal Processing” for how to use the correlation, covariance, and spectral analysis tools to estimate important functions of discrete random signals.

Once you understand the general principles and applications of the toolbox, learn how to use the interactive tools.

- Chapter 5, “Interactive Tools” for an overview of the interactive GUI environment and examples of how to use it for signal exploration, filter design and implementation, and spectral analysis.

Finally, see the following chapter for a discussion of various specialized toolbox functions.

- Chapter 4, “Special Topics” for a variety of specialized functions including filter windows, parametric modeling, resampling, cepstrum analysis, time-dependent Fourier transforms and spectrograms, median filtering, communications applications, deconvolution, and specialized transforms.

If you are an experienced toolbox user. See Chapter 5, “Interactive Tools” for an overview of the interactive GUI environment and examples of how to use it for signal viewing, filter design and implementation, and spectral analysis.

All toolbox users. Use Chapter 6, “Reference” for locating information on specific functions. Reference descriptions include a synopsis of the function’s syntax, as well as a complete explanation of options and operation. Many reference descriptions also include helpful examples, a description of the function’s algorithm, and references to additional reading material.

Use this manual in conjunction with the software to learn about the powerful features that MATLAB provides. Each chapter provides numerous examples that apply the toolbox to representative signal processing tasks.

Some examples use MATLAB’s random number generation function `randn`. In these cases, to duplicate the results in the example, type

```
randn('seed', 0)
```

before running the example.

Installation

To install this toolbox on a workstation, see the *MATLAB Installation Guide for UNIX*. To install the toolbox on a PC or Macintosh, see the *MATLAB Installation Guide for PC and Macintosh*.

To determine if the Signal Processing Toolbox is already installed on your system, check for a subdirectory named `signal` within the main toolbox directory or folder.

Typographical Conventions

To Indicate...	This Manual Uses...	Example
Example code	Monospace type.	To assign the value 5 to A, enter A = 5
MATLAB output	Monospace type.	MATLAB responds with A = 5
Function names	Monospace type.	The cos function finds the cosine of each array element.
New terms	<i>Italics.</i>	An <i>array</i> is an ordered collection of information.
Keys	Boldface with an initial capital letter.	Press the R eturn key.
Menu names, items, and GUI controls	Boldface with an initial capital letter.	Choose the F ile menu.
Mathematical expressions	Variables in <i>italics</i> . Functions, operators, and constants in standard type.	This vector represents the polynomial $p = x^2 + 2x + 3$

Technical Notations

This manual and the Signal Processing Toolbox functions use the following technical notations:

Nyquist frequency	One-half the sampling frequency. Most toolbox functions normalize this value to 1.
$x(1)$	The first element of a data sequence or filter, corresponding to zero lag.
Ω	Analog frequency in radians per second.
w	Digital frequency in radians per second.
f	Digital frequency in Hertz.

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Signal Processing Toolbox Central Features

This chapter describes how to begin using MATLAB and the Signal Processing Toolbox for your signal processing applications. It assumes a basic knowledge and understanding of signals and systems, including such topics as filter and linear system theory and basic Fourier analysis.

There are many examples throughout the chapter that demonstrate how to apply toolbox functions. If you are not already familiar with MATLAB's signal processing capabilities, use this chapter in conjunction with the software to try examples and learn about the powerful features available to you.

The Signal Processing Toolbox functions are algorithms, expressed mostly in M-files, that implement a variety of signal processing tasks. These toolbox functions are a specialized extension of the MATLAB computational and graphical environment.

Filtering and FFTs

Two of the most important functions for signal processing are not in the Signal Processing Toolbox at all, but are built-in MATLAB functions:

- `filter` applies a digital filter to a data sequence.
- `fft` calculates the discrete Fourier transform of a sequence.

The operations these functions perform are the main computational workhorses of classical signal processing. Both are described in this chapter. The Signal Processing Toolbox uses many other standard MATLAB functions and language features, including polynomial root finding, complex arithmetic, matrix inversion and manipulation, and graphics tools.

Signals and Systems

The basic entities that toolbox functions work with are signals and systems. The functions emphasize digital, or discrete, signals and filters, as opposed to analog, or continuous, signals. The principal filter type the toolbox supports is the linear, time-invariant digital filter with a single input and a single output. You can represent linear time-invariant systems using one of several models (such as transfer function, state-space, zero-pole-gain, and second-order section) and convert between representations.

Key Areas: Filter Design and Spectral Analysis

In addition to its core functions, the toolbox provides rich, customizable support for the key areas of filter design and spectral analysis. It is easy to implement a design technique that suits your application, design digital filters directly, or create analog prototypes and discretize them. Toolbox functions also estimate power spectral density and cross spectral density, using either parametric or nonparametric techniques. Chapters 2 and 3, respectively, detail toolbox functions for filter design and spectral analysis.

There are functions for computation and graphical display of frequency response, as well as functions for system identification; generating signals; discrete cosine, chirp- z , and Hilbert transforms; lattice filters; resampling; time-frequency analysis; and basic communication systems simulation.

Graphical User Interface (GUI)

The power of the Signal Processing Toolbox is greatly enhanced by its easy-to-use graphical user interface. The GUI provides an integrated set of interactive tools for performing a wide variety of signal processing tasks. These tools enable you to use the mouse and menus to manipulate a rich graphical environment for signal viewing, filter design and implementation, and spectral analysis.

Extensibility

Perhaps the most important feature of the MATLAB environment is that it is extensible: MATLAB lets you create your own M-files to meet numeric computation needs for research, design, or engineering of signal processing systems. Simply copy the M-files provided with the Signal Processing Toolbox and modify them as needed, or create new functions to expand the functionality of the toolbox.

Representing Signals

The central data construct in MATLAB is the *numeric array*, an ordered collection of real or complex numeric data with two or more dimensions. The basic data objects of signal processing (one-dimensional signals or sequences, multichannel signals, and two-dimensional signals) are all naturally suited to array representation.

Vector Representation

MATLAB represents ordinary one-dimensional sampled data signals, or sequences, as *vectors*. Vectors are 1-by- n or n -by-1 arrays, where n is the number of samples in the sequence. One way to introduce a sequence into MATLAB is to enter it as a list of elements at the command prompt. The statement

```
x = [4 3 7 -9 1]
```

creates a simple five-element real sequence in a row vector. Transposition turns the sequence into a column vector,

```
x = x'
```

resulting in

```
x =  
    4  
    3  
    7  
   -9  
    1
```

Column orientation is preferable for single channel signals because it extends naturally to the multichannel case. For multichannel data, each column of a matrix represents one channel. Each row of such a matrix then corresponds to a sample point. A three-channel signal that consists of x , $2x$, and x/π is

```
y = [x 2*x x/pi]
```


This results in

$y =$

4.0000	8.0000	1.2732
3.0000	6.0000	0.9549
7.0000	14.0000	2.2282
-9.0000	-18.0000	-2.8648
1.0000	2.0000	0.3183

Waveform Generation: Time Vectors and Sinusoids

There are a variety of toolbox functions for generating waveforms. Most require you to begin with a vector representing a time base. Consider generating data with a 1000 Hz sample frequency, for example. An appropriate time vector is

```
t = (0: .001: 1) ' ;
```

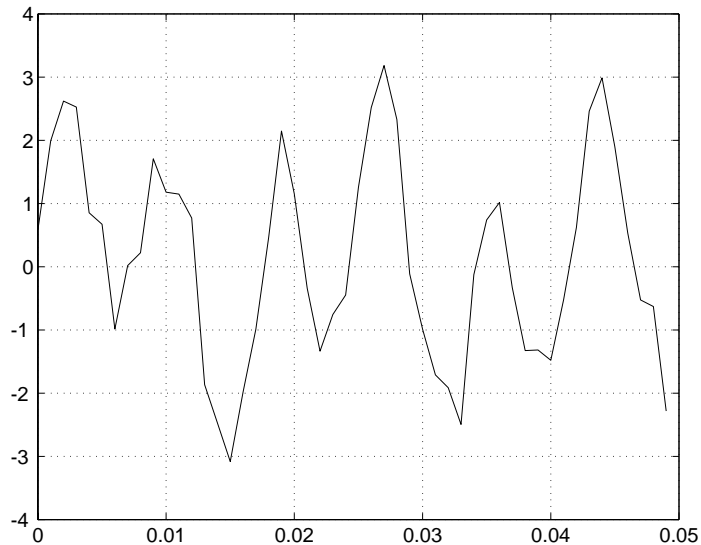
where MATLAB's colon operator creates a 1001-element row vector that represents time running from zero to one second in steps of one millisecond. The transpose operator (') changes the row vector into a column; the semicolon (;) tells MATLAB to compute but not display the result.

Given t you can create a sample signal y consisting of two sinusoids, one at 50 Hz and one at 120 Hz with twice the amplitude:

```
y = sin(2*pi*50*t) + 2*sin(2*pi*120*t) ;
```

The new variable y , formed from vector t , is also 1001 elements long. You can add normally distributed white noise to the signal and graph the first fifty points using

```
yn = y + 0.5*randn(size(t)) ;  
plot(t(1:50), yn(1:50))
```



Common Sequences: Unit Impulse, Unit Step, and Unit Ramp

Since MATLAB is a programming language, an endless variety of different signals is possible. Here are some statements that generate several commonly used sequences, including the unit impulse, unit step, and unit ramp functions:

```
t = (0: .001: 1)';
y = [1; zeros(99, 1)]; % impulse
y = ones(100, 1);      % step (filter assumes 0 initial cond.)
y = t;                  % ramp
y = t.^2;
y = square(4*t);
```

All of these sequences are column vectors – the last three inherit their shapes from `t`.

Multichannel Signals

Use standard MATLAB array syntax to work with multichannel signals. For example, a multichannel signal consisting of the last three signals generated above is

```
z = [t t.^2 square(4*t)];
```

You can generate a multichannel unit sample function using the outer product operator. For example, a six-element column vector whose first element is one, and whose remaining five elements are zeros, is

```
a = [1 zeros(1, 5)]';
```

To duplicate column vector `a` into a matrix without performing any multiplication, use MATLAB's colon operator and the `ones` function:

```
c = a(:, ones(1, 3));
```

Common Periodic Waveforms

The toolbox provides functions for generating widely used periodic waveforms:

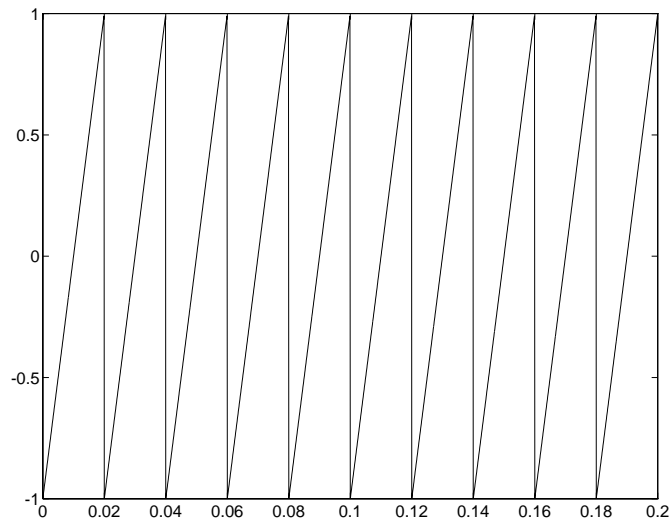
- `sawtooth` generates a sawtooth wave with peaks at ± 1 and a period of 2π . An optional `width` parameter specifies a fractional multiple of 2π at which the signal's maximum occurs.
- `square` generates a square wave with a period of 2π . An optional parameter specifies *duty cycle*, the percent of the period for which the signal is positive.

To generate 1.5 seconds of a 50 Hz sawtooth wave with a sample rate of 10 kHz and plot 0.2 seconds of the generated waveform, use

```

Fs = 10000;
t = 0: 1/Fs: 1.5;
x = sawtooth(2*pi*50*t);
plot(t, x), axis([0 0.2 -1 1])

```



Common Aperiodic Waveforms

The toolbox also provides functions for generating several widely used aperiodic waveforms:

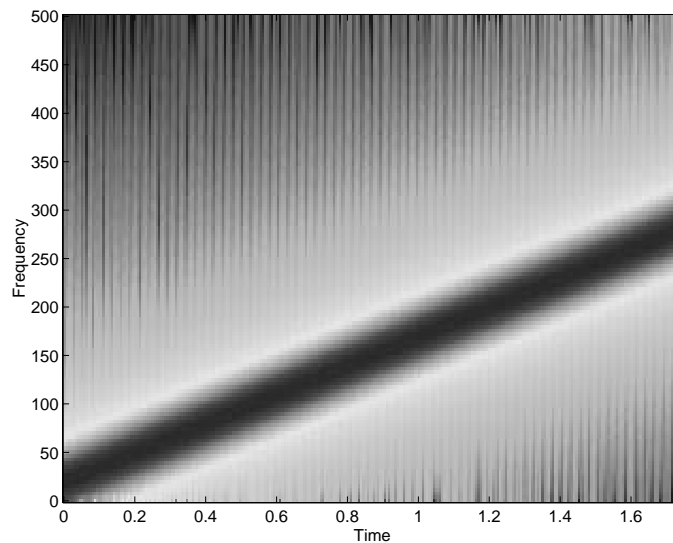
- `gauspuls` generates a Gaussian-modulated sinusoidal pulse with a specified time, center frequency, and fractional bandwidth. Optional parameters return in-phase and quadrature pulses, the RF signal envelope, and the cutoff time for the trailing pulse envelope.
- `chirp` generates a linear swept-frequency cosine signal. An optional parameter specifies alternative sweep methods. An optional parameter `phi` allows initial phase to be specified in degrees.

To compute 2 seconds of a linear chirp signal with a sample rate of 1 kHz, that starts at DC and crosses 150 Hz at 1 second, use

```
t = 0:1/1000:2;  
y = chirp(t, 0, 1, 150);
```

To plot the spectrogram, use

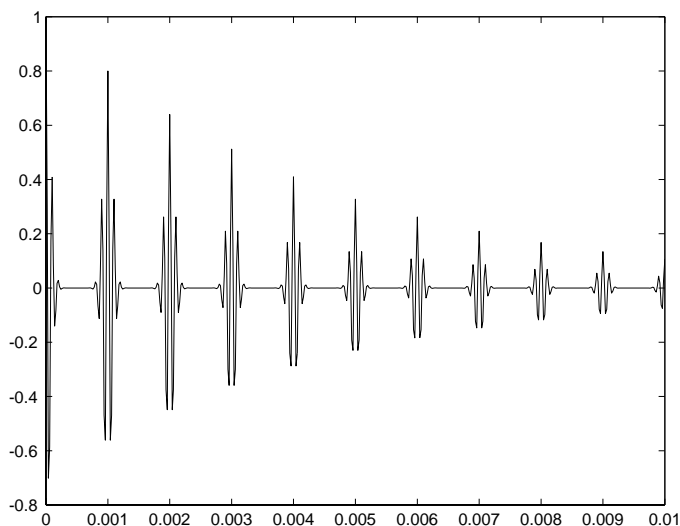
```
specgram(y, 256, 1000, 256, 250)
```



The pulstran Function

The `pulstran` function generates pulse trains from either continuous or sampled prototype pulses. The following example generates a pulse train consisting of the sum of multiple delayed interpolations of a Gaussian pulse. The pulse train is defined to have a sample rate of 50 kHz, a pulse train length of 10 ms, and a pulse repetition rate of 1 kHz; `D` specifies the delay to each pulse repetition in column 1 and an optional attenuation for each repetition in column 2. The pulse train is constructed by passing the name of the `gauspuls` function to `pulstran`, along with additional parameters that specify a 10 kHz Gaussian pulse with 50% bandwidth:

```
T = 0: 1/50E3: 10E-3;
D = [0: 1/1E3: 10E-3; 0.8.^(0: 10)]';
Y = pulstran(T, D, 'gauspuls', 10E3, 0.5);
plot(T, Y)
```



The sinc Function

The `sinc` function computes the mathematical sinc function for an input vector or matrix `x`. The sinc function is the continuous inverse Fourier transform of the rectangular pulse of width 2π and height 1:

$$\text{sinc}(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega t} d\omega$$

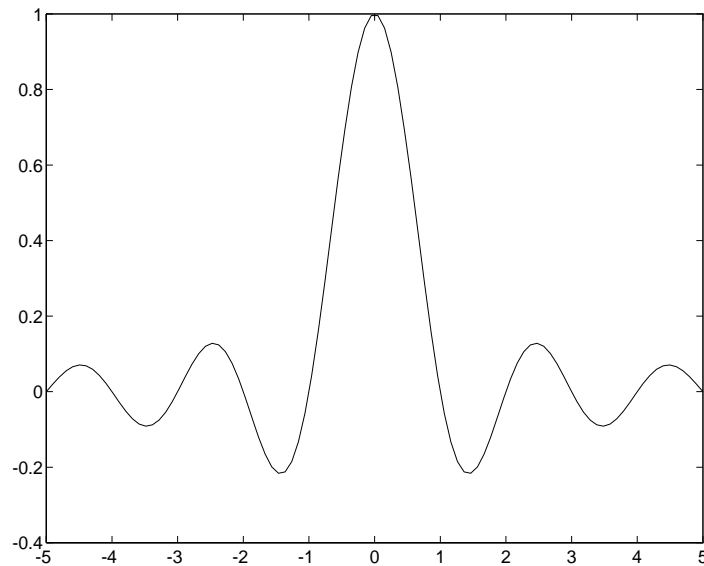
The sinc function has a value of 1 where x is zero, and a value of

$$\frac{\sin(\pi x)}{\pi x}$$

for all other elements of x .

To plot the sinc function for a linearly spaced vector with values ranging from -5 to 5,

```
x = linspace(-5, 5);
y = sinc(x);
plot(x, y)
```



The Dirichlet Function

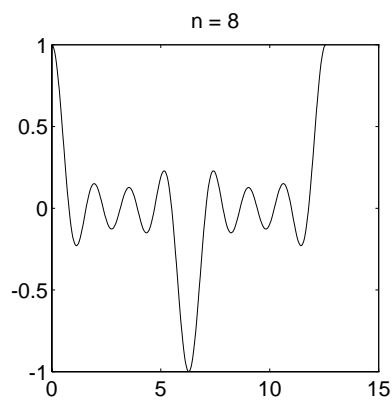
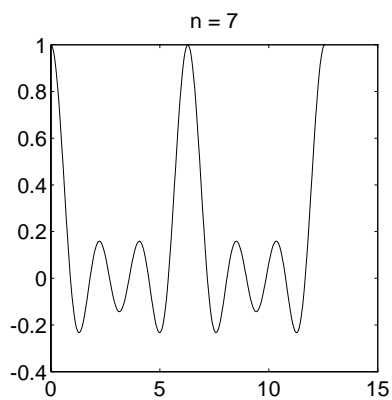
The toolbox function `diric` computes the Dirichlet function, sometimes called the *periodic sinc* or *aliased sinc* function, for an input vector or matrix x . The Dirichlet function is

$$\text{diric}(x) = \begin{cases} -1^{k(n-1)} & x = 2\pi k, \quad k = 0, \pm 1, \pm 2, \dots \\ \frac{\sin(nx/2)}{n \sin(x/2)} & \text{otherwise} \end{cases}$$

where n is a user-specified positive integer. For n odd, the Dirichlet function has a period of 2π ; for n even, its period is 4π . The magnitude of this function is $(1/n)$ times the magnitude of the discrete-time Fourier transform of the n -point rectangular window.

To plot the Dirichlet function over the range 0 to 4π for $n = 7$ and $n = 8$, use

```
x = linspace(0, 4*pi, 300);
plot(x, diric(x, 7))
plot(x, diric(x, 8))
```



Working with Data

The examples in the preceding sections obtain data in one of two ways:

- By direct input, that is, entering the data manually at the keyboard
- Using a MATLAB or toolbox function, such as `sin`, `cos`, `sawtooth`, `square`, or `sinc`

Some applications, however, may need to import data from outside MATLAB. Depending on your data format, you can do this in the following ways:

- Loading data from an ASCII file or MAT-file with MATLAB's `load` command
- Reading the data into MATLAB with a low-level file I/O function, such as `fopen`, `fread`, and `fscanf`
- Developing a MEX-file to read the data

Other resources are also useful, such as a high-level language program (in Fortran or C, for example) that converts your data into MAT-file format—see the *MATLAB Application Programming Interface* reference manual for details. MATLAB reads such files using the `load` command.

Similar techniques are available for exporting data generated within MATLAB. See *Using MATLAB* for more details on importing and exporting data, and see the online *MATLAB Function Reference* for descriptions of file loading and I/O routines.

Filter Implementation and Analysis

This section describes how to filter discrete signals using MATLAB's `filter` function and other functions in the Signal Processing Toolbox. It also discusses how to use the toolbox functions to analyze filter characteristics, including impulse response, magnitude and phase response, group delay, and zero-pole locations.

Convolution and Filtering

The mathematical foundation of filtering is convolution. MATLAB's `conv` function performs standard one-dimensional convolution, convolving one vector with another:

```
conv([1 1 1], [1 1 1])

ans =

     1     2     3     2     1
```

NOTE Convolve rectangular matrices for two-dimensional signal processing using the `conv2` function.

A digital filter's output $y(n)$ is related to its input $x(n)$ by convolution with its impulse response $h(n)$:

$$y(n) = h(n) * x(n) = \sum_{m=-\infty}^{\infty} h(n-m)x(m)$$

If a digital filter's impulse response $h(n)$ is finite length, and the input $x(n)$ is also finite length, you can implement the filter using `conv`. Store $x(n)$ in a vector `x`, $h(n)$ in a vector `h`, and convolve the two:

```
x = randn(5, 1);    % a random vector of length 5
h = [1 1 1 1]/4;    % length 4 averaging filter
y = conv(h, x);
```

Filters and Transfer Functions

In general, the z -transform $Y(z)$ of a digital filter's output $y(n)$ is related to the z -transform $X(z)$ of the input by:

$$Y(z) = H(z)X(z) = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}} X(z)$$

where $H(z)$ is the filter's *transfer function*. Here, the constants $b(i)$ and $a(i)$ are the filter coefficients and the order of the filter is the maximum of na and nb .

NOTE The filter coefficients start with subscript 1, rather than 0. This reflects MATLAB's standard indexing scheme for vectors.

MATLAB stores the coefficients in two vectors, one for the numerator and one for the denominator. By convention, MATLAB uses row vectors for filter coefficients.

Filter Coefficients and Filter Names

Many standard names for filters reflect the number of a and b coefficients present:

- When $nb = 0$ (that is, b is a scalar), the filter is an Infinite Impulse Response (IIR), all-pole, recursive, or autoregressive (AR) filter.
- When $na = 0$ (that is, a is a scalar), the filter is a Finite Impulse Response (FIR), all-zero, nonrecursive, or moving average (MA) filter.
- If both na and nb are greater than zero, the filter is an IIR, pole-zero, recursive, or autoregressive moving average (ARMA) filter.

The names AR, MA, and ARMA are usually applied to filters associated with filtered stochastic processes.

Filtering with the filter Function

It is simple to work back to a difference equation from the z -transform relation shown earlier. Assume that $a(1) = 1$. Move the denominator to the left-hand side and take the inverse z -transform:

$$y(n) + a(2)y(n-1) + \dots + a(na+1)y(n-na) = b(1)x(n) + b(2)x(n-1) + \dots + b(nb+1)x(n-nb)$$

In terms of current and past inputs, and past outputs, $y(n)$ is:

$$y(n) = b(1)x(n) + b(2)x(n-1) + \dots + b(nb+1)x(n-nb) - a(2)y(n-1) - \dots - a(na+1)y(n-na)$$

This is the standard time-domain representation of a digital filter, computed starting with $y(1)$ and assuming zero initial conditions. This representation's progression is

$$\begin{aligned} y(1) &= b(1)x(1) \\ y(2) &= b(1)x(2) + b(2)x(1) - a(2)y(1) \\ y(3) &= b(1)x(3) + b(2)x(2) + b(3)x(1) - a(2)y(2) - a(3)y(1) \\ &\vdots \end{aligned}$$

A filter in this form is easy to implement with the `filter` function. For example, a simple single-pole filter (lowpass) is:

```
b = 1;           % numerator
a = [1 -0.9];    % denominator
```

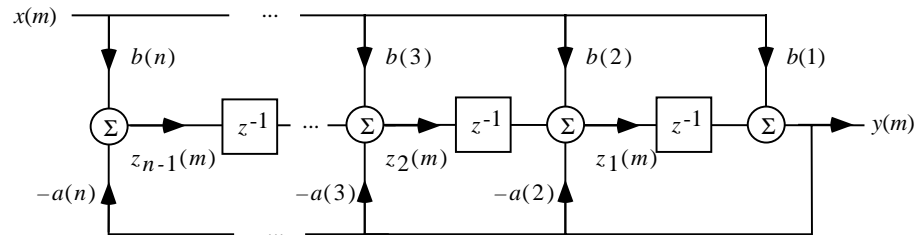
where the vectors `b` and `a` represent the coefficients of a filter in transfer function form. To apply this filter to your data:

```
y = filter(b, a, x);
```

`filter` gives you as many output samples as there are input samples, that is, the length of `y` is the same as the length of `x`. If the first element of `a` is not 1, `filter` divides the coefficients by `a(1)` prior to implementing the difference equation.

filter Function Implementation and Initial Conditions

`filter` is implemented as a transposed direct form II structure



where $n-1$ is the filter order. This is a canonical form that has the minimum number of delay elements.

At sample m , `filter` computes the difference equations

$$\begin{aligned} y(m) &= b(1)x(m) + z_1(m-1) \\ z_1(m) &= b(2)x(m) + z_2(m-1) - a(2)y(m) \\ \vdots &= \vdots \\ z_{n-2}(m) &= b(n-1)x(m) + z_{n-1}(m-1) - a(n-1)y(m) \\ z_{n-1}(m) &= b(n)x(m) - a(n)y(m) \end{aligned}$$

In its most basic form, `filter` initializes the delay outputs $z_i(1)$, $i = 1, \dots, n-1$ to 0. This is equivalent to assuming both past inputs and outputs are zero. Set the initial delay outputs using a fourth input parameter to `filter`, or access the final delay outputs using a second output parameter:

$$[y, zf] = \text{filter}(b, a, x, zi)$$

Access to initial and final conditions is useful for filtering data in sections, especially if memory limitations are a consideration. Suppose you have collected data in two segments of 5000 points each:

```
x1 = randn(5000, 1); % two random sequences to
x2 = randn(5000, 1); % serve as simulated data
```

Perhaps the first sequence, $x1$, corresponds to the first 10 minutes of data and the second, $x2$, to an additional 10 minutes. The whole sequence is $x = [x1; x2]$. If there is not sufficient memory to hold the combined sequence, filter the subsequences $x1$ and $x2$ one at a time. To ensure continuity of the

filtered sequences, use the final conditions from `x1` as initial conditions to filter `x2`:

```
[y1, zf] = filter(b, a, x1);  
y2 = filter(b, a, x2, zf);
```

The `filtic` function generates initial conditions for `filter`. `filtic` computes the delay vector to make the behavior of the filter reflect past inputs and outputs that you specify. To obtain the same output delay values `zf` as above using `filtic`:

```
zf = filtic(b, a, flipud(y1), flipud(x1));
```

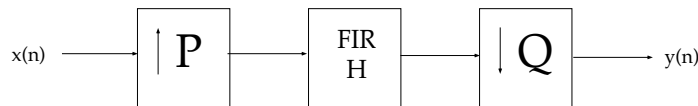
This can be useful when filtering short data sequences, as appropriate initial conditions help reduce transient startup effects.

Other Functions for Filtering

In addition to `filter`, there are several other functions in the Signal Processing Toolbox that perform the basic filtering operation. These functions are `upfirdn`, which performs FIR filtering with resampling, `filtfilt`, which eliminates phase distortion in the filtering process, and `fftfilter`, which performs the FIR filtering operation in the frequency domain.

Multirate Filter Bank Implementation

The function `upfirdn` alters the sampling rate of a signal by an integer ratio P/Q . It computes the result of the cascade of three systems: (1) upsampling (zero insertion) by integer factor p , (2) filtering by FIR filter h , and (3) downsampling by integer factor q :



For example, to change the sample rate of a signal from 44.1 kHz to 48 kHz, we first find the smallest integer conversion ratio p/q :

```

d = gcd(48000, 44100);
p = 48000/d;
q = 44100/d;
  
```

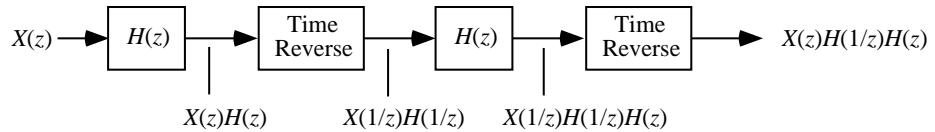
where we find that $p = 160$ and $q = 147$. Sample rate conversion is then accomplished by $y = \text{upfirdn}(x, h, p, q)$. This cascade of operations is implemented in an efficient manner using polyphase filtering techniques, and it is a central concept of multirate filtering (see reference [1] for details on multirate filter theory). Note that the quality of the resampling result relies on the quality of the FIR filter h .

Filter banks may be implemented using `upfirdn` by allowing the filter h to be a matrix, with one FIR filter per column. A signal vector is passed independently through each FIR filter, resulting in a matrix of output signals.

Anti-Causal, Zero-Phase Filter Implementation

In the case of FIR filters, it is possible to design linear phase filters that, when applied to data (using `filter` or `conv`), simply delay the output by a fixed number of samples. For IIR filters, however, the phase distortion is usually highly nonlinear. The `filtfilt` function uses the information in the signal at points before and after the current point, in essence “looking into the future,” to eliminate phase distortion.

To see how `filtfilt` does this, recall that if the z -transform of a real sequence $x(n)$ is $X(z)$, the z -transform of the time reversed sequence $x(n)$ is $X(1/z)$. Consider the processing scheme



When $|z| = 1$, that is $z = e^{j\omega}$, the output reduces to $X(e^{j\omega}) |H(e^{j\omega})|^2$. Given all the samples of the sequence $x(n)$, a doubly filtered version of x that has zero-phase distortion is possible.

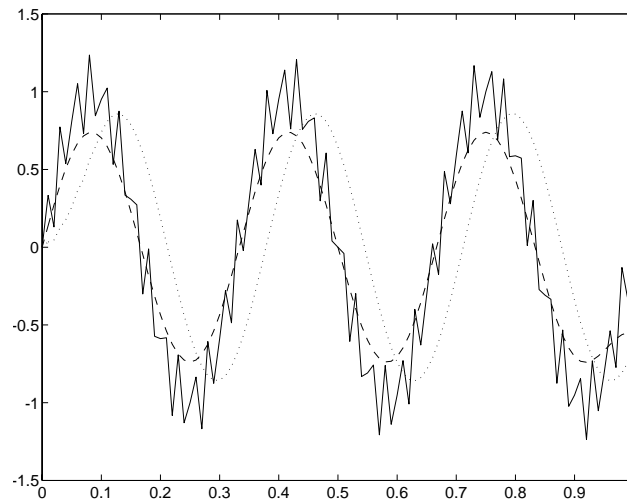
For example, a one-second duration signal sampled at 100 Hz, composed of two sinusoidal components at 3 Hz and 40 Hz, is

```

Fs = 100;
t = 0: 1/Fs: 1;
x = sin(2*pi*t*3) + .25*sin(2*pi*t*40);
  
```


Now create a 10-point averaging FIR filter, and filter `x` using both `filter` and `filtfilt` for comparison:

```
b = ones(1, 10) / 10;           % 10 point averaging filter
y = filtfilt(b, 1, x);          % non-causal filtering
yy = filter(b, 1, x);           % normal filtering
plot(t, x, t, y, '--', t, yy, ':')
```



Both filtered versions eliminate the 40 Hz sinusoid evident in the original, solid line. The plot also shows how `filter` and `filtfilt` differ; the dashed (`filtfilt`) line is in phase with the original 3 Hz sinusoid, while the dotted (`filter`) line is delayed by about five samples. Also, the amplitude of the dashed line is smaller due to the magnitude squared effects of `filtfilt`.

`filtfilt` reduces filter startup transients by carefully choosing initial conditions, and by prepending onto the input sequence a short, reflected piece of the input sequence. For best results, make sure the sequence you are filtering has length at least three times the filter order, and that it tapers to zero on both edges.

Frequency Domain Filter Implementation

Duality between the time domain and the frequency domain makes it possible to perform any operation in either domain. Usually one domain or the other is more convenient for a particular operation, but you can always accomplish a given operation in either domain.

To implement general IIR filtering in the frequency domain, multiply the discrete Fourier transform (DFT) of the input sequence with the quotient of the DFT of the filter,

```
n = length(x);  
y = ifft(fft(x) .* fft(b, n) ./ fft(a, n));
```

This computes results that are identical to `filter`, but with different startup transients (edge effects). For long sequences, this computation is very inefficient because of the large zero-padded FFT operations on the filter coefficients, and because the FFT algorithm becomes less efficient as the number of points `n` increases.

For FIR filters, however, it is possible to break longer sequences into shorter, computationally efficient FFT lengths. The function

```
y = fftfilt(b, x)
```

uses the overlap add method (see reference [1] at the end of this chapter) to filter a long sequence with multiple medium-length FFTs. Its output is equivalent to `filter(b, 1, x)`.

Impulse Response

The impulse response of a digital filter is the output arising from the input sequence

$$x(n) = \begin{cases} 1, & n = 1 \\ 0, & n \neq 1 \end{cases}$$

In MATLAB, you can generate an impulse sequence a number of ways; one straightforward way is

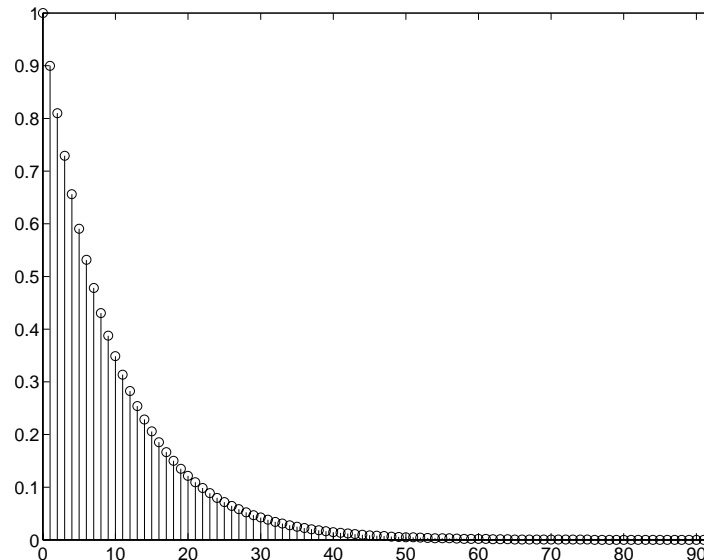
```
imp = [1; zeros(49, 1)];
```

The impulse response of the simple filter $b = 1$ and $a = [1 \ -0.9]$ is

```
h = filter(b, a, imp);
```

The `impz` function in the toolbox simplifies this operation, choosing the number of points to generate and then making a stem plot (using the `stem` function):

```
impz(b, a)
```



The plot shows the exponential decay $h(n) = 0.9^n$ of the single pole system.

Frequency Response

The Signal Processing Toolbox enables you to perform frequency domain analysis of both analog and digital filters.

Digital Domain

`freqz` uses an FFT-based algorithm to calculate the z -transform frequency response of a digital filter. Specifically, the statement

```
[h, w] = freqz(b, a, n)
```

returns the n -point complex frequency response, $H(e^{jw})$, of the digital filter:

$$H(e^{j\omega}) = \frac{b(1) + b(2)e^{-j\omega} + \dots + b(nb+1)e^{-j\omega(nb)}}{a(1) + a(2)e^{-j\omega} + \dots + a(na+1)e^{-j\omega(na)}}$$

In its simplest form, `freqz` accepts the filter coefficient vectors `b` and `a`, and an integer `n` specifying the number of points at which to calculate the frequency response. `freqz` returns the complex frequency response in vector `h`, and the actual frequency points in vector `w` in radians/second.

`freqz` can accept other parameters, such as a sampling frequency or a vector of arbitrary frequency points. The example below finds the 256-point frequency response for a 12th-order Chebyshev type I filter. The call to `freqz` specifies a sampling frequency F_s of 1000 Hz:

```
[b, a] = cheby1(12, 0.5, 200/500);  
[h, f] = freqz(b, a, 256, 1000);
```

Because the parameter list includes a sampling frequency, `freqz` returns a vector `f` that contains the 256 frequency points between 0 and $F_s/2$ used in the frequency response calculation.

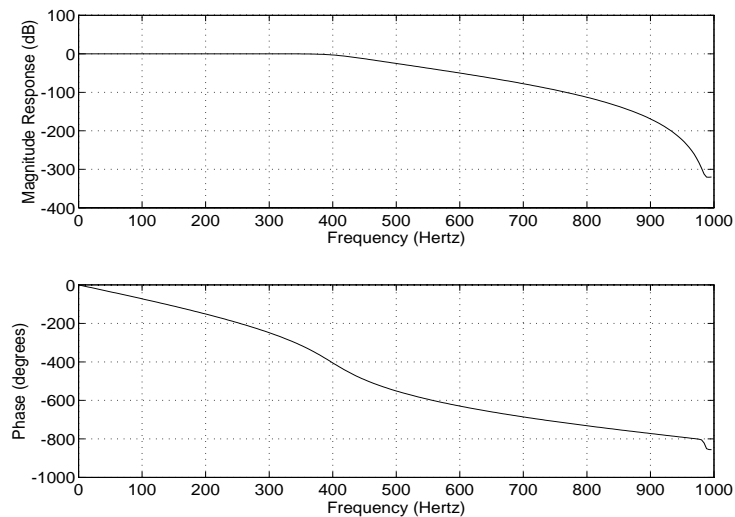
Frequency Normalization This toolbox uses the convention that unit frequency is the Nyquist frequency, defined as half the sampling frequency. The cutoff frequency parameter for all basic filter design functions is normalized by the Nyquist frequency. For a system with a 1000 Hz sampling frequency, for example, 300 Hz is $300/500 = 0.6$. To convert normalized frequency to angular frequency around the unit circle, multiply by π . To convert normalized frequency back to Hertz, multiply by half the sample frequency.

If you call `freqz` with no output arguments, it automatically plots both magnitude versus frequency and phase versus frequency. For example, a ninth-order Butterworth lowpass filter with a cutoff frequency of 400 Hz, based on a 2000 Hz sampling frequency, is

```
[b, a] = butter(9, 400/1000);
```

Now calculate the 256-point complex frequency response for this filter, and plot the magnitude and phase with a call to `freqz`:

```
freqz(b, a, 256, 2000)
```



`freqz` can also accept a vector of arbitrary frequency points for use in the frequency response calculation. For example,

```
w = linspace(0, pi);
h = freqz(b, a, w);
```

calculates the complex frequency response at the frequency points in `w` for the filter defined by vectors `b` and `a`. The frequency points can range from 0 to 2π . To specify a frequency vector that ranges from zero to your sampling frequency, include both the frequency vector and the sampling frequency value in the parameter list.

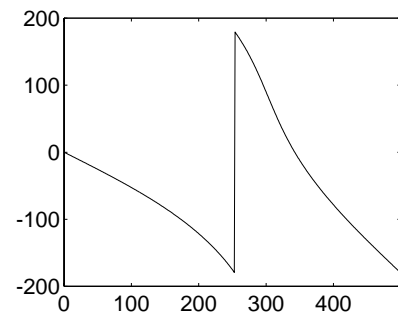
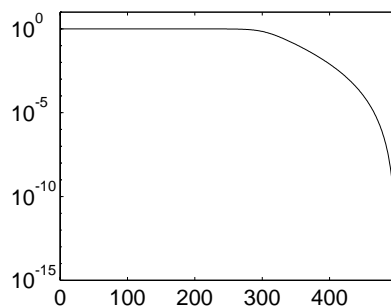
Analog Domain

`freqs` evaluates frequency response for an analog filter defined by two input coefficient vectors `b` and `a`. Its operation is similar to that of `freqz`; you can specify a number of frequency points to use (by default, the function uses 200), supply a vector of arbitrary frequency points, and plot the magnitude and phase response of the filter.

Magnitude and Phase

MATLAB provides functions to extract magnitude and phase from a frequency response vector `h`. The function `abs` returns the magnitude of the response; `angle` returns the phase angle in radians. To extract and plot the magnitude and phase of a Butterworth filter:

```
[b, a] = butter(6, 300/500); [h, w] = freqz(b, a, 512, 1000);
m = abs(h); p = angle(h);
semilogy(w, m);
plot(w, p*180/pi)
```

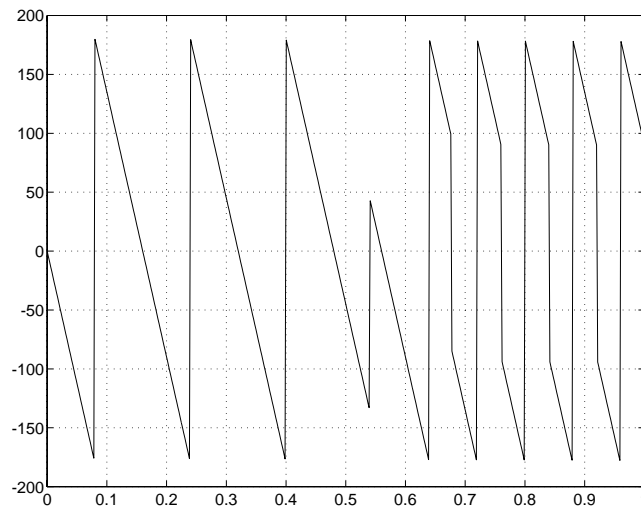


The `unwrap` function is also useful in frequency analysis. `unwrap` unwraps the phase to make it continuous across 360° phase discontinuities by adding multiples of $\pm 360^\circ$, as needed. To see how `unwrap` is useful, design a 25th-order lowpass FIR filter:

```
h = fir1(25, 0.4);
```

Obtain the filter's frequency response with `freqz`, and plot the phase in degrees:

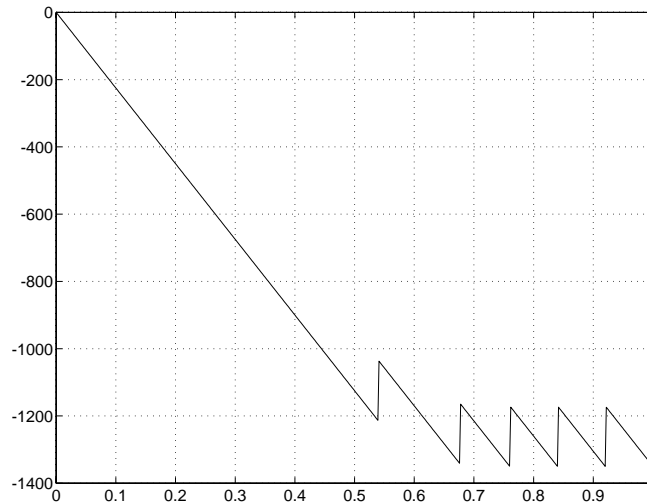
```
[H, f] = freqz(h, 1, 512, 2);  
plot(f, angle(H) * 180/pi); grid
```



It is difficult to distinguish the 360° jumps (an artifact of the arctangent function inside `angle`) from the 180° jumps that signify zeros in the frequency response.

Use `unwrap` to eliminate the 360° jumps:

```
plot(f, unwrap(angle(H)) * 180/pi); grid
```



Delay

The *group delay* of a filter is a measure of the average delay of the filter as a function of frequency. It is defined as the negative first derivative of a filter's phase response. If the complex frequency response of a filter is $H(e^{j\omega})$, then the group delay is

$$\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$$

where θ is the phase angle of $H(e^{j\omega})$. Compute group delay with

```
[gd, w] = grpdelay(b, a, n)
```

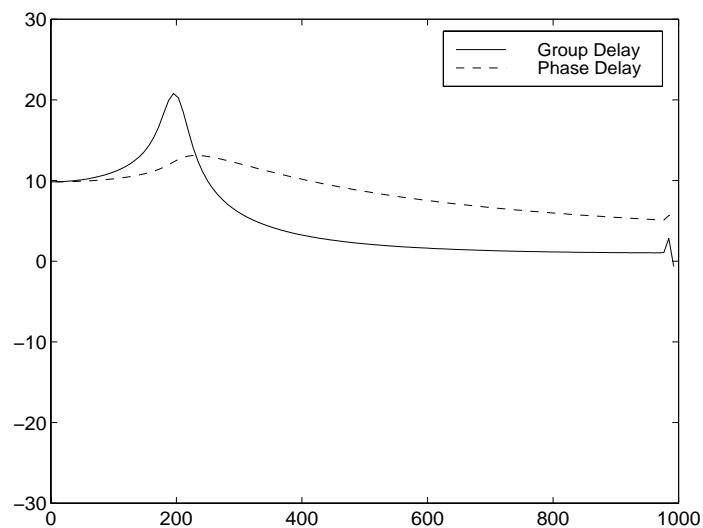
which returns the n -point group delay, $\tau_g(\omega)$, of the digital filter specified by b and a , evaluated at the frequencies in vector w .

The *phase delay* of a filter is the negative of phase divided by frequency

$$\tau_p(\omega) = -\frac{\theta(\omega)}{\omega}$$

To plot both the group and phase delays of a system on the same graph:

```
[b, a] = butter(10, 200/1000);
gd = grpdelay(b, a, 128);
[h, f] = freqz(b, a, 128, 2000);
pd = -unwrap(angle(h)) * (2000 / (2 * pi)) / f;
plot(f, gd, '-', f, pd, '- -')
axis([0 1000 -30 30])
legend('Group Delay', 'Phase Delay')
```



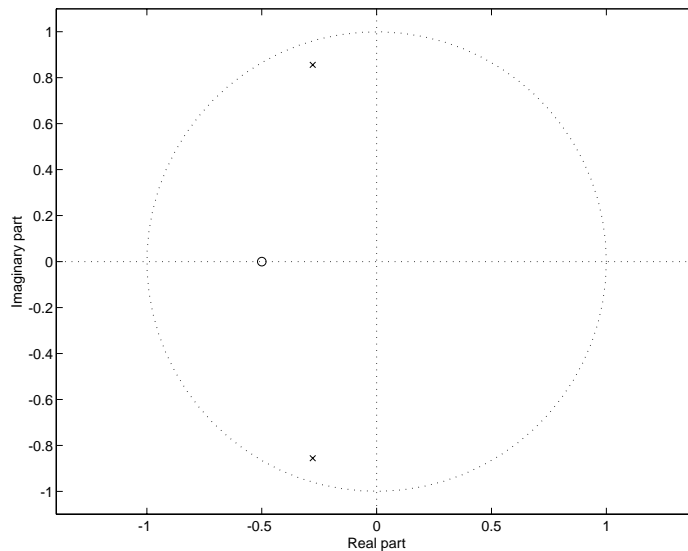
Zero-Pole Analysis

The `zplane` function plots poles and zeros of a linear system. For example, a simple filter with a 0 at $-1/2$ and a complex pole pair at $0.9e^{j2\pi(0.3)}$ and $0.9e^{-j2\pi(0.3)}$ is

```
zer = -0.5;
pol = .9*exp(j*2*pi*[-0.3 .3]')
```

The zero-pole plot for the filter is:

```
zplane(zer, pol)
```



For a system in zero-pole form, supply column vector arguments `z` and `p` to `zplane`:

```
zplane(z, p)
```

For a system in transfer function form, supply row vectors `b` and `a` as arguments to `zplane`:

```
zplane(b, a)
```

In this case `zplane` finds the roots of `b` and `a` using the `roots` function and plots the resulting zeros and poles.

See “Linear System Models” on page 1-32 for details on zero-pole and transfer function representation of systems.

Linear System Models

The Signal Processing Toolbox provides several models for representing linear time-invariant systems. This flexibility lets you choose the representational scheme that best suits your application and, within the bounds of numeric stability, convert freely to and from most other models. This section provides a brief overview of supported linear system models and describes how to work with these models in MATLAB.

Discrete-Time System Models

The discrete-time system models are representational schemes for digital filters. MATLAB supports several discrete-time system models:

- Transfer function
- Zero-pole-gain form
- State-space form
- Partial fraction expansion
- Second-order section form
- Lattice structure form
- Convolution matrices

Transfer Function

The *transfer function* is a basic z -domain representation of a digital filter, expressing the filter as a ratio of two polynomials. It is the principal discrete-time model for this toolbox. The transfer function model description for the z -transform of a digital filter's difference equation is

$$Y(z) = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}} X(z)$$

Here, the constants $b(i)$ and $a(i)$ are the filter coefficients, and the order of the filter is the maximum of na and nb . In MATLAB, you store these coefficients in two vectors (row vectors by convention), one row vector for the numerator and one for the denominator. See “Filters and Transfer Functions” on page 1-15 for more details on the transfer function form.

Zero-Pole-Gain

The factored or *zero-pole-gain* form of a transfer function is

$$H(z) = \frac{q(z)}{p(z)} = k \frac{(z - q(1))(z - q(2)) \dots (z - q(n))}{(z - p(1))(z - p(2)) \dots (z - p(n))}$$

By convention, MATLAB stores polynomial coefficients in row vectors and polynomial roots in column vectors. In zero-pole-gain form, therefore, the zero and pole locations for the numerator and denominator of a transfer function reside in column vectors. The factored transfer function gain k is a MATLAB scalar.

The `poly` and `roots` functions convert between polynomial and zero-pole-gain representations. For example, a simple IIR filter is

```
b = [2 3 4];
a = [1 3 3 1];
```

The zeros and poles of this filter are

```
q = roots(b)

q =

    -0.7500 + 1.1990i
    -0.7500 - 1.1990i

p = roots(a)

p =

    -1.0000
    -1.0000 + 0.0000i
    -1.0000 - 0.0000i

k = b(1)/a(1)

k =

    2
```

Returning to the original polynomials,

$$bb = k * \text{poly}(q)$$

$$bb =$$

$$2.0000 \quad 3.0000 \quad 4.0000$$

$$aa = \text{poly}(p)$$

$$aa =$$

$$1.0000 \quad 3.0000 \quad 3.0000 \quad 1.0000$$

Note that b and a in this case represent the transfer function

$$H(z) = \frac{2 + 3z^{-1} + 4z^{-2}}{1 + 3z^{-1} + 3z^{-2} + z^{-3}} = \frac{2z^3 + 3z^2 + 4z}{z^3 + 3z^2 + 3z + 1}$$

For $b = [2 \ 3 \ 4]$, the `roots` function misses the zero for z equal to 0. In fact, it misses poles and zeros for z equal to 0 whenever the input transfer function has more poles than zeros, or vice versa. This is acceptable in most cases. To circumvent the problem, however, simply append zeros to make the vectors the same length before using the `roots` function, for example, $b = [b \ 0]$.

State-Space

It is always possible to represent a digital filter, or a system of difference equations, as a set of first-order difference equations. In matrix or *state-space* form, you can write the equations as

$$x(n+1) = Ax(n) + Bu(n)$$

$$y(n) = Cx(n) + Du(n)$$

where u is the input, x is the state vector, and y is the output. For single-channel systems, A is an m -by- m matrix where m is the order of the filter, B is a column vector, C is a row vector, and D is a scalar. State-space notation is especially convenient for multichannel systems where input u and output y become vectors, and B , C , and D become matrices.

State-space representation extends easily to the MATLAB environment. In MATLAB, A , B , C , and D are rectangular arrays; MATLAB treats them as individual variables.

Taking the z -transform of the state-space equations and combining them shows the equivalence of state-space and transfer function form:

$$Y(z) = H(z)U(z), \quad \text{where } H(z) = C(zI - A)^{-1}B + D$$

Don't be concerned if you are not familiar with the state-space representation of linear systems. Some of the filter design algorithms use state-space form internally, but do not require any knowledge of state-space concepts to use them successfully. If your applications use state-space based signal processing extensively, however, consult the Control System Toolbox for a comprehensive library of state-space tools.

Partial Fraction Expansion (Residue Form)

Each transfer function also has a corresponding *partial fraction expansion* or *residue* form representation, given by

$$\frac{b(z)}{a(z)} = \frac{r(1)}{1 - p(1)z^{-1}} + \cdots + \frac{r(n)}{1 - p(n)z^{-1}} + k(1) + k(2)z^{-1} + \cdots + k(m - n + 1)z^{-(m-n)}$$

provided $H(z)$ has no repeated poles. Here, n is the degree of the denominator polynomial of the rational transfer function $b(z)/a(z)$. If r is a pole of multiplicity s_r , then $H(z)$ has terms of the form

$$\frac{r(j)}{1 - p(j)z^{-1}} + \frac{r(j+1)}{(1 - p(j)z^{-1})^2} + \cdots + \frac{r(j + s_r - 1)}{(1 - p(j)z^{-1})^{s_r}}$$

The `residuez` function in the Signal Processing Toolbox converts transfer functions to and from the partial fraction expansion form. The “ z ” on the end of `residuez` stands for z -domain, or discrete domain. `residuez` returns the poles in a column vector `p`, the residues corresponding to the poles in a column vector `r`, and any improper part of the original transfer function in a row vector `k`. `residuez` determines that two poles are the same if the magnitude of their difference is smaller than 0.1 percent of either of the poles' magnitudes.

Partial fraction expansion arises in signal processing as one method of finding the inverse z -transform of a transfer function. For example, the partial fraction expansion of

$$H(z) = \frac{-4 + 8z^{-1}}{1 + 6z^{-1} + 8z^{-2}}$$

is

```
b = [-4 8];
a = [1 6 8];
[r, p, k] = residuez(b, a)
```

```
r =
```

```
    -12
     8
```

```
p =
```

```
    -4
    -2
```

```
k =
```

```
    []
```

corresponding to

$$H(z) = \frac{-12}{1+4z^{-1}} + \frac{8}{1+2z^{-1}}$$

To find the inverse z -transform of $H(z)$, find the sum of the inverse z -transforms of the two addends of $H(z)$, giving the causal impulse response:

$$h(n) = -12(-4)^n + 8(-2)^n, \quad n = 0, 1, 2, \dots$$

To verify this in MATLAB,

```
imp = [1 0 0 0 0];
resptf = filter(b, a, imp)
```

```
resptf =
```

```
    -4     32    -160     704    -2944
```

```
respres = filter(r(1), [1 -p(1)], imp) + filter(r(2), [1 -p(2)], imp)
```

```
respres =
```

```
    -4     32    -160     704    -2944
```


Second-Order Sections (SOS)

Any transfer function $H(z)$ has a second-order sections representation,

$$H(z) = \prod_{k=1}^L H_k(z) = \prod_{k=1}^L \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}}$$

where L is the number of second-order sections that describe the system. MATLAB represents the second-order section form of a discrete-time system as an L -by-6 array `sos`. Each row of `sos` contains a single second-order section, where the row elements are the three numerator and three denominator coefficients that describe the second-order section:

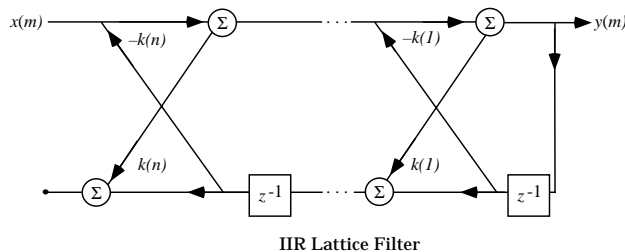
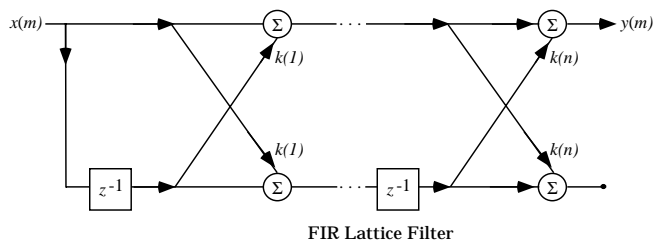
$$\text{sos} = \begin{bmatrix} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{bmatrix}$$

There are an uncountable number of ways to represent a filter in second order sections form. Through careful pairing of the pole and zero pairs, ordering of the sections in the cascade, and multiplicative scaling of the sections, it is possible to reduce quantization noise gain and avoid overflow in some fixed-point filter implementations. The functions `zp2sos` and `ss2sos`, described later in “Linear System Transformations,” perform pole-zero pairing, section scaling, and section ordering.

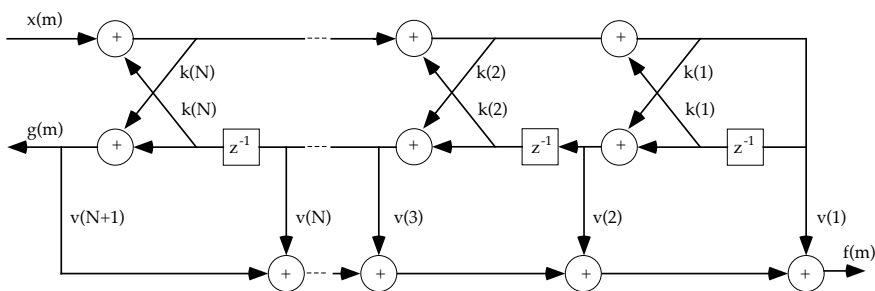
Lattice Structure

For a discrete N th order all-pole or all-zero filter described by the polynomial coefficients $a(n)$, $n = 1, 2, \dots, N+1$, there are N corresponding lattice structure coefficients $k(n)$, $n = 1, 2, \dots, N$. The parameters $k(n)$ are also called the *reflection*

coefficients of the filter. Given these reflection coefficients, you can implement a discrete filter as



For a general pole-zero IIR filter described by polynomial coefficients a and b , there are both lattice coefficients $k(n)$ for the denominator a and ladder coefficients $v(n)$ for the numerator b . The lattice/ladder filter may be implemented as



The toolbox function `tf2latc` accepts an FIR or IIR filter in polynomial form and returns the corresponding reflection coefficients. An example IIR filter in polynomial form is

```
a = [ 1. 0000    0. 6149    0. 9899    0. 0000    0. 0031   -0. 0082];
```

This filter's lattice (reflection coefficient) representation is

```
k = tf2latc(a)
```

```
k =  
    0. 3090  
    0. 9800  
    0. 0031  
    0. 0081  
   -0. 0082
```

The magnitude of the reflection coefficients provides an easy stability check for a filter. If all the reflection coefficients corresponding to a polynomial have magnitude less than 1, all of that polynomial's roots are inside the unit circle.

The function `latc2tf` calculates the polynomial coefficients for a filter from its lattice (reflection) coefficients. Given the reflection coefficient vector `k` (above), the corresponding polynomial form is

```
a = latc2tf(k)  
  
a =  
    1. 0000    0. 6149    0. 9899   -0. 0000    0. 0031   -0. 0082
```

The lattice or lattice/ladder coefficients can be used to implement the filter using the function `latcfilt`.

Convolution Matrix

In signal processing, convolving two vectors or matrices is equivalent to filtering one of the input operands by the other. This relationship permits the representation of a digital filter as a *convolution matrix*.

Given any vector, the toolbox `convmtx` function generates a matrix whose inner product with another vector is equivalent to the convolution of the two vectors. The generated matrix represents a digital filter that you can apply to any

vector of appropriate length; the inner dimension of the operands must agree to compute the inner product.

The convolution matrix for a vector b , representing the numerator coefficients for a digital filter, is

```
b = [ 1 2 3]; x = randn(3, 1);
C = convmtx(b', 3)
```

```
C =

     1     0     0
     2     1     0
     3     2     1
     0     3     2
     0     0     3
```

Two ways to convolve b with x are

```
y1 = C*x;
y2 = conv(b, x);
```

Type this example into MATLAB; the results for $y1$ and $y2$ are equal.

Continuous-Time System Models

The continuous-time system models are representational schemes for analog filters. Many of the discrete-time system models described earlier are also appropriate for the representation of continuous-time systems:

- State-space form
- Partial fraction expansion
- Transfer function
- Zero-pole-gain form

It is possible to represent any system of linear time-invariant differential equations as a set of first-order differential equations. In matrix or *state-space* form, you can express the equations as

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

where u is a vector of nu inputs, x is an nx -element state vector, and y is a vector of ny outputs. In MATLAB, store A, B, C, and D in separate rectangular arrays.

An equivalent representation of the state-space system is the Laplace transform transfer function description

$$Y(s) = H(s)U(s)$$

where

$$H(s) = C(sI - A)^{-1}B + D$$

For single-input, single-output systems, this form is given by

$$H(s) = \frac{b(s)}{a(s)} = \frac{b(1)s^{nb} + b(2)s^{nb-1} + \dots + b(nb+1)}{a(1)s^{na} + a(2)s^{na-1} + \dots + a(na+1)}$$

Given the coefficients of a Laplace transform transfer function, *residue* determines the partial fraction expansion of the system. See the description of *residue* in the *MATLAB Language Reference Manual* for details.

The factored zero-pole-gain form is

$$H(s) = \frac{z(s)}{p(s)} = k \frac{(s - z(1))(s - z(2)) \dots (s - z(n))}{(s - p(1))(s - p(2)) \dots (s - p(n))}$$

As in the discrete-time case, MATLAB stores polynomial coefficients in row vectors in descending powers of s . MATLAB stores polynomial roots, or zeros and poles, in column vectors.

Linear System Transformations

The Signal Processing Toolbox provides a number of functions that convert between the various linear system models; see the reference description in Chapter 6 for a complete description of each. You can use the following chart to find an appropriate transfer function: find the row of the model to convert *from* on the left side of the chart and the column of the model to convert *to* on the top of the chart and read the function name(s) at the intersection of the row and column:

	Transfer function	State space	Zero-pole gain	Partial fraction	Lattice filter	Second-order sections	Convolution matrix
Transfer function		tf2ss	tf2zp roots	residuez residue	tf2l at c		convmtx
State space	ss2tf		ss2zp			ss2sos	
Zero-pole gain	zp2tf poly	zp2ss				zp2sos	
Partial fraction	residuez residue						
Lattice filter	latc2tf						
Second-order sections	sos2tf	sos2ss	sos2zp				
Convolution matrix							

Many of the toolbox filter design functions use these functions internally. For example, the `zp2ss` function converts the poles and zeros of an analog prototype into the state-space form required for creation of a Butterworth, Chebyshev, or elliptic filter. Once in state-space form, the filter design function performs any required frequency transformation, that is, it transforms the initial lowpass design into a bandpass, highpass, or bandstop filter, or a lowpass filter with the desired cutoff frequency. See Chapter 6 and the reference descriptions of the individual filter design functions for more details.

Discrete Fourier Transform

The discrete Fourier transform, or DFT, is the primary tool of digital signal processing. The foundation of the Signal Processing Toolbox is the Fast Fourier Transform (FFT), a method for computing the DFT with reduced execution time. Many of the toolbox functions (including *z*-domain frequency response, spectrum and cepstrum analysis, and some filter design and implementation functions) incorporate the FFT.

MATLAB provides the functions `fft` and `ifft` to compute the discrete Fourier transform and its inverse, respectively. For the input sequence x and its transformed version X (the discrete-time Fourier transform at equally spaced frequencies around the unit circle), the two functions implement the relationships

$$X(k+1) = \sum_{n=0}^{N-1} x(n+1)W_N^{kn}$$

$$x(n+1) = \frac{1}{N} \sum_{k=0}^{N-1} X(k+1)W_N^{-kn}$$

In these equations, the series subscripts begin with 1 instead of 0 because of MATLAB's vector indexing scheme, and

$$W_N = e^{-j\left(\frac{2\pi}{N}\right)}$$

NOTE MATLAB uses a negative j for the `fft` function. This is an engineering convention; physics and pure mathematics typically use a positive j .

`fft`, with a single input argument x , computes the DFT of the input vector or matrix. If x is a vector, `fft` computes the DFT of the vector; if x is a rectangular array, `fft` computes the DFT of each array column.

For example, create a time vector and signal:

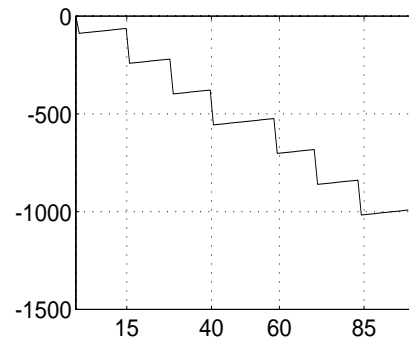
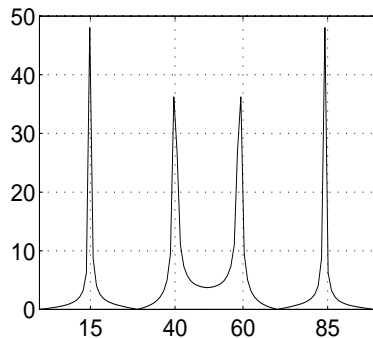
```
t = (0:1/99:1); % time vector
x = sin(2*pi*15*t) + sin(2*pi*40*t); % signal
```

The DFT of the signal, and the magnitude and phase of the transformed sequence, are then

```
y = fft(x); % Compute DFT of x.
m = abs(y); p = unwrap(angle(y)); % mag. and phase
```

To plot the magnitude and phase,

```
f = (0:length(y)-1)*99/length(y); % frequency vector
plot(f, m)
set(gca, 'XTick', [15 40 60 85]);
plot(f, p*180/pi)
set(gca, 'XTick', [15 40 60 85]);
```



A second argument to `fft` specifies a number of points `n` for the transform, representing DFT length:

```
y = fft(x, n);
```

In this case, `fft` pads the input sequence with zeros if it is shorter than `n`, or truncates the sequence if it is longer than `n`. If `n` is not specified, it defaults to the length of the input sequence.

Execution time for `fft` depends on the length `n` of the DFT it performs:

- For any `n` that is a power of two, `fft` uses the high-speed radix-2 algorithm. This results in the fastest execution time. Additionally, the algorithm for power of two `n` is highly optimized for real `x`, providing a 40% speed-up over the complex case.
- For any composite number `n` that is not a power of two, `fft` uses a prime factor algorithm. The speed of this algorithm depends on both the size of `n` and number of prime factors it has. Although 1013 and 1000 are close in magnitude, `fft` transforms a sequence of length 1000 much more quickly than a sequence of length 1013.
- For a prime number `n`, `fft` cannot use an FFT algorithm, and instead performs the slower, computation-intensive DFT directly.

The inverse discrete Fourier transform function `ifft` also accepts an input sequence and, optionally, the number of desired points for the transform. Try the example below; the original sequence `x` and the reconstructed sequence are identical (within rounding error):

```
t = (0: 1/255: 1);
x = sin(2*pi * 120*t);
y = real (ifft(fft(x)));
```

This toolbox also includes functions for the two-dimensional FFT and its inverse, `fft2` and `ifft2`. These functions are useful for two-dimensional signal or image processing; see the reference descriptions in Chapter 6 for details.

It is sometimes convenient to rearrange the output of the `fft` or `fft2` function so the zero frequency component is at the center of the sequence. The MATLAB function `fftshift` moves the zero frequency component to the center of a vector or matrix.

References

Algorithm development for the Signal Processing Toolbox has drawn heavily upon the references listed below. All are recommended to the interested reader who needs to know more about signal processing than is covered in this manual.

- 1 Crochiere, R.E., and L.R. Rabiner. *Multi-Rate Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1983. Pgs. 88-91.
- 2 IEEE. *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979.
- 3 Jackson, L.B. *Digital Filters and Signal Processing*. Third Ed. Boston: Kluwer Academic Publishers, 1989.
- 4 Kay, S.M. *Modern Spectral Estimation*. Englewood Cliffs, NJ: Prentice Hall, 1988.
- 5 Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989.
- 6 Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987.
- 7 Pratt, W.K. *Digital Image Processing*. New York: John Wiley & Sons, 1991.
- 8 Percival, D.B., and A.T. Walden. *Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques*. Cambridge: Cambridge University Press, 1993.
- 9 Proakis, J.G., and D.G. Manolakis. *Digital Signal Processing: Principles, Algorithms, and Applications*. Upper Saddle River, NJ: Prentice Hall, 1996.
- 10 Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1975.
- 11 Welch, P.D. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms." *IEEE Trans. Audio Electroacoust.* Vol. AU-15 (June 1967). Pgs. 70-73.

Filter Design

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Filter Requirements and Specification

The Signal Processing Toolbox provides functions that support a range of filter design methodologies. This chapter explains how to apply the filter design tools to *Infinite Impulse Response* (IIR) and *Finite Impulse Response* (FIR) filter design problems.

The goal of filter design is to perform frequency dependent alteration of a data sequence. A possible requirement might be to remove noise above 30 Hz from a data sequence sampled at 100 Hz. A more rigorous specification might call for a specific amount of passband ripple, stopband attenuation, or transition width. A very precise specification could ask to achieve the performance goals with the minimum filter order, or it could call for an arbitrary magnitude shape, or it might require an FIR filter.

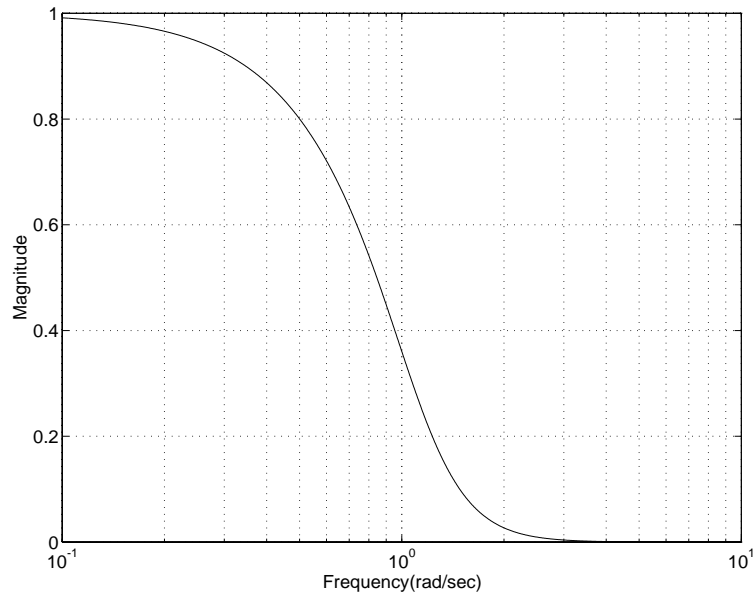
Filter design methods differ primarily in how performance is specified. For “loosely specified” requirements, as in the first case above, a Butterworth IIR filter is often sufficient. To design a fifth-order 30 Hz lowpass Butterworth filter and apply it to the data in vector x ,

```
[b, a] = butter(5, 30/50);  
y = filter(b, a, x);
```

The second input argument to `butter` indicates the cutoff frequency, normalized to half the sampling frequency (the Nyquist frequency).

Frequency Normalization in the Signal Processing Toolbox All of the filter design functions operate with normalized frequencies, so they do not require the system sampling rate as an extra input argument. This toolbox uses the convention that unit frequency is the Nyquist frequency, defined as half the sampling frequency. The normalized frequency, therefore, is always in the interval $0 \leq f \leq 1$. For a system with a 1000 Hz sampling frequency, 300 Hz is $300/500 = 0.6$. To convert normalized frequency to angular frequency around the unit circle, multiply by π . To convert normalized frequency back to Hertz, multiply by half the sample frequency.

More rigorous filter requirements traditionally include passband ripple (R_p , in decibels), stopband attenuation (R_s , in decibels), and transition width ($W_s - W_p$, in Hertz).



You can design Butterworth, Chebyshev type I, Chebyshev type II, and elliptic filters that meet this type of performance specification. The toolbox order selection functions estimate the minimum filter order that meets a given set of requirements.

To meet specifications with more rigid constraints like linear phase or arbitrary filter shape, use the FIR and direct IIR filter design routines.

IIR Filter Design

The primary advantage of IIR filters over FIR filters is that they typically meet a given set of specifications with a much lower filter order than a corresponding FIR filter. Although IIR filters have nonlinear phase, data processing within MATLAB is commonly performed “off-line,” that is, the entire data sequence is available prior to filtering. This allows for a noncausal, zero-phase filtering approach (via the `filtfilt` function), which eliminates the nonlinear phase distortion of an IIR filter.

The classical IIR filters, Butterworth, Chebyshev types I and II, elliptic, and Bessel, all approximate the ideal “brickwall” filter in different ways. This toolbox provides functions to create all these types of classical IIR filters in both the analog and digital domains (except Bessel, for which only the analog case is supported), and in lowpass, highpass, bandpass, and bandstop configurations. For most filter types, you can also find the lowest filter order that fits a given filter specification in terms of passband and stopband attenuation, and transition width(s).

The direct filter design function `yulewalk` finds a filter with magnitude response approximating a desired function. This is one way to create a multiband bandpass filter.

You can also use the parametric modeling or system identification functions to design IIR filters. These functions are discussed in the “Parametric Modeling” section of Chapter 4.

The generalized Butterworth design function `maxflat` is discussed in the section “Generalized Butterworth Filter Design” on page 2-14.

The following table summarizes the various filter methods in the toolbox and lists the functions available to implement these methods:

Method	Description	Functions
Analog Prototyping	Using the poles and zeros of a classical lowpass prototype filter in the continuous (Laplace) domain, obtain a digital filter through frequency transformation and filter discretization.	<p>Complete design functions:</p> <p><code>bessel f</code>, <code>butter</code>, <code>cheby1</code>, <code>cheby2</code>, <code>ellip</code></p> <p>Order estimation functions:</p> <p><code>buttord</code>, <code>cheb1ord</code>, <code>cheb2ord</code>, <code>ellipord</code></p> <p>Lowpass analog prototype functions:</p> <p><code>bessel ap</code>, <code>buttap</code>, <code>cheb1ap</code>, <code>cheb2ap</code>, <code>ellipap</code></p> <p>Frequency transformation functions:</p> <p><code>lp2bp</code>, <code>lp2bs</code>, <code>lp2hp</code>, <code>lp2lp</code></p> <p>Filter discretization functions:</p> <p><code>bilinear</code>, <code>impinvar</code></p>
Direct Design	Design digital filter directly in the discrete domain by approximating a piecewise linear magnitude response.	<code>yulewalk</code>
Parametric Modeling*	Find a digital filter that approximates a prescribed time or frequency domain response.	<p>Time-domain modeling functions:</p> <p><code>lpc</code>, <code>prony</code>, <code>stmcb</code></p> <p>Frequency-domain modeling functions:</p> <p><code>invfreqs</code>, <code>invfreqz</code></p>
Generalized Butterworth Design	Design lowpass Butterworth filters with more zeros than poles.	<code>maxflat</code>

* See the System Identification Toolbox for an extensive collection of parametric modeling tools.

Classical IIR Filter Design Using Analog Prototyping

The principal IIR digital filter design technique this toolbox provides is based on the conversion of classical lowpass analog filters to their digital equivalents. The following sections describe how to design filters and summarize the characteristics of the supported filter types. See “Special Topics in IIR Filter Design” on page 2-37 for detailed steps on the filter design process.

Complete Classical IIR Filter Design

You can easily create a filter of any order with a lowpass, highpass, bandpass, or bandstop configuration using the filter design functions:

Filter Type	Design Function
Butterworth	<code>[b, a] = butter(n, Wn, options)</code> <code>[z, p, k] = butter(n, Wn, options)</code> <code>[A, B, C, D] = butter(n, Wn, options)</code>
Chebyshev type I	<code>[b, a] = cheby1(n, Rp, Wn, options)</code> <code>[z, p, k] = cheby1(n, Rp, Wn, options)</code> <code>[A, B, C, D] = cheby1(n, Rp, Wn, options)</code>
Chebyshev type II	<code>[b, a] = cheby2(n, Rs, Wn, options)</code> <code>[z, p, k] = cheby2(n, Rs, Wn, options)</code> <code>[A, B, C, D] = cheby2(n, Rs, Wn, options)</code>
Elliptic	<code>[b, a] = ellip(n, Rp, Rs, Wn, options)</code> <code>[z, p, k] = ellip(n, Rp, Rs, Wn, options)</code> <code>[A, B, C, D] = ellip(n, Rp, Rs, Wn, options)</code>
Bessel (analog only)	<code>[b, a] = besself(n, Wn, options)</code> <code>[z, p, k] = besself(n, Wn, options)</code> <code>[A, B, C, D] = besself(n, Wn, options)</code>

By default, each of these functions returns a lowpass filter; you need only specify the desired cutoff frequency W_n in normalized frequency (Nyquist frequency = 1 Hz). For a highpass filter, append the string 'high' to the function's parameter list. For a bandpass or bandstop filter, specify W_n as a two-element vector containing the passband edge frequencies, appending the string 'stop' for the bandstop configuration.

Here are some example digital filters:

```
[b, a] = butter(5, .4);           % lowpass Butterworth
[b, a] = cheby1(4, 1, [.4 .7]);   % bandpass Chebyshev type I
[b, a] = cheby2(6, 60, .8, 'high'); % highpass Chebyshev type II
[b, a] = ellip(3, 1, 60, [.4 .7], 'stop'); % bandstop elliptic
```

To design an analog filter, perhaps for simulation, use a trailing 's' and specify cutoff frequencies in radians/second:

```
[b, a] = butter(5, .4, 's'); % analog Butterworth filter
```

All filter design functions return a filter in the transfer function, zero-pole-gain, or state-space linear system model representation, depending on how many output arguments are present.

Designing IIR Filters to Frequency Domain Specifications

This toolbox provides order selection functions that calculate the minimum filter order that meets a given set of requirements:

Filter Type	Order Estimation Function
Butterworth	[n, Wn] = buttord(Wp, Ws, Rp, Rs)
Chebyshev type I	[n, Wn] = cheb1ord(Wp, Ws, Rp, Rs)
Chebyshev type II	[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs)
Elliptic	[n, Wn] = ellipord(Wp, Ws, Rp, Rs)

These are useful in conjunction with the filter design functions. Suppose you want a bandpass filter with a passband from 1000 to 2000 Hz, stopbands starting 500 Hz away on either side, a 10 kHz sampling frequency, at most 1 dB

of passband ripple, and at least 60 dB of stopband attenuation. To meet these specifications with the `butter` function:

```
[n, Wn] = buttord([1000 2000]/5000, [500 2500]/5000, 1, 60)

n =

    12

Wn =

    0.1951    0.4080

[b, a] = butter(n, Wn);
```

An elliptic filter that meets the same requirements is given by

```
[n, Wn] = ellipord([1000 2000]/5000, [500 2500]/5000, 1, 60)

n =

     5

Wn =

    0.2000    0.4000

[b, a] = ellip(n, 1, 60, Wn);
```

These functions also work with the other standard band configurations, as well as for analog filters; see Chapter 6 for details.

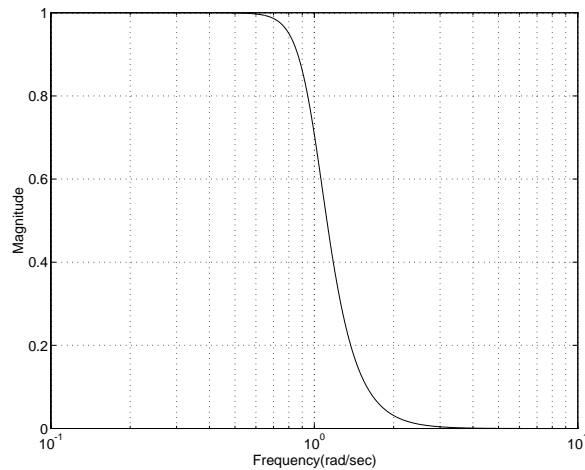
Comparison of Classical IIR Filter Types

The toolbox provides five different types of classical IIR filters, each optimal in some way. This section shows the basic analog prototype form for each and summarizes major characteristics.

Butterworth Filter

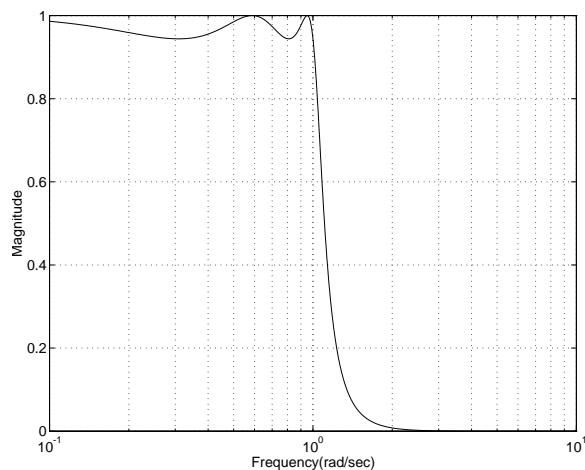
The Butterworth filter provides the best Taylor Series approximation to the ideal lowpass filter response at $\Omega = 0$ and $\Omega = \infty$; for any order N , the magnitude squared response has $2N-1$ zero derivatives at these locations (*maximally flat*

at $\Omega = 0$ and $\Omega = \infty$). Response is monotonic overall, decreasing smoothly from $\Omega = 0$ to $\Omega = \infty$. $|H(j\Omega)| = \sqrt{1/2}$ at $\Omega = 1$.



Chebyshev Type I Filter

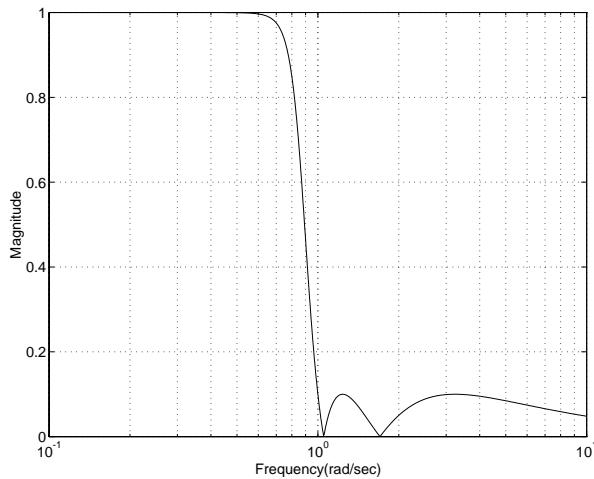
The Chebyshev type I filter minimizes the absolute difference between the ideal and actual frequency response over the entire passband by incorporating an equal ripple of R_p dB in the passband. Stopband response is maximally flat. The transition from passband to stopband is more rapid than for the Butterworth filter. $|H(j\Omega)| = 10^{-R_p/20}$ at $\Omega = 1$.



Chebyshev Type II Filter

The Chebyshev type II filter minimizes the absolute difference between the ideal and actual frequency response over the entire stopband, by incorporating an equal ripple of R_s dB in the stopband. Passband response is maximally flat.

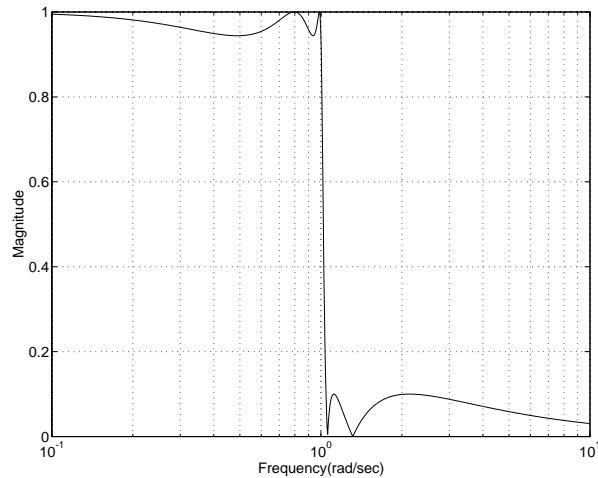
The stopband does not approach zero as quickly as the type I filter (and does not approach zero at all for even-valued n). The absence of ripple in the passband, however, is often an important advantage. $|H(j\Omega)| = 10^{-R_s/20}$ at $\Omega = 1$.



Elliptic Filter

Elliptic filters are equiripple in both the passband and stopband. They generally meet filter requirements with the lowest order of any supported filter type. Given a filter order n , passband ripple R_p in decibels, and stopband ripple

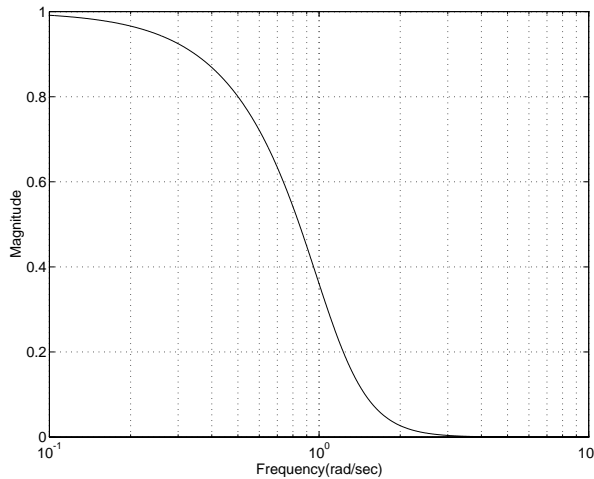
Rs in decibels, elliptic filters minimize transition width. $|H(j\Omega)| = 10^{-R_p/20}$ at $\Omega = 1$.



Bessel Filter

Analog Bessel lowpass filters have maximally flat group delay at zero frequency and retain nearly constant group delay across the entire passband. Filtered signals therefore maintain their waveshapes in the passband frequency range. Frequency mapped and digital Bessel filters, however, do not have this maximally flat property; this toolbox supports only the analog case for the complete Bessel filter design function.

Bessel filters generally require a higher filter order than other filters for satisfactory stopband attenuation. $|H(j\Omega)| < 1/\sqrt{2}$ at $\Omega = 1$ and decreases as n increases.



NOTE The lowpass filters shown above were created with the analog prototype functions `besselap`, `butter`, `cheb1ap`, `cheb2ap`, and `ellipap`. These functions find the zeros, poles, and gain of an order n analog filter of the appropriate type with cutoff frequency of 1 rad/sec. The complete filter design functions (`bessel`, `butter`, `cheby1`, `cheby2`, and `ellip`) call the prototyping functions as a first step in the design process. See “Special Topics in IIR Filter Design” on page 2-37 for details.

To create similar plots, use $n = 5$ and, as needed, $R_p = 0.5$ and $R_s = 20$. For example, to create the elliptic filter plot:

```
[z, p, k] = ellipap(5, .5, 20);
w = logspace(-1, 1, 1000);
h = freqs(k*poly(z), poly(p), w);
semilogx(w, abs(h)), grid
```

Direct IIR Filter Design

This toolbox uses the term *direct methods* to describe techniques for IIR design that find a filter based on specifications in the discrete domain. Unlike the analog prototyping method, direct design methods are not constrained to the standard lowpass, highpass, bandpass, or bandstop configurations. Rather, these functions design filters with an arbitrary, perhaps multiband, frequency response. This section discusses the `yulewalk` function, which is intended specifically for filter design; “Parametric Modeling” in Chapter 4 discusses other methods that may also be considered direct, such as Prony’s method, Linear Prediction, the Steiglitz-McBride method, and inverse frequency design.

`yulewalk` designs recursive IIR digital filters by fitting a specified frequency response. `yulewalk`’s name reflects its method for finding the filter’s denominator coefficients: it finds the inverse FFT of the ideal desired power spectrum and solves the “modified Yule-Walker equations” using the resulting autocorrelation function samples. The statement

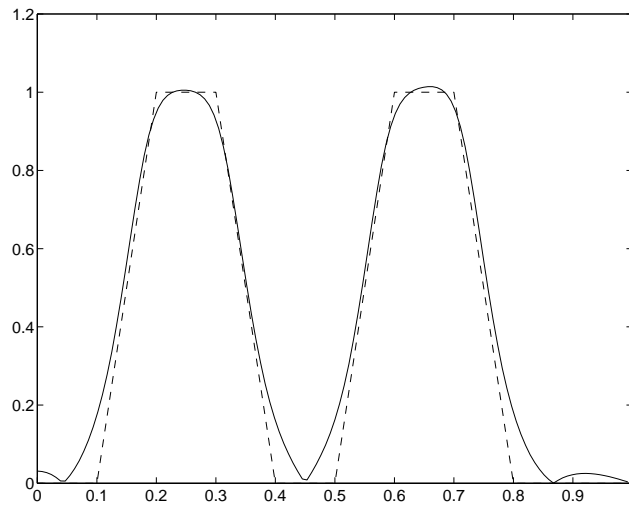
```
[b, a] = yulewalk(n, f, m)
```

returns row vectors `b` and `a` containing the $n+1$ numerator and denominator coefficients of the order n IIR filter whose frequency-magnitude characteristics approximate those given in vectors `f` and `m`. `f` is a vector of frequency points ranging from 0 to 1, where 1 represents the Nyquist frequency. `m` is a vector containing the desired magnitude response at the points in `f`. `f` and `m` can describe any piecewise linear shape magnitude response, including a multiband response. The FIR counterpart of this function is `fir2`, which also designs a filter based on an arbitrary piecewise linear magnitude response. See “FIR Filter Design” on page 2-16 for details.

Note that `yulewalk` does not accept phase information, and no statements are made about the optimality of the resulting filter.

Design a multiband filter with `yul ewal k`, and plot the desired and actual frequency response:

```
m = [0 0 1 1 0 0 1 1 0 0];
f = [0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 1];
[b, a] = yul ewal k(10, f, m);
[h, w] = freqz(b, a, 128);
pl ot(f, m, w/pi, abs(h))
```



Generalized Butterworth Filter Design

The toolbox function `maxflat` enables you to design generalized Butterworth filters, that is, Butterworth filters with differing numbers of zeros and poles. This is desirable in some implementations where poles are more expensive computationally than zeros. `maxflat` is just like the `butter` function, except that it you can specify *two* orders (one for the numerator and one for the denominator) instead of just one. These filters are *maximally flat*. This means that the resulting filter is optimal for any numerator and denominator orders, with the maximum number of derivatives at 0 and the Nyquist frequency $\omega=\pi$ both set to 0.

For example, when the two orders are the same, `maxflat` is the same as `butter`:

```
[b, a] = maxflat(3, 3, 0.25)

b =
    0.0317    0.0951    0.0951    0.0317

a =
    1.0000   -1.4590    0.9104   -0.1978

[b, a] = butter(3, 0.25)

b =
    0.0317    0.0951    0.0951    0.0317

a =
    1.0000   -1.4590    0.9104   -0.1978
```

However, `maxflat` is more versatile, because you can design a filter with more zeros than poles:

```
[b, a] = maxflat(3, 1, 0.25)

b =
    0.0950    0.2849    0.2849    0.0950

a =
    1.0000   -0.2402
```

The third input to `maxflat` is the *half-power frequency*, a frequency between 0 and 1 with a desired magnitude response of $1/\sqrt{2}$.

You can also design linear phase filters that have the maximally flat property using the 'sym' option:

```
maxflat(4, 'sym', 0.3)

ans =
    0.0331    0.2500    0.4337    0.2500    0.0331
```

For complete details of the `maxflat` algorithm, see Selesnick and Burrus [2].

FIR Filter Design

Digital filters with finite-duration impulse response (all-zero, or FIR filters) have both advantages and disadvantages compared to infinite-duration impulse response (IIR) filters.

FIR filters have the following primary advantages:

- They can have exactly linear phase.
- They are always stable.
- The design methods are generally linear.
- They can be realized efficiently in hardware.
- The filter startup transients have finite duration.

The primary disadvantage of FIR filters is that they often require a much higher filter order than IIR filters to achieve a given level of performance.

Correspondingly, the delay of these filters is often much greater than for an equal performance IIR filter.

Method	Description	Functions
Windowing	Apply window to truncated inverse Fourier transform of desired “brickwall” filter	<code>fir1</code> , <code>fir2</code> , <code>kaiserord</code>
Multiband with Transition Bands	Equiripple or least squares approach over sub-bands of the frequency range	<code>firls</code> , <code>remez</code> , <code>remezord</code>
Constrained Least Squares	Minimize squared integral error over entire frequency range subject to maximum error constraints	<code>fircls</code> , <code>fircls1</code>
Arbitrary Response	Arbitrary responses, including nonlinear phase and complex filters	<code>cremez</code>
Raised Cosine	Lowpass response with smooth, sinusoidal transition	<code>firrcos</code>

Linear Phase Filters

Except for `cremez`, all of the FIR filter design functions design linear phase filters only. The filter coefficients, or “taps,” of such filters obey either an even or odd symmetry relation. Depending on this symmetry, and on whether the order n of the filter is even or odd, a linear phase filter (stored in length $n+1$ vector b) has certain inherent restrictions on its frequency response:

Linear Phase Filter Type	Filter Order n	Symmetry of Coefficients	Response $H(f)$, $f = 0$	Response $H(f)$, $f = 1$ (Nyquist)
Type I	Even	Even:	No restriction	No restriction
Type II	Odd	$b(k) = b(n+2-k)$, $k = 1, \dots, n+1$	No restriction	$H(1) = 0$
Type III	Even	Odd:	$H(0) = 0$	$H(1) = 0$
Type IV	Odd	$b(k) = -b(n+2-k)$, $k = 1, \dots, n+1$	$H(0) = 0$	No restriction

The phase delay and group delay of linear phase FIR filters are equal and constant over the frequency band. For an order n linear phase FIR filter, the group delay is $n/2$, and the filtered signal is simply delayed by $n/2$ time steps (and the magnitude of its Fourier transform is scaled by the filter's magnitude response). This property preserves the wave shape of signals in the passband, that is, there is no phase distortion.

The functions `fir1`, `fir2`, `firls`, `remez`, `fircls`, `fircls1`, and `firrcos` all design type I and II linear phase FIR filters by default. Both `firls` and `remez` design type III and IV linear phase FIR filters given a 'hilbert' or 'differentiator' flag. `remez` can design any type of linear phase filter, and nonlinear phase filters as well.

NOTE Because the frequency response of a type II filter is zero at the Nyquist rate ("high" frequency), `fir1` does not design type II highpass and bandstop filters. For odd-valued n in these cases, `fir1` adds 1 to the order and returns a type I filter.

Windowing Method

Consider the ideal, or "brick-wall," digital lowpass filter with a cutoff frequency of ω_0 rad/sec. This filter has magnitude 1 at all frequencies with magnitude less

than ω_0 , and magnitude 0 at frequencies with magnitude between ω_0 and π . Its impulse response sequence $h(n)$ is

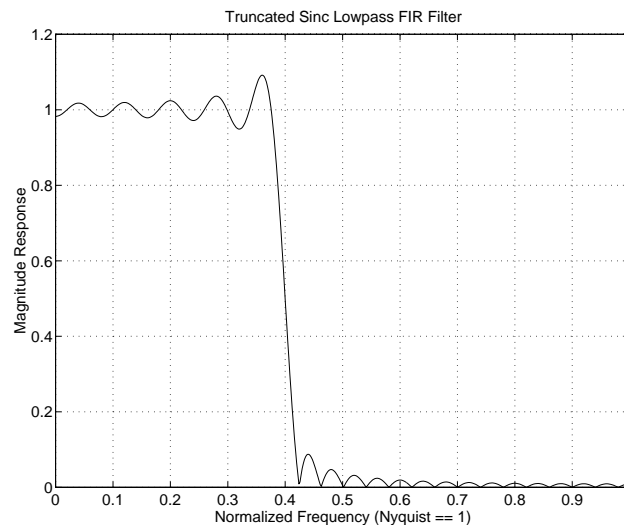
$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} e^{j\omega n} d\omega = \frac{\omega_0}{\pi} \text{sinc}\left(\frac{\omega_0}{\pi} n\right)$$

This filter is not implementable since its impulse response is infinite and noncausal. To create a finite-duration impulse response, truncate it by applying a window. By retaining the central section of impulse response in this truncation, you obtain a linear phase FIR filter. For example, a length 51 filter with a lowpass cutoff frequency ω_0 of 0.4π rad/sec is

$$b = 0.4 * \text{sinc}(0.4 * (-25:25));$$

The window applied here is a simple rectangular or “boxcar” window. By Parseval’s theorem, this is the length 51 filter that best approximates the ideal lowpass filter, in the integrated least squares sense. To view its frequency response,

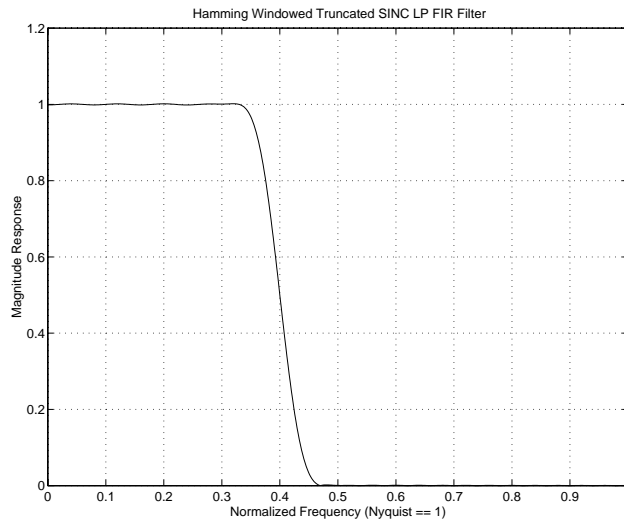
```
[H, w] = freqz(b, 1, 512, 2);  
plot(w, abs(H)), grid
```



Note the ringing and ripples in the response, especially near the band edge. This “Gibbs effect” does not vanish as the filter length increases, but a

nonrectangular window reduces its magnitude. Multiplication by a window in the time domain causes a convolution or smoothing in the frequency domain. Apply a length 51 Hamming window to the filter:

```
b = b.*hamming(51)';
[H,w] = freqz(b,1,512,2);
plot(w,abs(H)), grid
```



As you can see, this greatly reduces the ringing. This improvement is at the expense of transition width (the windowed version takes longer to ramp from passband to stopband) and optimality (the windowed version does not minimize the integrated squared error).

The functions `fir1` and `fir2` are based on this windowing process. Given a filter order and description of an ideal desired filter, these functions return a windowed inverse Fourier transform of that ideal filter. Both use a Hamming window by default, but they accept any window function. See the “Windows” section of Chapter 4 for an overview of windows and their properties.

Standard Band FIR Filter Design: `fir1`

`fir1` implements the classical method of windowed linear phase FIR digital filter design. It resembles the IIR filter design functions in that it is formulated to design filters in standard band configurations: lowpass, bandpass, highpass, and bandstop.

The statements

```

n = 50;
Wn = 0.4;
b = fir1(n, Wn);

```

create row vector **b** containing the coefficients of the order **n** Hamming-windowed filter. This is a lowpass, linear phase FIR filter with cutoff frequency **Wn**. **Wn** is a number between 0 and 1, where 1 corresponds to the Nyquist frequency, half the sampling frequency. (Unlike other methods, here **Wn** corresponds to the 6 dB point.) For a highpass filter, simply append the string ' **high** ' to the function's parameter list. For a bandpass or bandstop filter, specify **Wn** as a two-element vector containing the passband edge frequencies; append the string ' **stop** ' for the bandstop configuration.

b = fir1(n, Wn, window) uses the window specified in column vector **window** for the design. The vector **window** must be **n+1** elements long. If you do not specify a window, **fir1** applies a Hamming window.

Kaiser Window Order Estimation. The **kaiserord** function estimates the filter order, cutoff frequency, and Kaiser window beta parameter needed to meet a given set of specifications. Given a vector of frequency band edges and a corresponding vector of magnitudes, as well as maximum allowable ripple, **kaiserord** returns appropriate input parameters for the **fir1** function. For details on **kaiserord**, see the reference description in Chapter 6.

Multiband FIR Filter Design: fir2

The function **fir2** also designs windowed FIR filters, but with an arbitrarily shaped piecewise linear frequency response. This is in contrast to **fir1**, which only designs filters in standard lowpass, highpass, bandpass, and bandstop configurations.

The commands

```

n = 50;
f = [0 .4 .5 1];
m = [1 1 0 0];
b = fir2(n, f, m);

```

return row vector **b** containing the **n+1** coefficients of the order **n** FIR filter whose frequency-magnitude characteristics match those given by vectors **f** and **m**. **f** is a vector of frequency points ranging from 0 to 1, where 1 represents the

Nyquist frequency. `m` is a vector containing the desired magnitude response at the points specified in `f`. (The IIR counterpart of this function is `yul ewal k`, which also designs filters based on arbitrary piecewise linear magnitude responses. See “IIR Filter Design” for details.)

Multiband FIR Filter Design with Transition Bands

The `fi r l s` and `remez` functions provide a more general means of specifying the ideal desired filter than the `fi r 1` and `fi r 2` functions. These functions design Hilbert transformers, differentiators, and other filters with odd symmetric coefficients (type III and type IV linear phase). They also let you include transition or “don’t care” regions in which the error is not minimized, and perform band dependent weighting of the minimization.

`fi r l s` is an extension of the `fi r 1` and `fi r 2` functions in that it minimizes the integral of the square of the error between the desired frequency response and the actual frequency response.

`remez` implements the Parks-McClellan algorithm, which uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with optimal fits between the desired and actual frequency responses. The filters are optimal in the sense that they minimize the maximum error between the desired frequency response and the actual frequency response; they are sometimes called *minimax* filters. Filters designed in this way exhibit an equiripple behavior in their frequency response, and hence are also known as *equiripple* filters. The Parks-McClellan FIR filter design algorithm is perhaps the most popular and widely used FIR filter design methodology.

The syntax for `fi r l s` and `remez` is the same; the only difference is their minimization schemes. The next example shows how filters designed with `fi r l s` and `remez` reflect these different schemes.

Basic Configurations

The default mode of operation of `fi r l s` and `remez` is to design type I or type II linear phase filters, depending on whether the order you desire is even or odd, respectively. A lowpass example with approximate amplitude 1 from 0 to 0.4 Hz, and approximate amplitude 0 from 0.5 to 1.0 Hz is

```
n = 20;           % filter order
f = [0 .4 .5 1]; % frequency band edges
a = [1 1 0 0];    % desired amplitudes
b = remez(n, f, a);
```

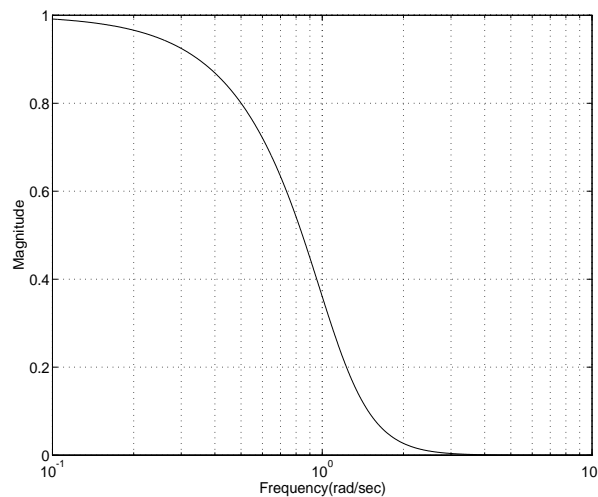

From 0.4 to 0.5 Hz, `remez` performs no error minimization; this is a transition band or “don’t care” region. A transition band minimizes the error more in the bands that you do care about, at the expense of a slower transition rate. In this way, these types of filters have an inherent trade-off similar to FIR design by windowing.

To compare least squares to equiripple filter design, use `fi r l s` to create a similar filter:

```
bb = fi r l s(n, f, a);
```

and compare their frequency responses:

```
[H, w]=freqz(b);  
[HH, w]=freqz(bb);  
plot(w/pi, abs(H), w/pi, abs(HH), '--'), grid
```



You can see that the filter designed with `remez` exhibits equiripple behavior. Also note that the `fi r l s` filter has a better response over most of the passband and stopband, but at the band edges ($f = 0.4$ and $f = 0.5$), the response is further away from the ideal than the `remez` filter. This shows that the `remez` filter’s *maximum* error over the pass- and stopbands is smaller and, in fact, it is the smallest possible for this band edge configuration and filter length.

Think of frequency bands as lines over short frequency intervals. `remez` and `fi r l s` use this scheme to represent any piecewise linear desired function with

any transition bands. `firls` and `remez` design lowpass, highpass, bandpass, and bandstop filters; a bandpass example is

```
f = [0 .3 .4 .7 .8 1]; % band edges in pairs
a = [0 0 1 1 0 0]; % bandpass filter amplitude
```

Technically, these `f` and `a` vectors define five bands:

- Two stopbands, from 0.0 to 0.3 and from 0.8 to 1.0
- A passband from 0.4 to 0.7
- Two transition bands, from 0.3 to 0.4 and from 0.7 to 0.8

Example highpass and bandstop filters are

```
f = [0 .7 .8 1]; % band edges in pairs
a = [0 0 1 1]; % highpass filter amplitude

f = [0 .3 .4 .5 .8 1]; % band edges in pairs
a = [1 1 0 0 1 1]; % bandstop filter amplitude
```

An example multiband bandpass filter is

```
f = [0 .1 .15 .25 .3 .4 .45 .55 .6 .7 .75 .85 .9 1];
a = [1 1 0 0 1 1 0 0 1 1 0 0 1 1];
```

Another possibility is a filter that has as a transition region the line connecting the passband with the stopband; this can help control “runaway” magnitude response in wide transition regions:

```
f = [0 .4 .42 .48 .5 1];
a = [1 1 .8 .2 0 0]; % passband, linear transition, stopband
```

The Weight Vector

Both `firls` and `remez` allow you to place more or less emphasis on minimizing the error in certain frequency bands relative to others. To do this, specify a weight vector following the frequency and amplitude vectors. An example

lowpass equiripple filter with 10 times less ripple in the stopband than the passband is

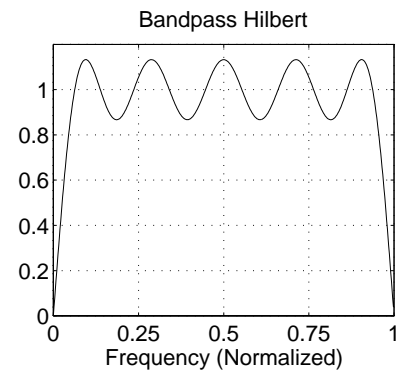
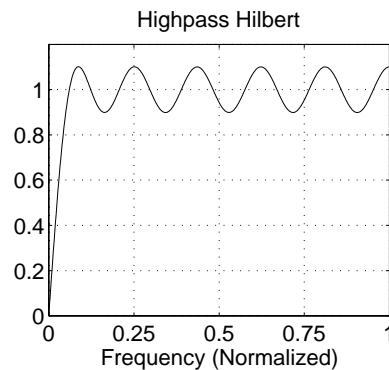
```
n = 20;                % filter order
f = [0 .4 .5 1];      % frequency band edges
a = [1 1 0 0];        % desired amplitudes
w = [1 10];           % weight vector
b = remez(n, f, a, w);
```

A legal weight vector is always half the length of the *f* and *a* vectors; there must be exactly one weight per band.

Anti-Symmetric Filters / Hilbert Transformers

When called with a trailing 'h' or 'Hilbert' option, *remez* and *firls* design FIR filters with odd symmetry, that is, type III (for even order) or type IV (for odd order) linear phase filters. An ideal Hilbert transformer has this anti-symmetry property and an amplitude of 1 across the entire frequency range. Try the following approximate Hilbert transformers:

```
b = remez(21, [.05 1], [1 1], 'h');    % highpass Hilbert
bb = remez(20, [.05 .95], [1 1], 'h'); % bandpass Hilbert
```



You can find the delayed Hilbert transform of a signal *x* by passing it through these filters:

```
Fs = 1000;            % sampling frequency
t = (0:1/Fs:2)';      % two second time vector
x = sin(2*pi*300*t);   % 300 Hz sine wave example signal
xh = filter(bb, 1, x); % Hilbert transform of x
```

The analytic signal corresponding to x is the complex signal that has x as its real part and the Hilbert transform of x as its imaginary part. For this FIR method (an alternative to the `hilbert` function), you must delay x by half the filter order to create the analytic signal:

```
xd = [zeros(10, 1); x(1:length(x)-10)]; % delay 10 samples
xa = xd + j*xh; % analytic signal
```

This method does not work directly for filters of odd order, which require a noninteger delay. In this case, the `hilbert` function, described in the “Specialized Transforms” section in Chapter 4, estimates the analytic signal. Alternately, use the `resample` function to delay the signal by a noninteger number of samples.

Differentiators

Differentiation of a signal in the time domain is equivalent to multiplication of the signal's Fourier transform by an imaginary ramp function. That is, to differentiate a signal, pass it through a filter that has a response $H(w) = jw$. Approximate the ideal differentiator (with a delay) using `remez` or `firls` with a 'd' or 'differentiator' option:

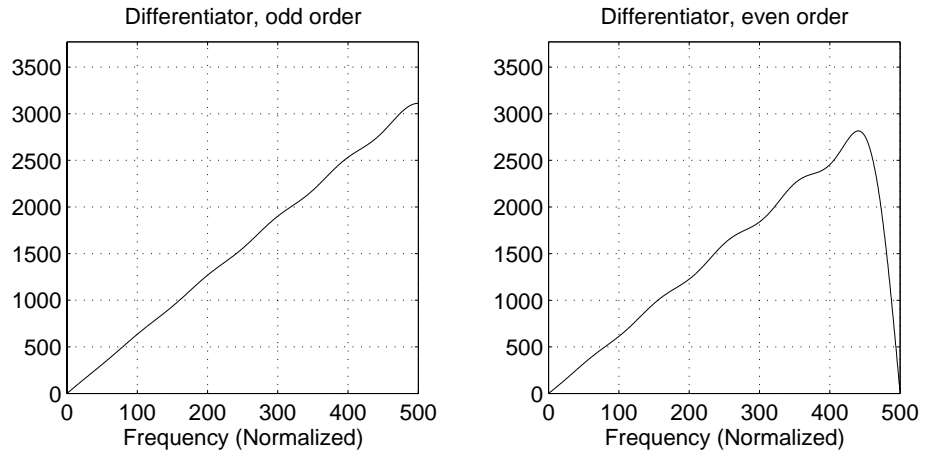
```
b = remez(21, [0 1], [0 pi*Fs], 'd');
```

To obtain the correct derivative, scale by $\pi * F_s$ rad/sec, where F_s is the sampling frequency in Hertz. For a type III filter, the differentiation band should stop short of the Nyquist frequency, and the amplitude vector must reflect that change to ensure the correct slope:

```
bb = remez(20, [0 .9], [0 .9*pi*Fs], 'd');
```

In the 'd' mode, `remez` weights the error by $1/w$ in nonzero amplitude bands to minimize the maximum *relative* error. `firls` weights the error by $(1/w)^2$ in nonzero amplitude bands in the 'd' mode.

The magnitude response plots for the differentiators shown above are



Constrained Least Squares FIR Filter Design

The Constrained Least Squares (CLS) FIR filter design functions implement a technique that enables you to design FIR filters without explicitly defining the transition bands for the magnitude response. The ability to omit the specification of transition bands is useful in several situations. For example, it may not be clear where a rigidly defined transition band should appear if noise and signal information appear together in the same frequency band. Similarly, it may make sense to omit the specification of transition bands if they appear only to control the results of Gibbs phenomena that appear in the filter's response. See Selesnick, Lang, and Burrus [2] for discussion of this method.

Instead of defining passbands, stopbands, and transition regions, the CLS method accepts a cutoff frequency (for the highpass, lowpass, bandpass, or bandstop cases), or passband and stopband edges (for multiband cases), for the desired response. In this way, the CLS method defines transition regions implicitly, rather than explicitly.

The key feature of the CLS method is that it enables you to define upper and lower thresholds that contain the maximum allowable ripple in the magnitude response. Given this constraint, the technique applies the least square error minimization technique over the frequency range of the filter's response, instead of over specific bands. The error minimization includes any areas of discontinuity in the ideal, "brick wall" response. An additional benefit is that

the technique enables you to specify arbitrarily small peaks resulting from Gibbs' phenomena.

There are two toolbox functions that implement this design technique:

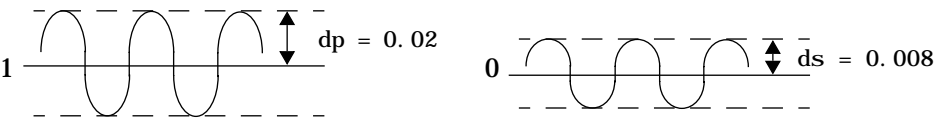
Description	Function
Constrained least square multiband FIR filter design.	<code>fircls</code>
Constrained least square filter design for lowpass and highpass linear phase filters	<code>fircls1</code>

For details on the calling syntax for these functions, see their reference descriptions in Chapter 6.

Basic Lowpass and Highpass CLS Filter Design

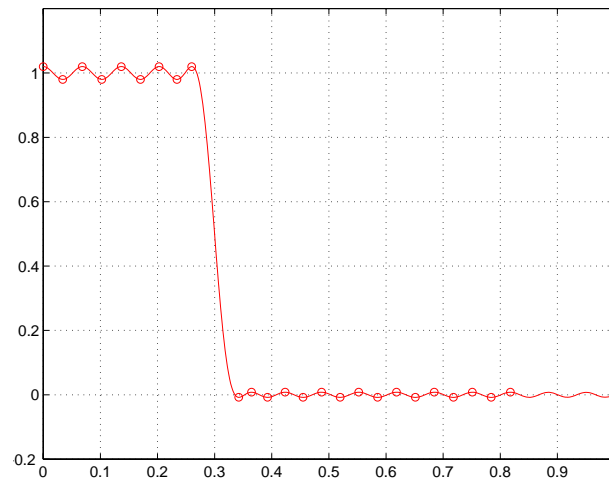
The most basic of the CLS design functions, `fircls1`, uses this technique to design lowpass and highpass FIR filters. As an example, consider designing a filter with order 61 impulse response and cutoff frequency of 0.3 (normalized). Further, define the upper and lower bounds that constrain the design process as

- Maximum passband deviation from 1 (passband ripple) of 0.02.
- Maximum stopband deviation from 0 (stopband ripple) of 0.008.



To approach this design problem using `fircls1`:

```
n = 61;
wo = 0.3;
dp = 0.02;
ds = 0.008;
h = fircls1(n, wo, dp, ds, 'plot');
```



Multiband CLS Filter Design

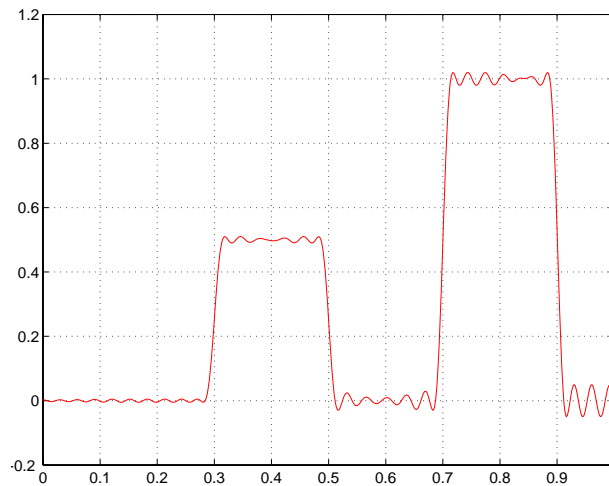
`fircls` uses the same technique to design FIR filters with a desired piecewise constant magnitude response. In this case, you can specify a vector of band edges and a corresponding vector of band amplitudes. In addition, you can specify the maximum amount of ripple for each band.

For example, assume the specifications for a filter call for

- From 0 to 0.3 (normalized): amplitude 0, upper bound 0.005, lower bound -0.005
- From 0.3 to 0.5: amplitude 0.5, upper bound 0.51, lower bound 0.49
- From 0.5 to 0.7: amplitude 0, upper bound 0.03, lower bound -0.03
- From 0.7 to 0.9: amplitude 1, upper bound 1.02, lower bound 0.98
- From 0.9 to 1: amplitude 0, upper bound 0.05, lower bound -0.05

Design a CLS filter with impulse response order 129 that meets these specifications:

```
n = 129;
f = [0 0.3 0.5 0.7 0.9 1];
a = [0 0.5 0 1 0];
up = [0.005 0.51 0.03 1.02 0.05];
lo = [-0.005 0.49 -0.03 0.98 -0.05];
h = fircls(n, f, a, up, lo, 'plot');
```



Weighted CLS Filter Design

Weighted CLS filter design lets you design lowpass or highpass FIR filters with relative weighting of the error minimization in each band. The `fircls1` function enables you to specify the passband and stopband edges for the least squares weighting function, as well as a constant k that specifies the ratio of the stopband to passband weighting.

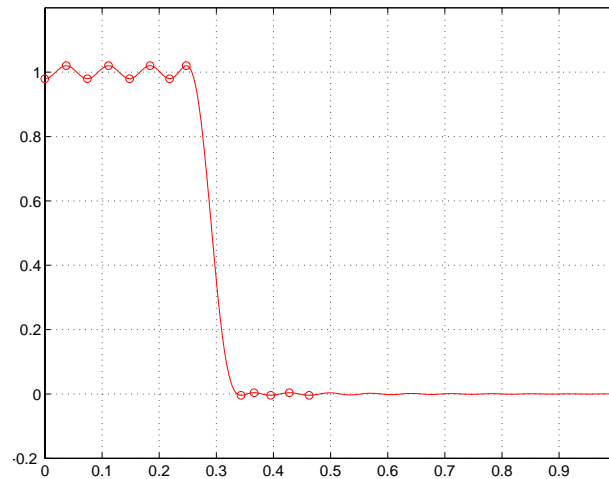
For example, consider specifications that call for an FIR filter with impulse response order of 55 and cutoff frequency of 0.3 (normalized). Also assume

maximum allowable passband ripple of 0.02 and maximum allowable stopband ripple of 0.004. In addition, add weighting requirements:

- Passband edge for the weight function of 0.28 (normalized)
- Stopband edge for the weight function of 0.32
- Weight error minimization 10 times as much in the stopband as in the passband

To approach this using `fircls1`:

```
n = 55;
wo = 0.3;
dp = 0.02;
ds = 0.004;
wp = 0.28;
ws = 0.32;
k = 10;
h = fircls1(n, wo, dp, ds, wp, ws, k, 'plot');
```



Arbitrary-Response Filter Design

The `cremez` filter design function provides a tool for designing FIR filters with arbitrary complex responses. It differs from the other filter design functions in how the frequency response of the filter is specified: it accepts the name of a function which returns the filter response calculated over a grid of frequencies.

This capability makes `cremez` a highly versatile and powerful technique for filter design.

This design technique may be used to produce nonlinear-phase FIR filters, asymmetric frequency-response filters (with complex coefficients), or more symmetric filters with custom frequency responses.

The design algorithm optimizes the Chebyshev (or minimax) error using an extended Remez-exchange algorithm for an initial estimate. If this exchange method fails to obtain the optimal filter, the algorithm switches to an ascent-descent algorithm that takes over to finish the convergence to the optimal solution.

For details on the calling syntax for `cremez`, see the reference description in Chapter 6.

Multiband Filter Design

Consider a multiband filter with the following special frequency-domain characteristics:

Band	Amplitude	Optimization Weighting
[-1 -0.5]	[5 1]	1
[-0.4 +0.3]	[2 2]	10
[+0.4 +0.8]	[2 1]	5

A linear-phase multiband filter may be designed using the predefined frequency-response function `multiband`, as follows:

```
b = cremez(38, [-1 -0.5 -0.4 0.3 0.4 0.8], ...
               {'multiband', [5 1 2 2 2 1]}, [1 10 5]);
```

For the specific case of a multiband filter, we can use a shorthand filter design notation similar to the syntax for `remez`:

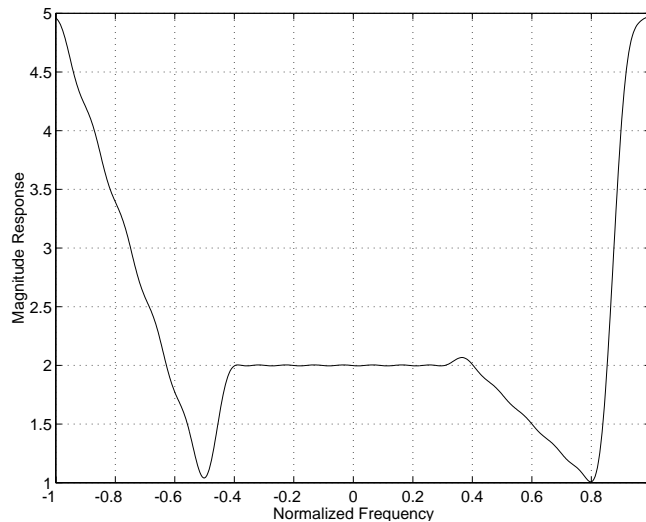
```
b = cremez(38, [-1 -0.5 -0.4 0.3 0.4 0.8], ...
               [5 1 2 2 2 1], [1 10 5]);
```

As with `remez`, a vector of band edges is passed to `cremez`. This vector defines the frequency bands over which optimization is performed; note that there are two transition bands, from -0.5 to -0.4 and from 0.3 to 0.4.

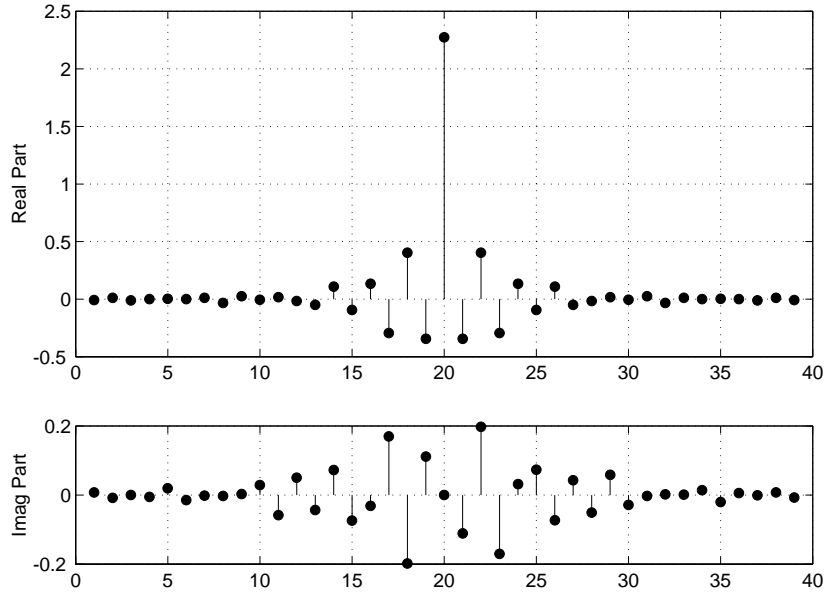
In either case, the frequency response is obtained and plotted using linear scale:

```
[h, w] = freqz(b, 1, 512, 'whole');  
plot(w/pi-1, fftshift(abs(h)));
```

Note that the frequency response has been calculated over the entire normalized frequency range $[-1, +1]$ by passing the option `'whole'` to `freqz`. In order to plot the negative frequency information in a natural way, the response has been “wrapped,” just as FFT data is, using `fftshift`:



The filter response for this multiband filter is complex, which is expected because of the asymmetry in the frequency domain. The filter response is



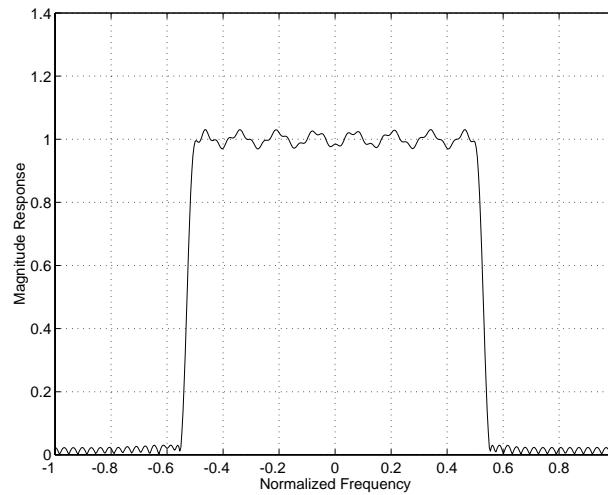
Filter Design with Reduced Delay

Consider the design of a 62-tap lowpass filter with a half-Nyquist cutoff. If we specify a negative offset value to the `lowpass` filter design function, the group delay offset for the design is significantly less than that obtained for a standard linear-phase design. This filter design may be computed as follows:

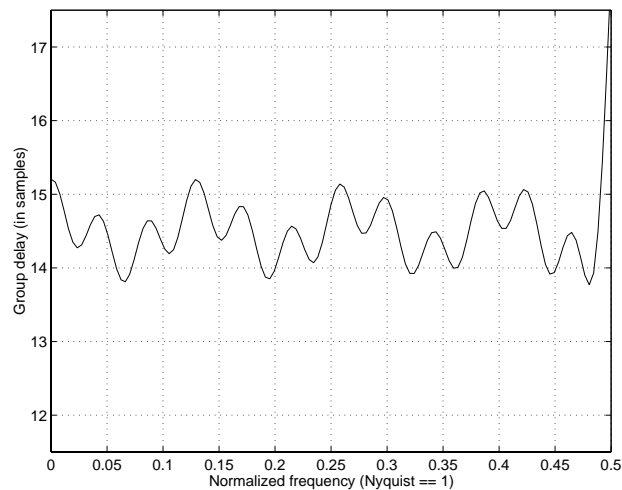
```
b = cremez(61, [0 0.5 0.55 1], {'lowpass', -16});
```

The resulting magnitude response is:

```
[h, w] = freqz(b, 1, 512, 'whole');
plot(w/pi-1, fftshift(abs(h)));
```



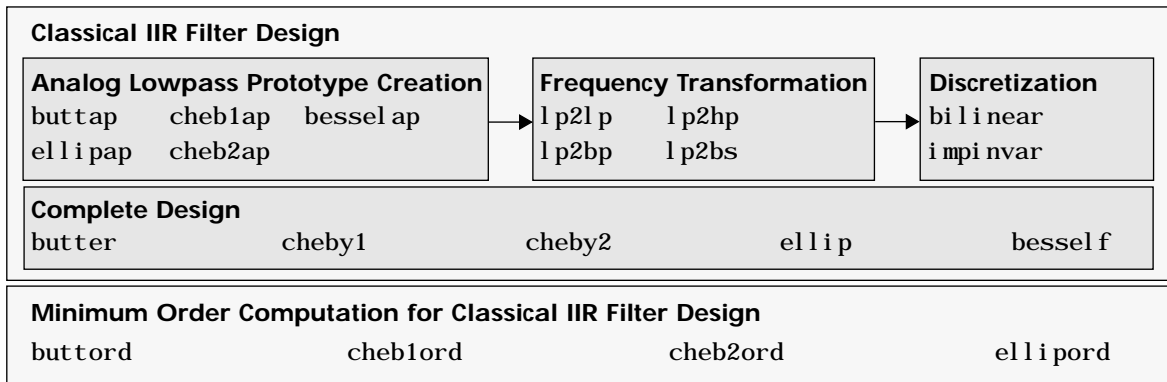
The group delay of the filter reveals that the offset has been reduced from $N/2=30.5$ to $N/2-16=14.5$. Now, however, the group delay is no longer flat in the passband region (plotted over the normalized frequency range 0 to 0.5 for clarity):



If we compare this nonlinear-phase filter to a linear-phase filter that has exactly 14.5 samples of group delay, the resulting filter is of order 2×14.5 or 29. Using `b = cremez(29, [0 0.5 0.55 1], 'lowpass')`, the passband and stopband ripple is much greater for the order 29 filter. These comparisons can assist you in deciding which filter is more appropriate for a specific application.

Special Topics in IIR Filter Design

The classic IIR filter design technique finds an analog lowpass filter with cutoff frequency of 1, translates this “prototype” filter to the desired band configuration, then transforms the filter to the digital domain. The toolbox provides functions for each step of this process:



The butter, cheby1, cheby2, and ellip functions are sufficient for many design problems, and the lower level functions are generally not needed. But if you do have an application where you need to transform the band edges of an analog filter, or discretize a rational transfer function, this section describes tools to do so.

Analog Prototype Design

This toolbox provides a number of functions to create lowpass analog prototype filters with cutoff frequency of 1, the first step in the classical approach to IIR filter design. The table below summarizes the analog prototype design functions for each supported filter type; plots for each type are shown in the “IIR Filter Design” section above.

Filter Type	Analog Prototype Function
Bessel	[z, p, k] = <code>besselap(n)</code>
Butterworth	[z, p, k] = <code>buttap(n)</code>
Chebyshev type I	[z, p, k] = <code>cheb1ap(n, Rp)</code>
Chebyshev type II	[z, p, k] = <code>cheb2ap(n, Rs)</code>
Elliptic	[z, p, k] = <code>ellipap(n, Rp, Rs)</code>

Frequency Transformation

The second step in the analog prototyping design technique is the frequency transformation of a lowpass prototype. The toolbox provides a set of functions to transform analog lowpass prototypes (with cutoff frequency of 1 rad/sec) into bandpass, highpass, bandstop, and lowpass filters of the desired cutoff frequency:

Freq. Transformation	Transformation Function
Lowpass to lowpass $s' = s / \omega_0$	$[\text{numt}, \text{dent}] = \text{l p2l p}(\text{num}, \text{den}, \text{Wo})$ $[\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{l p2l p}(\text{A}, \text{B}, \text{C}, \text{D}, \text{Wo})$
Lowpass to highpass $s' = \frac{\omega_0}{s}$	$[\text{numt}, \text{dent}] = \text{l p2hp}(\text{num}, \text{den}, \text{Wo})$ $[\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{l p2hp}(\text{A}, \text{B}, \text{C}, \text{D}, \text{Wo})$
Lowpass to bandpass $s' = \frac{\omega_0}{B_w} \frac{(s / \omega_0)^2 + 1}{s / \omega_0}$	$[\text{numt}, \text{dent}] = \text{l p2bp}(\text{num}, \text{den}, \text{Wo}, \text{Bw})$ $[\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{l p2bp}(\text{A}, \text{B}, \text{C}, \text{D}, \text{Wo}, \text{Bw})$
Lowpass to bandstop $s' = \frac{B_w}{\omega_0} \frac{s / \omega_0}{(s / \omega_0)^2 + 1}$	$[\text{numt}, \text{dent}] = \text{l p2bs}(\text{num}, \text{den}, \text{Wo}, \text{Bw})$ $[\text{At}, \text{Bt}, \text{Ct}, \text{Dt}] = \text{l p2bs}(\text{A}, \text{B}, \text{C}, \text{D}, \text{Wo}, \text{Bw})$

As shown, all of the frequency transformation functions can accept two linear system models: transfer function and state-space form. For the bandpass and bandstop cases,

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

and

$$B_w = \omega_2 - \omega_1$$

where ω_1 is the lower band edge and ω_2 is the upper band edge.

The frequency transformation functions perform frequency variable substitution. In the case of `l p2bp` and `l p2bs`, this is a second-order substitution, so the output filter is twice the order of the input. For `l p2l p` and `l p2hp`, the output filter is the same order as the input.

To begin designing an order 10 bandpass Chebyshev type I filter with a value of 3 dB for passband ripple:

$$[z, p, k] = \text{cheb1ap}(5, 3);$$

z , p , and k contain the poles, zeros, and gain of a lowpass analog filter with cutoff frequency Ω_c equal to 1 rad/sec. Use the `lp2bp` function to transform this lowpass prototype to a bandpass analog filter with band edges $W_1 = \pi/5$ and $W_2 = \pi$. First, convert the filter to state-space form so the `lp2bp` function can accept it:

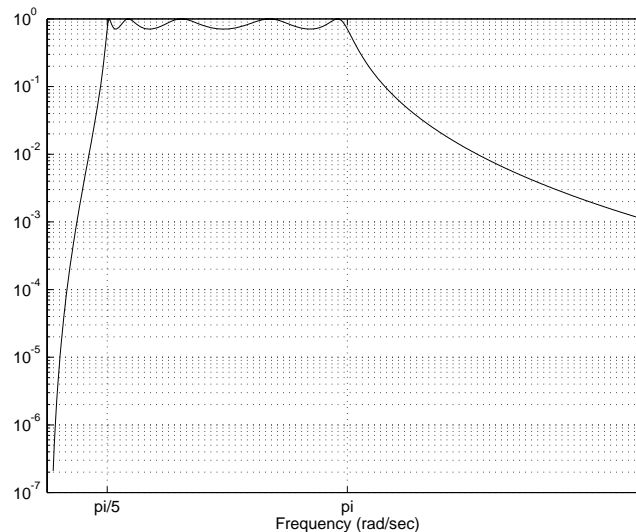
```
[A, B, C, D] = zp2ss(z, p, k); % Convert to state-space form.
```

Now, find the bandwidth and center frequency, and call `lp2bp`:

```
u1 = 0.1*2*pi; u2 = 0.5*2*pi; % in radians per second
Bw = u2-u1;
Wo = sqrt(u1*u2);
[At, Bt, Ct, Dt] = lp2bp(A, B, C, D, Wo, Bw);
```

Finally, calculate the frequency response and plot its magnitude:

```
[b, a] = ss2tf(At, Bt, Ct, Dt); % Convert to TF form.
w = linspace(.01, 1, 500)*2*pi; % Generate frequency vector.
h = freqs(b, a, w); % Compute frequency response.
semilog(w/2/pi, abs(h)), grid % Plot log magnitude vs. freq.
```



Filter Discretization

The third step in the analog prototyping technique is the transformation of the filter to the discrete-time domain. The toolbox provides two methods for this: the impulse invariant and bilinear transformations. The filter design functions `butter`, `cheby1`, `cheby2`, and `ellip` use the bilinear transformation for discretization in this step.

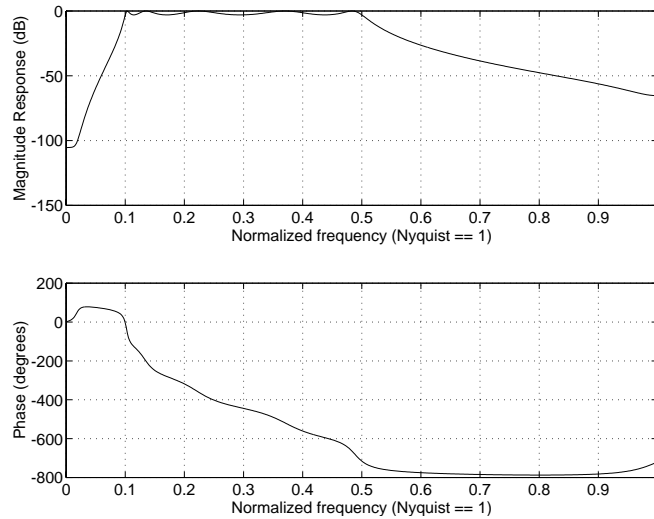
Analog to Digital Transformation	Transformation Function
Impulse invariance	<code>[numd, dend] = impinvar(num, den, Fs)</code>
Bilinear transform	<code>[zd, pd, kd] = bilinear(z, p, k, Fs, Fp)</code> <code>[numd, dend] = bilinear(num, den, Fs, Fp)</code> <code>[Ad, Bd, Cd, Dd] = bilinear(At, Bt, Ct, Dt, Fs, Fp)</code>

Impulse Invariance

The toolbox function `impinvar` creates a digital filter whose impulse response is the samples of the continuous impulse response of an analog filter. This function only works on filters in transfer function form. For best results, the analog filter should have negligible frequency content above half the sampling frequency, because such high frequency content is aliased into lower bands upon sampling. Impulse invariance works for some lowpass and bandpass filters, but is not appropriate for highpass and bandstop filters.

Design a Chebyshev type I filter and plot its frequency response:

```
[bz, az] = impzvar(b, a, 2);
freqz(bz, az)
```



Impulse invariance retains the cutoff frequencies of 0.1 Hz and 0.5 Hz.

Bilinear Transformation

The bilinear transformation is a nonlinear mapping of the continuous domain to the discrete domain; it maps the s -plane into the z -plane by

$$H(z) = H(s) \Big|_{s=k \frac{z-1}{z+1}}$$

Bilinear transformation maps the $j\Omega$ axis of the continuous domain to the unit circle of the discrete domain according to

$$\omega = 2 \tan^{-1} \left(\frac{\Omega}{k} \right)$$

The toolbox function `bilinear` implements this operation, where the frequency warping constant k is equal to twice the sampling frequency ($2 \cdot F_s$) by default and equal to $2 \cdot \pi \cdot F_p / \tan(\pi \cdot F_p / F_s)$ if you give `bilinear` a trailing argument that represents a “match” frequency F_p . If a match frequency F_p (in Hertz) is

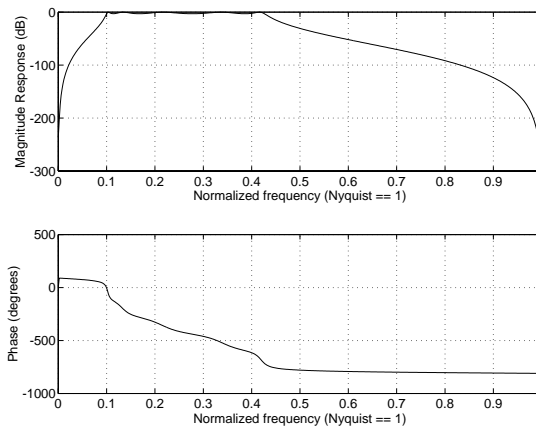
present, `biilinear` maps the frequency $\Omega = 2\pi f_p$ (in radians/second) to the same frequency in the discrete domain, normalized to the sampling rate: $\omega = 2\pi f_p/f_s$ (also in radians/second).

The `biilinear` function can perform this transformation on three different linear system representations: zero-pole-gain, transfer function, and state-space form. Try calling `biilinear` with the state-space matrices that describe the Chebyshev type I filter from the previous section, using a sampling frequency of 2 Hz, and retaining the lower band edge of 0.1 Hz:

```
[Ad, Bd, Cd, Dd] = biilinear(At, Bt, Ct, Dt, 2, 0.1);
```

The frequency response of the resulting digital filter is

```
[bz, az] = ss2tf(Ad, Bd, Cd, Dd); % convert to TF
freqz(bz, az)
```



The lower band edge is at 0.1 Hz as expected. Notice, however, that the upper band edge is slightly less than 0.5 Hz, although in the analog domain it was exactly 0.5 Hz. This illustrates the nonlinear nature of the bilinear transformation. To counteract this nonlinearity, it is necessary to create analog domain filters with “prewarped” band edges, which map to the correct locations

upon bilinear transformation. Here the prewarped frequencies u_1 and u_2 generate B_w and W_o for the `lp2bp` function:

```

Fs = 2;                                % sampling frequency (Hertz)
u1 = 2*Fs*tan(.1*(2*pi/Fs)/2); % lower band edge (radians/second)
u2 = 2*Fs*tan(.5*(2*pi/Fs)/2); % upper band edge (radians/second)
Bw = u2-u1;                            % bandwidth
Wo = sqrt(u1*u2);                      % center frequency
[At, Bt, Ct, Dt] = lp2bp(A, B, C, D, Wo, Bw);

```

A digital bandpass filter with correct band edges 0.1 and 0.5 times the Nyquist frequency is

```
[Ad, Bd, Cd, Dd] = bilinear(At, Bt, Ct, Dt, Fs);
```

The example bandpass filters from the last two sections could also be created in one statement using the complete IIR design function `cheby1`. For instance, an analog version of the example Chebyshev filter is

```
[b, a] = cheby1(5, 3, [0.1 0.5]*2*pi, 's');
```

Note that the band edges are in radians/second for analog filters, whereas for the digital case, frequency is normalized (the Nyquist frequency is equal to 1 Hz):

```
[bz, az] = cheby1(5, 3, [0.1 0.5]);
```

All of the complete design functions call `bilinear` internally. They prewarp the band edges as needed to obtain the correct digital filter. See Chapter 6 for more on these functions.

References

- 1 Karam, L.J., and J.H. McClellan. "Complex Chebyshev Approximation for FIR Filter Design." *IEEE Trans. on Circuits and Systems II*. March 1995.
- 2 Selesnick, I.W., and C.S. Burrus. "Generalized Digital Butterworth Filter Design." *Proceedings of the IEEE Int. Conf. Acoust., Speech, Signal Processing*. Vol. 3 (May 1996).
- 3 Selesnick, I.W., M. Lang, and C.S. Burrus. "Constrained Least Square Design of FIR Filters without Specified Transition Bands." *Proceedings of the IEEE Int. Conf. Acoust., Speech, Signal Processing*. Vol. 2 (May 1995). Pgs. 1260-1263.

Statistical Signal Processing

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Correlation and Covariance

The Signal Processing Toolbox provides tools for estimating important functions of random signals. In particular, there are tools to estimate correlation and covariance sequences and spectral density functions of discrete signals. This chapter explains the correlation and covariance functions, and discusses the mathematically related functions for estimating the power spectrum.

The functions `xcorr` and `xcov` estimate the cross-correlation and cross-covariance sequences of random processes. They also handle autocorrelation and autocovariance as special cases.

The true cross-correlation sequence is a statistical quantity defined as

$$\gamma_{xy}(m) = E\{x_n y_{n+m}^*\}$$

where x_n and y_n are stationary random processes, $-\infty < n < \infty$, and $E\{\}$ is the expected value operator. The cross-covariance sequence is the mean-removed cross-correlation sequence:

$$C_{xy}(m) = E\{(x_n - \mu_x)(y_{n+m}^* - \mu_y^*)\}$$

or, in terms of the cross-correlation:

$$C_{xy}(m) = \gamma_{xy}(m) - \mu_x \mu_y^*$$

In practice, you must estimate these sequences, because it is possible to access only a finite segment of the infinite-length random process. A common estimate based on N samples of x_n and y_n is the deterministic cross-correlation sequence (also called the time-ambiguity function):

$$\hat{R}_{xy}(m) = \begin{cases} \sum_{n=0}^{N-|m|-1} x_n y_{n+m}^* & m \geq 0 \\ \hat{R}_{yx}^*(-m) & m < 0 \end{cases}$$

where we assume for this discussion that x_n and y_n are indexed from 0 to $N-1$, and $\hat{R}_{xy}(m)$ from $-(N-1)$ to $N-1$. The `xcorr` function evaluates this sum with an efficient FFT-based algorithm, given inputs x_n and y_n stored in length N vectors x and y . Its operation is equivalent to convolution with one of the two subsequences reversed in time.

For example,

```
x = [1 1 1 1 1]';
y = x;
xyc = xcorr(x, y)

xyc =

    1.0000
    2.0000
    3.0000
    4.0000
    5.0000
    4.0000
    3.0000
    2.0000
    1.0000
```

Notice that the resulting sequence is twice the length of the input sequence minus 1. Thus, the N th element is the correlation at lag 0. Also notice the triangular pulse of the output that results when convolving two square pulses.

The `xcov` function estimates autocovariance and cross-covariance sequences. This function has the same options and evaluates the same sum as `xcorr`, but first removes the means of x and y .

Bias and Normalization

An estimate of a quantity is *biased* if its expected value is not equal to the quantity it estimates. The expected value of the output of `xcorr` is

$$E\{\hat{R}_{xy}(m)\} = \sum_{n=0}^{N-|m|-1} E\{x_n y_{n+m}^*\} = (N-|m|)\gamma_{xy}(m)$$

`xcorr` provides the unbiased estimate, dividing by $N - |m|$, when you specify an 'unbiased' flag after the input sequences:

```
xcorr(x, y, 'unbiased')
```

Although this estimate is unbiased, the end points (near $-(N-1)$ and $N-1$) suffer from large variance because `xcorr` computes them using only a few data points. A possible trade-off is to simply divide by N using the 'biased' flag:

```
xcorr(x, y, 'biased')
```

With this scheme, only the sample of the correlation at 0 lag (the N th output element) is unbiased. This estimate is often more desirable than the unbiased one because it avoids random large variations at the end points of the correlation sequence.

`xcorr` provides one other normalization scheme. The syntax

```
xcorr(x, y, 'coeff')
```

divides the output by $\text{norm}(x) * \text{norm}(y)$ so that, for autocorrelations, the sample at 0 lag is 1.

Multiple Channels

For a multichannel signal, `xcorr` and `xcov` estimate the autocorrelation and cross-correlation and covariance sequences for all of the channels at once. If S is an M -by- N signal matrix representing N channels in its columns, `xcorr(S)` returns a $(2M-1)$ -by- N^2 matrix with the autocorrelations and cross-correlations of the channels of S in its N^2 columns. If S is a 3-channel signal

```
S = [s1 s2 s3]
```

then the result of `xcorr(S)` is organized as

```
R = [Rs1s1 Rs1s2 Rs1s3 Rs2s1 Rs2s2 Rs2s3 Rs3s1 Rs3s2 Rs3s3]
```

Two related functions, `cov` and `corrcoef`, are available in the standard MATLAB environment. They estimate covariance and normalized covariance respectively between the different channels at lag 0 and arrange them in a square matrix.

Spectral Analysis

Spectral analysis seeks to describe the frequency content of a signal, random process, or system, based on a finite set of data. Estimation of power spectra is useful in a variety of applications, including the detection of signals buried in wide-band noise.

The *power spectral density* (PSD) of a stationary random process x_n is related mathematically to the correlation sequence by the discrete-time Fourier transform:

$$P_{xx}(\omega) = \sum_{m=-\infty}^{\infty} \gamma_{xx}(m) e^{-j\omega m}$$

This function of frequency has the property that its integral over a frequency band is equal to the power of the signal x_n in that band. The PSD is a special case of the *cross spectral density* (CSD) function, defined between two signals x_n and y_n as

$$P_{xy}(\omega) = \sum_{m=-\infty}^{\infty} \gamma_{xy}(m) e^{-j\omega m}$$

As is the case for the correlation and covariance sequences, the toolbox *estimates* the PSD and CSD because signal lengths are finite.

The various methods of PSD estimation can be identified as *parametric* or *nonparametric*. One technique offered in the Signal Processing Toolbox is the popular nonparametric scheme developed by Welch [5]. This is complemented by more modern nonparametric techniques such as the *multitaper method* (MTM) and the *multiple signal classification* (MUSIC) or *eigenvector* (EV) *method*, which is well suited for line spectra (data made up of sinusoids). The *Yule-Walker autoregressive* (AR) *method* is a parametric method that estimates the autocorrelation function to solve for the AR model parameters. The *Burg method* is another parametric spectral estimation method that minimizes the forward and backward linear prediction errors while satisfying the Levinson-Durbin recursion. These methods are listed in the table below together with the corresponding toolbox function name. The number below each method name indicates the page that describes the method in greater

detail. See “Parametric Modeling” in Chapter 4 for details about `lpc` and other parametric estimation functions.

Method	Description	Functions
Burg (3-20)	Autoregressive (AR) spectral estimation of a time-series by minimization of linear prediction errors	<code>pburg</code>
Multitaper (3-16)	Spectral estimate from combination of multiple orthogonal windows (or “tapers”)	<code>pmtm</code>
MUSIC (3-22)	Multiple signal classification or eigenvector method	<code>pmusic</code>
Welch (3-6)	Averaged periodograms of overlapped, windowed signal sections	<code>psd</code> , <code>csd</code> , <code>tfe</code> , <code>cohere</code>
Yule-Walker AR (3-19)	Autoregressive (AR) spectral estimate of a time-series from its estimated autocorrelation function	<code>pyulear</code>

Welch’s Method

One way of estimating the power spectrum of a process is to simply find the discrete-time Fourier transform of the samples of the process (usually done on a grid with an FFT) and take the magnitude squared of the result. An example 1001-element signal `xn`, which consists of two sinusoids plus noise, is given by

```

Fs = 1000;           % sampling frequency
t = 0:1/Fs:1;        % one second worth of samples
xn = sin(2*pi*50*t) + 2*sin(2*pi*120*t) + randn(size(t));

```

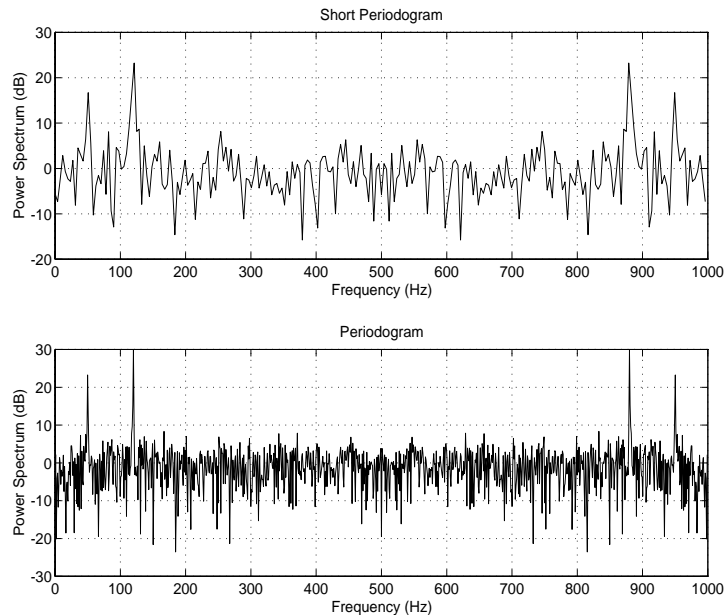
A crude estimate of the PSD of `xn` is

```
Pxx = abs(fft(xn, 1024)).^2/1001;
```

This estimate is called the *periodogram*. Scale the magnitude squared of the FFT by the square of the norm of the data window applied to the signal (in this

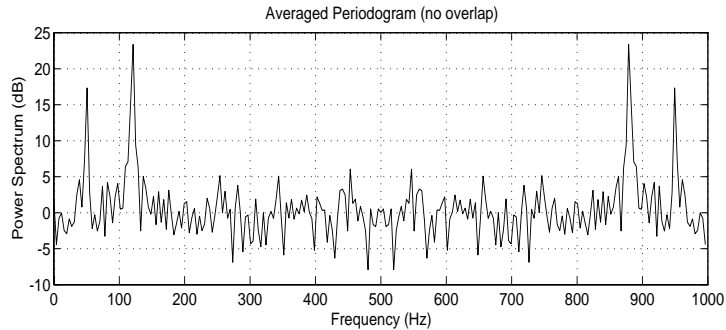
case, a length 1001 rectangular window) to ensure that the estimate is *asymptotically unbiased*. That is, as the number of samples increases, the expected value of the periodogram approaches the true PSD. The problem with the periodogram estimate is that its variance is large (on the order of the PSD squared) and does not decrease as the number of samples increases. The following two examples show this; as FFT length increases, the periodogram does not become smoother:

```
Pxx_short = abs(fft(xn, 256)).^2/256;
plot((0:255)/256*Fs, 10*log10(Pxx_short))
plot((0:1023)/1024*Fs, 10*log10(Pxx))
```



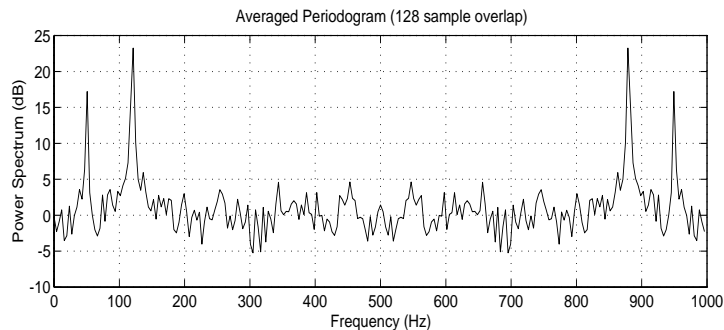
Reduce the variance of the PSD estimate by breaking the signal into nonoverlapping sections and averaging the periodograms of these sections:

```
Pxx = (abs(fft(xn( 1:256))).^2 + abs(fft(xn(257:512))).^2 + ...
        abs(fft(xn(513:768))).^2 ) / (256*3);
plot((0:255)/256*Fs, 10*log10(Pxx))
```



This averaged estimate has one third the variance of the length 256 periodogram shown earlier. The more sections you average, the lower the variance of the result. However, the signal length limits the number of sections possible (to three sections of length 256 in the previous example). To obtain more sections, break the signal into overlapping sections:

```
Pxx = (abs(fft(xn( 1:256))).^2 + abs(fft(xn(129:384))).^2 + ...
        abs(fft(xn(257:512))).^2 + abs(fft(xn(385:640))).^2 + ...
        abs(fft(xn(513:768))).^2 + ...
        abs(fft(xn(641:896))).^2 ) / (256*6);
plot((0:255)/256*Fs, 10*log10(Pxx))
```

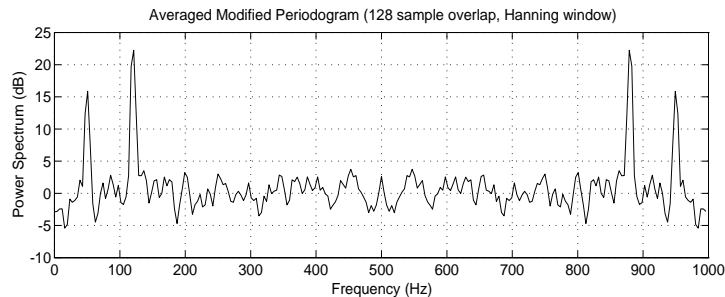


In this case the sections are statistically dependent, resulting in higher variance; thus there is a trade-off between the number of sections and the overlap rate.

Another way to improve the periodogram estimate is to apply a nonrectangular data window to the sections prior to computing the periodogram, resulting in a *modified periodogram*. This reduces the effect of section dependence due to overlap, because the window is tapered to 0 on the edges. Also, a nonrectangular window diminishes the side-lobe interference or “spectral leakage” while increasing the width of spectral peaks. With a suitable window (such as Hamming, Hanning, or Kaiser), overlap rates of about half the section length have been found to lower the variance of the estimate significantly.

The application of a Hanning window results in

```
w = hann(256)';
Pxx = ( abs(fft(w.*xn( 1:256))).^2 + ...
        abs(fft(w.*xn(129:384))).^2 + ...
        abs(fft(w.*xn(257:512))).^2 + ...
        abs(fft(w.*xn(385:640))).^2 + ...
        abs(fft(w.*xn(513:768))).^2 + ...
        abs(fft(w.*xn(641:896))).^2 ) / (norm(w)^2*6);
plot((0:255)/256*Fs, 10*log10(Pxx))
```



Notice in this plot that the spectral peaks have widened, and the *noise floor*, or level of the noise, seems to be the flattest of any estimate so far. This method of averaged, modified periodograms is Welch's method of PSD estimation.

The functions `psd` and `csd` provide control over all the parameters discussed so far (FFT length, window, and amount of overlap) in computing the PSD and CSD of one or two signals using Welch's method.

For a more detailed discussion of Welch's method of PSD estimation, see Kay [1] and Welch [5].

Power Spectral Density Function

The `psd` function averages and scales the modified periodograms of detrended sections of a signal. Simply specify the parameters that control the algorithm as arguments to the function.

An estimate for the PSD of a sequence `xn` using `psd`'s default FFT length (256), window (Hanning of length 256), overlap samples (none), and detrending option (remove best linear fit from sections) is

```
Pxx = psd(xn);
```

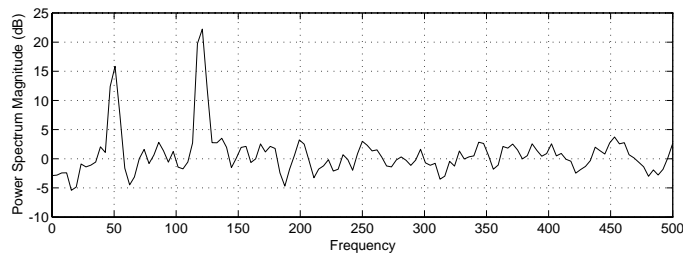
If the original sequence `xn` has units of volts, `Pxx` has units of volts^2/Hz .

To recreate the last example accurately, specify 128 as the number of samples to overlap and ask for no detrending:

```
nfft = 256;           % length of FFT
Fs = 1000;            % sampling frequency
window = hann(256);   % window function
noverlap = 128;        % number of samples overlap
dflag = 'none';        % detrending option
Pxx = psd(xn, nfft, Fs, window, noverlap, dflag);
```

The order of the inputs to `psd` is important, except for the `dflag` string, which can be in any position as long as it is last. The sampling frequency doesn't affect the PSD estimate but helps `psd` scale the frequency axis for plotting. `psd` without any outputs generates a plot:

```
psd(xn, nfft, Fs, window, noverlap, dflag)
```



If you want to plot the PSD yourself, obtain a frequency vector through an additional output argument:

```
[Pxx, f] = psd(xn, nfft, Fs, 256, noverlap, df lag);
plot(f, 10*log10(Pxx))
```

Since the signal x_n is real, `psd` returns only the frequencies from 0 through the Nyquist frequency. In contrast, the earlier FFT example generated PSD estimates ranging from 0 through the sampling frequency.

Bias and Normalization in Welch's Method

In studying the output of `psd` shown earlier, several revealing characteristics about the signal x_n are evident. The noise floor is flat at 0 decibels (dB), implying white noise of variance 1. Furthermore, the “signal” part of x_n is concentrated in two peaks at 50 and 120 Hz. The relation of the peak heights is meaningful. For instance, the 50 Hz peak is 6 dB below the 120 Hz peak, verifying that the higher frequency sinusoid has twice the magnitude as the lower ($10^{6/20} = 2.0$). Unlike the relative heights, the actual height of the peaks does not tell us much about the original amplitude of the sinusoids without some more analysis.

To obtain useful information about the peak amplitudes of the underlying sinusoids, note that the expected value of the estimated PSD is

$$E\{\hat{P}_{xx}(\omega)\} = \frac{1}{2\pi\|w\|^2} \int_{-\pi}^{\pi} P_{xx}(\theta) |W(\omega - \theta)|^2 d\theta$$

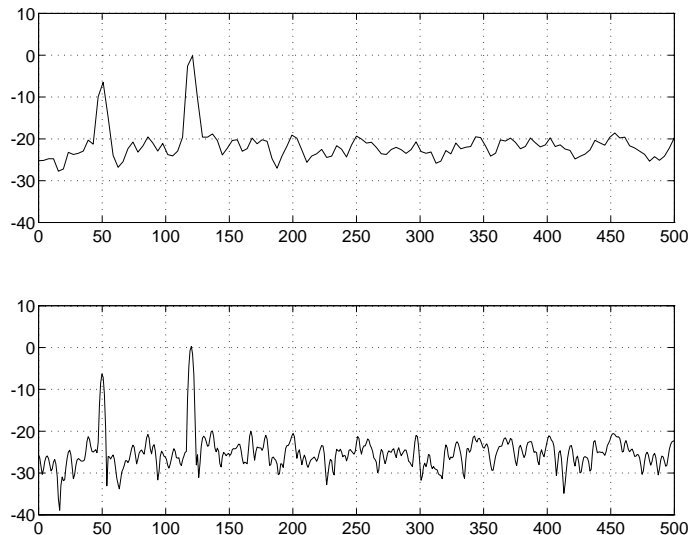
Since the expected value is not equal to the true PSD, the estimate is *biased*. This quantity is the convolution of the true PSD with the squared magnitude of the window's discrete-time Fourier transform $W(\omega)$, scaled by the squared norm of the window. The scaling factor is the sum of the squares of the window function:

$$\|w\|^2 = \sum w(n)^2$$

This says that if $P_{xx}(\omega)$ has a peak of height 1 at a particular frequency ω_0 , the estimate will have approximate height $|W(0)|^2 / \|w\|^2$ at that frequency, provided the window $W(\omega)$ is narrow with respect to the spacing between the peak and other spectral features. So, to obtain an estimate of the height of the original peaks, multiply the result of `psd` by $\text{norm}(w)^2 / \text{sum}(w)^2$, where w is the

window vector. This scaling is independent of window length and shape. For example:

```
w1 = hanning(256); w2 = hanning(500);
[Pxx1, f1] = psd(xn, 256, Fs, w1, 128, 'none');
[Pxx2, f2] = psd(xn, 1024, Fs, w2, 250, 'none');
plot(f1, 10*log10(Pxx1*norm(w1)^2/sum(w1)^2))
plot(f2, 10*log10(Pxx2*norm(w2)^2/sum(w2)^2))
```



In both plots, which show the spectrum at positive frequencies only (the negative frequencies are the same), the higher frequency peak has a value of 0 dB, and the lower frequency peak is at -6 dB. The 120 Hz sinusoid height of 0 dB corresponds to a squared amplitude of 1. This results from the sinusoid of amplitude 2 having complex exponential components of amplitude 1 at both positive and negative frequency. Similarly, the 50 Hz sinusoid has both positive and negative frequency components with squared amplitude of $(1)^2 = 1$, or $10 \cdot \log_{10}(1) = 0$ dB, as shown in the plot. Also, note that the second plot reflects a slightly lower noise floor, which is the result of a longer window length.

Parseval's Relation

According to Parseval's relation, the integral of the PSD across the entire band is a measure of the total energy of the signal. The results of psd can approximately verify this:

```
[Pxx, f] = psd(xn, 256, Fs, 256, 128, 'none');
format long
sum(xn.^2)/length(xn)
```

```
ans =
```

```
3.60376766505040
```

```
sum(Pxx)/length(Pxx)
```

```
ans =
```

```
3.68430077838701
```

To approximate the percentage of energy the signal has in a given frequency band, sum the PSD estimate at the desired frequency points only. The percentage of energy of xn in the band from 40 to 60 Hz is

```
ind = find(f>40 & f<60)
```

```
ind =
```

```
12
```

```
13
```

```
14
```

```
15
```

```
16
```

```
sum(Pxx(ind))/sum(Pxx)
```

```
ans =
```

```
0.13369450463404
```

Cross-Spectral Density Function

To estimate the cross-spectral density of two equal length signals x and y using Welch's method, the csd function forms the periodogram as the product of the FFT of x and the conjugate of the FFT of y. Unlike the real-valued PSD, the

CSD is a complex function. `csd` handles the sectioning, detrending, and windowing of x and y in the same way as the `psd` function:

$$P_{xy} = \text{csd}(x, y, \text{nfft}, F_s, \text{window}, \text{noverlap}, \text{dfлаг})$$

Confidence Intervals

Both the `psd` and `csd` functions can compute confidence intervals. Simply provide an input argument p , which specifies the percentage of the confidence interval:

$$\begin{aligned} [P_{xx}, P_{xxc}, f] &= \text{psd}(x, \text{nfft}, F_s, \text{window}, \text{noverlap}, p, \text{dfлаг}) \\ [P_{xy}, P_{xyc}, f] &= \text{csd}(x, y, \text{nfft}, F_s, \text{window}, \text{noverlap}, p, \text{dfлаг}) \end{aligned}$$

p must be a scalar between 0 and 1. The functions assume chi-squared distributed periodograms of nonoverlapping sections in computing the confidence intervals. (This assumption is valid when the signal is a Gaussian distributed random process.) Provided these assumptions are correct, there is a $p \cdot 100\%$ probability that the confidence interval

$$[P_{xx} - P_{xxc}(:, 1), P_{xx} + P_{xxc}(:, 2)]$$

covers the true PSD. If the sections overlap, the confidence interval is not reliable and the functions display a warning message.

Transfer Function Estimate

One application of Welch's method is nonparametric system identification. Assume that H is a linear, time invariant system, and $x(n)$ and $y(n)$ are the input to and output of H , respectively. Then the PSD of $x(n)$ is related to the CSD of $x(n)$ and $y(n)$ as

$$P_{xy}(\omega) = H(\omega)P_{xx}(\omega)$$

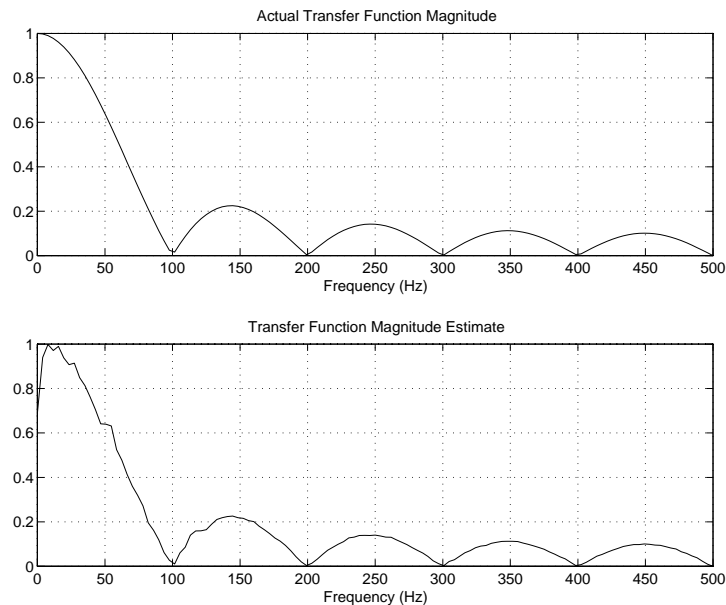
An estimate of the transfer function between $x(n)$ and $y(n)$ is

$$\hat{H}(\omega) = \frac{\hat{P}_{xy}(\omega)}{\hat{P}_{xx}(\omega)}$$

This method estimates both magnitude and phase information. The `tfe` function uses Welch's method to compute the CSD and PSD and then forms their quotient for the transfer function estimate. Use `tfe` the same way that you use the `csd` function.

Filter the signal x_n with an FIR filter, then plot the actual magnitude response and the estimated response:

```
h = ones(1, 10) / 10; % moving average filter
yn = filter(h, 1, xn);
[HEST, f] = tfe(xn, yn, 256, Fs, 256, 128, 'none');
H = freqz(h, 1, f, Fs);
plot(f, abs(H)); plot(f, abs(HEST));
```



Coherence Function

The magnitude-squared coherence between two signals $x(n)$ and $y(n)$ is

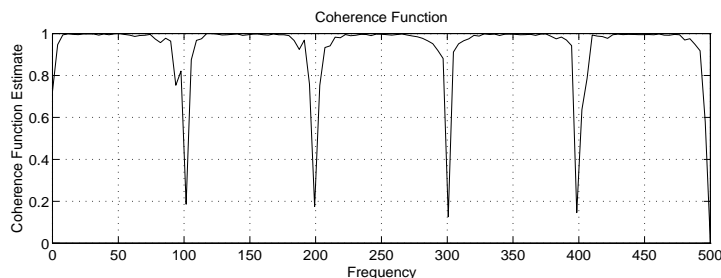
$$C_{xy}(\omega) = \frac{|P_{xy}(\omega)|^2}{P_{xx}(\omega)P_{yy}(\omega)}$$

This quotient is a real number between 0 and 1 that measures the correlation between $x(n)$ and $y(n)$ at the frequency ω .

The `cohere` function takes sequences x and y , computes their PSDs and CSD, and returns the quotient of the magnitude squared of the CSD and the product of the PSDs. Its options and operation are similar to the `csd` and `tfe` functions.

The coherence function of x_n and the filter output y_n versus frequency is

```
cohere(xn, yn, 256, Fs, 256, 128, 'none')
```



If the input sequence length, window length, and overlap are such that `cohere` operates on only a single record, the function returns all ones.

Multitaper Method

The *multitaper method* (MTM) uses orthogonal windows (or “tapers”) to obtain approximately independent estimates of the power spectrum and then combines them to yield an estimate. This estimate exhibits more degrees of freedom and allows for easier quantification of the bias and variance trade-offs, compared to conventional periodogram methods. Many conventional spectral estimates use a single taper (or “window”), with some irretrievable loss of information at the beginning and the end of the series. In the multitaper method, additional tapers are used to recover some of the lost information.

This brief discussion of the multitaper method provides an intuitive look at the algorithm to assist in determining when to use it. For a more detailed and thorough explanation, see Percival and Walden [3].

The simple parameter for the multitaper method is the time-bandwidth product, NW . This parameter is a “resolution” parameter directly related to the number of tapers used to compute the spectrum. There are always $2 \cdot NW - 1$ tapers used to form the estimate. This means that as NW increases, there are more estimates of the power spectrum and the variance of the estimate decreases. However, the bandwidth of each taper is also proportional to NW , so

as NW increases, each estimate exhibits more spectral leakage (i.e., wider peaks) and the overall spectral estimate is more biased. For each data set, there is usually a value for NW that allows an optimal trade-off between bias and variance.

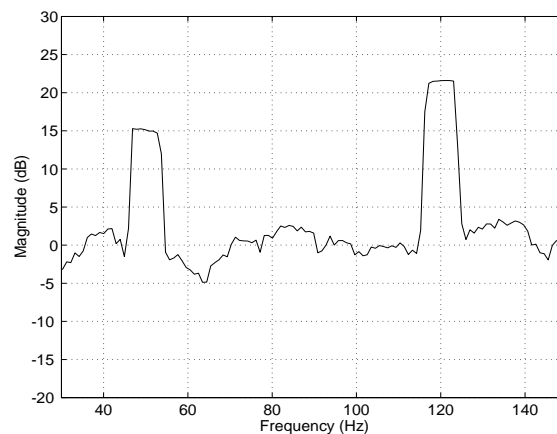
Using `pmtm` on the data from the previous section, `xn`, yields

```

Fs = 1000;
t = 0:1/Fs:1;
randn('seed', 0)
xn = sin(2*pi*50*t) + 2*sin(2*pi*120*t) + randn(size(t));

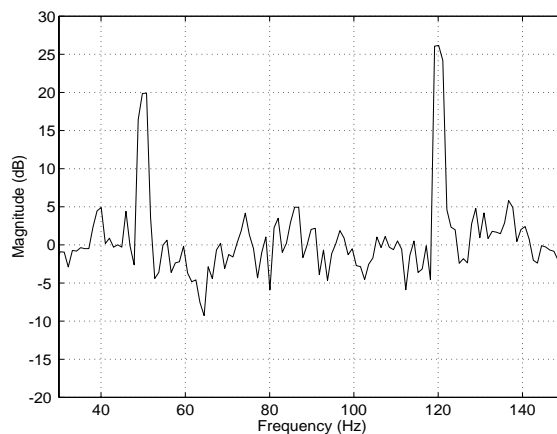
[P, f] = pmtm(xn, 4, 1024, Fs);
plot(f, 10*log10(P))           % plot in decibels
axis([30 150 -20 30])

```



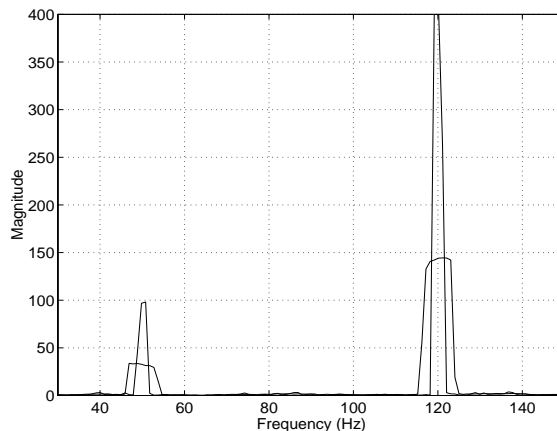
By lowering the time-bandwidth product, the peaks become narrower:

```
[P1, f] = pmtm(xn, 3/2, 1024, Fs);
plot(f, 10*log10(P1))           % plot in decibels
axis([30 150 -20 30])
```



Note that the area under the peaks remains about the same, as can be seen when both are plotted together on a linear scale:

```
plot(f, [P P1])
axis([30 150 0 400])
```



This conservation of total power is verifiable numerically:

```
sum(P)

ans =

    1.8447e+03

sum(P1)

ans =

    1.8699e+03
```

Note that total power is only approximately conserved in this case. This is because the adaptive weighting procedure that is used to minimize leakage does not strictly conserve total power.

This method is more expensive computationally than Welch's method, because of the cost of computing the discrete prolate spheroidal sequences (DPSSs, also known as Slepian sequences). For long data series (10,000 points or more), it is useful to compute the DPSSs once and save them in a MAT-file. The M-files `dpsssave`, `dpssload`, `dpssdir`, and `dpssclear` are provided, to keep a database of saved DPSSs in the MAT-file `dpss.mat`.

Yule-Walker AR Method

The *Yule-Walker AR method* is an autoregressive technique for spectral density estimation (see Marple [2], Chapter 7, and Proakis[4], Section 12.3.2). This method solves for the AR model parameters by the autocorrelation method.

The Yule-Walker AR estimate is obtained by solution of the normal equations:

$$\begin{bmatrix} r(1) & r(2)^* & \cdots & r(n)^* \\ r(2) & r(1) & \cdots & r(n-1)^* \\ \vdots & \vdots & \ddots & \vdots \\ r(n) & r(n-1) & \cdots & r(1) \end{bmatrix} \begin{bmatrix} a(2) \\ a(3) \\ \vdots \\ a(n+1) \end{bmatrix} = \begin{bmatrix} -r(2) \\ -r(3) \\ \vdots \\ -r(n+1) \end{bmatrix}$$

Here, $a = [1 \ a(2) \ \dots \ a(n+1)]$ is a vector of autoregressive coefficients, the elements of vector $r = [r(1) \ r(2) \ \dots \ r(n+1)]$ are correlations, and the left-hand side autocorrelation matrix is Hermitian Toeplitz and positive definite.

The spectral density estimate is

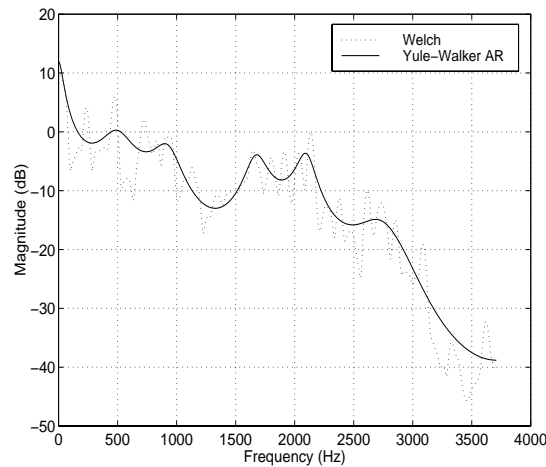
$$P_{YuleAR}(f) = \frac{1}{|a^H \mathbf{e}(f)|^2}$$

where $\mathbf{e}(f)$ is a complex sinusoid.

The toolbox function `pyulear` implements the Yule-Walker AR method.

For example, compare the spectrum of a speech signal using Welch's method and Yule-Walker AR:

```
load mtlb
[P1, f] = psd(mtlb, 1024, Fs, 256);
[P2, f] = pyulear(mtlb, 14, 1024, Fs); % 14th order model
plot(f, 10*log10(P1), ':', f, 10*log10(P2))
legend('Welch', 'Yule-Walker AR')
```



The solid Yule-Walker AR spectrum is smoother than the periodogram because of the simple underlying all-pole model.

Burg Method

The Burg method for AR spectral estimation is based on minimizing the forward and backward prediction errors while satisfying the Levinson-Durbin recursion (see Marple[2], Chapter 7, and Proakis[4], Section 12.3.3). In

contrast to other AR estimation methods, the Burg method avoids calculating the autocorrelation function, and instead estimates the reflection coefficients directly.

The primary advantages of the Burg method are resolving closely spaced sinusoids in signals with low noise levels, and estimating short data records, in which case the AR power spectral density estimates are very close to the true values. In addition, the Burg method ensures a stable AR model and is computationally efficient.

The accuracy of the Burg method is lower for high-order models, long data records, and high signal-to-noise ratios (which can cause *line splitting*, or the generation of extraneous peaks in the spectrum estimate). The spectral density estimate computed by the Burg method is also susceptible to frequency shifts (relative to the true frequency) resulting from the initial phase of noisy sinusoidal signals. This effect is magnified when analyzing short data sequences.

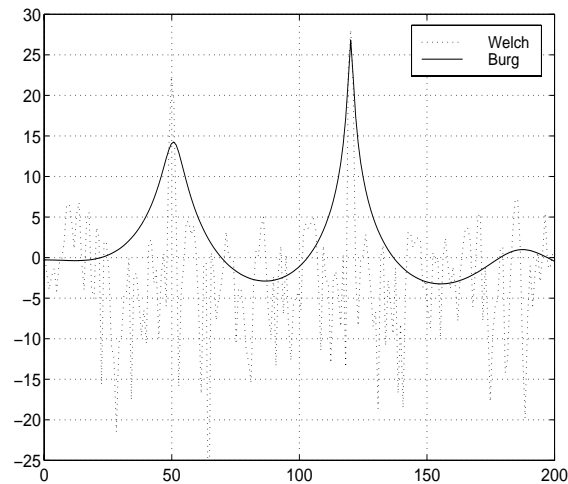
The toolbox function `pburg` implements the Burg method.

Compare the spectrum of a noisy signal computed using the Burg method and the Welch method:

```

Fs = 1000;
t = 0: 1/Fs: 1;
xn = sin(2*pi*50*t) + 2*sin(2*pi*120*t) + randn(size(t));
[P1, f] = psd(xn, 1024, Fs);
[P2, f] = pburg(xn, 17, 1024, Fs); % 17th order model
plot(f, 10*log10(P1), ':', f, 10*log10(P2)), grid
axis([0 200 -25 30])
legend('Welch', 'Burg')

```



Note that as the model order for the Burg method is reduced, a frequency shift due to the initial phase of the sinusoids will become apparent.

MUSIC and Eigenvector Analysis Methods

The `pmusic` function provides two related spectral analysis methods:

- The multiple signal classification method (MUSIC) developed by Schmidt
- The eigenvector (EV) method developed by Johnson

See Marple [2] (pgs. 373-378) for a summary of these methods.

Both of these methods are frequency estimator techniques based on eigenanalysis of the autocorrelation matrix. This type of spectral analysis

categorizes the information in a correlation or data matrix, assigning information to either a signal subspace or a noise subspace.

Eigenanalysis Overview

Consider a number of complex sinusoids embedded in white noise. You can write the autocorrelation matrix R for this system as the sum of the signal autocorrelation matrix (S) and the noise autocorrelation matrix (W).

$$R = S + W$$

There is a close relationship between the eigenvectors of the signal autocorrelation matrix and the signal and noise subspaces. The eigenvectors \mathbf{v} of S span the same signal subspace as the signal vectors. If the system contains M complex sinusoids and the order of the autocorrelation matrix is p , eigenvectors \mathbf{v}_{M+1} through \mathbf{v}_{p+1} span the noise subspace of the autocorrelation matrix.

Frequency Estimator Functions. To generate their frequency estimates, eigenanalysis methods calculate functions of the vectors in the signal and noise subspaces. Both the MUSIC and EV techniques choose a function that theoretically goes to infinity at one of the sinusoidal frequencies in the input signal. Using digital technology, the resulting estimate has sharp peaks at the frequencies of interest; this means that there won't be infinity values in the vectors.

The MUSIC estimate is given by the formula

$$P_{music}(f) = \frac{1}{\mathbf{e}^H(f) \left(\sum_{k=p+1}^N \mathbf{v}_k \mathbf{v}_k^H \right) \mathbf{e}(f)} = \frac{1}{\sum_{k=p+1}^N |\mathbf{v}_k^H \mathbf{e}(f)|^2}$$

where N is the size of the eigenvectors and $\mathbf{e}(f)$ is a complex sinusoid vector:

$$\mathbf{e}(f) = [1 \exp(j2\pi f) \exp(j2\pi f \cdot 2) \exp(j2\pi f \cdot 4) \dots \exp(j2\pi f \cdot (n-1))]^H$$

\mathbf{v} represents the eigenvectors of the input signal's correlation matrix; \mathbf{v}_k is the k^{th} eigenvector. H is the conjugate transpose operator. The eigenvectors used in the sum correspond to the smallest eigenvalues and span the noise subspace (p is the size of the signal subspace).

The expression

$$\mathbf{v}_k^H \mathbf{e}(f)$$

is equivalent to a Fourier transform (the vector $\mathbf{e}(f)$ consists of complex exponentials). This form is useful for numeric computation because the FFT can be computed for each \mathbf{v}_k and then the squared magnitudes can be summed.

The EV method weights the summation by the eigenvalues of the correlation matrix:

$$P_{ev}(f) = \frac{1}{\left(\sum_{k=p+1}^N |\mathbf{v}_k^H \mathbf{e}(f)|^2 \right) / \lambda_k}$$

The `pmusic` function in this toolbox uses the `svd` (singular value decomposition) function in the signal case and the `eig` function for analyzing the correlation matrix and assigning eigenvectors to the signal or noise subspaces. When `svd` is used, `pmusic` never computes the correlation matrix explicitly, but the singular values are the eigenvalues.

Controlling Subspace Thresholds

To provide user control over the assignments of eigenvectors to the signal and noise subspaces, the `pmusic` function accepts a threshold argument `thresh`. `thresh` is a two-element vector where the first element is the number of eigenvectors spanning the signal subspace and the second element is a threshold test:

- If `thresh(2) ≤ 1`, then `thresh(1)` specifies the number of eigenvectors spanning the signal subspace. In this case the values of `thresh(1)` must be in the range $[0, N)$, where N is
 - The column length of `xR` if `xR` is a data matrix
 - The matrix size if `xR` is a correlation matrix
 - The window length if `xR` is a signal vector
- If `thresh(1) ≥ N`, then `thresh(2)` is a value greater than or equal to 1 that specifies the absolute threshold for splitting the eigenvalues between the signal and noise subspaces. That is, if a given eigenvalue is less than or equal

to the product $\text{thresh}(2) \min\{\lambda_k\}$, then the given eigenvector is assigned to the noise subspace.

- If $\text{thresh}(1) < N$ and $\text{thresh}(2) \geq 1$, $\text{thresh}(1)$ still specifies the maximum number of eigenvectors in the signal subspace. However, the threshold test specified by $\text{thresh}(2)$ can also assign eigenvectors to the noise subspace.
- If $\text{thresh}(1) \geq N$ and $\text{thresh}(2) < 1$, there are no noise eigenvectors. This is an invalid case and `pmusic` generates an error.

For complete details on using the `thresh` parameter, see the reference description of `pmusic` in Chapter 6.

References

- 1 Kay, S.M. *Modern Spectral Estimation*. Englewood Cliffs, NJ: Prentice Hall, 1988.
- 2 Marple, S.L. *Digital Spectral Analysis*. Englewood Cliffs, NJ: Prentice Hall, 1987.
- 3 Percival, D.B., and A.T. Walden. *Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques*. Cambridge: Cambridge University Press, 1993.
- 4 Proakis, J.G., and D.G. Manolakis. *Digital Signal Processing: Principles, Algorithms, and Applications*. Englewood Cliffs, NJ: Prentice Hall, 1996.
- 5 Welch, P.D. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms." *IEEE Trans. Audio Electroacoust.* Vol. AU-15 (June 1967). Pgs. 70-73.

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Windows

In both digital filter design and power spectrum estimation, the choice of a windowing function can play an important role in determining the quality of overall results. The main role of the window is to damp out the effects of the Gibbs phenomenon that results from truncation of an infinite series.

The toolbox window functions are

Window	Function
Bartlett window	<code>bartlett</code>
Blackman window	<code>blackman</code>
Rectangular window	<code>boxcar</code>
Chebyshev window	<code>chebwin</code>
Hamming window	<code>hamming</code>
Hanning window	<code>hanning</code>
Kaiser window	<code>kaiser</code>
Triangular window	<code>triang</code>

Basic Shapes

The basic window is the *rectangular window*, a vector of ones of the appropriate length. A rectangular window of length 50 is

```
n = 50;
w = boxcar(n);
```

This toolbox stores windows in column vectors by convention, so an equivalent expression is

```
w = ones(50, 1);
```

The *Bartlett* (or triangular) *window* is the convolution of two rectangular windows. The functions `bartlett` and `triang` compute similar triangular windows, with three important differences. The `bartlett` function always

returns a window with two zeros on the ends of the sequence, so that for n odd, the center section of `bartlett(n+2)` is equivalent to `triang(n)`:

```
bartlett(7)
```

```
ans =
```

```

    0
0.3333
0.6667
1.0000
0.6667
0.3333
    0
```

```
triang(5)
```

```
ans =
```

```

0.3333
0.6667
1.0000
0.6667
0.3333
```

For n even, `bartlett` is still the convolution of two rectangular sequences. There is no standard definition for the triangular window for n even; the slopes of the line segments of `triang`'s result are slightly steeper than those of `bartlett`'s in this case:

```
w = bartlett(8);
[w(2:7) triang(6)]
```

```
ans =
```

```

0.2857    0.1667
0.5714    0.5000
0.8571    0.8333
0.8571    0.8333
0.5714    0.5000
0.2857    0.1667
```

The final difference between the Bartlett and triangular windows is evident in the Fourier transforms of these functions. The Fourier transform of a Bartlett

window is negative for n even. The Fourier transform of a triangular window, however, is always nonnegative.

This difference can be important when choosing a window for some spectral estimation techniques, such as the Blackman-Tukey method. Blackman-Tukey forms the spectral estimate by calculating the Fourier transform of the autocorrelation sequence. The resulting estimate might be negative at some frequencies if the window's Fourier transform is negative (see Kay [1], pg. 80).

Generalized Cosine Windows

Blackman, Hamming, Hanning, and rectangular windows are all special cases of the *generalized cosine window*. These windows are combinations of sinusoidal sequences with frequencies 0, $2\pi/(N-1)$, and $4\pi/(N-1)$, where N is the window length. One way to generate them is

$$\begin{aligned} \text{ind} &= (0:n-1)' * 2 * \pi / (n-1); \\ w &= A - B * \cos(\text{ind}) + C * \cos(2 * \text{ind}); \end{aligned}$$

where A , B , and C are constants you define. The concept behind these windows is that by summing the individual terms to form the window, the low frequency peaks in the frequency domain combine in such a way as to decrease sidelobe height. This has the side effect of increasing the mainlobe width.

The Hamming and Hanning windows are two-term generalized cosine windows, given by $A = 0.54$, $B = 0.46$ for Hamming and $A = 0.5$, $B = 0.5$ for Hanning ($C = 0$ in both cases). The `hamming` and `hanning` functions, respectively, compute these windows.

Note that the definition of the generalized cosine window shown in the earlier MATLAB code yields zeros at samples 1 and n for $A = 0.5$ and $B = 0.5$. To eliminate these zeros on the edges of the window, `hanning` uses a cosine of frequency $2\pi/(N+1)$ instead of $2\pi/(N-1)$.

The Blackman window is a popular three-term window, given by $A = 0.42$, $B = 0.5$, $C = 0.08$. The `blackman` function computes this window.

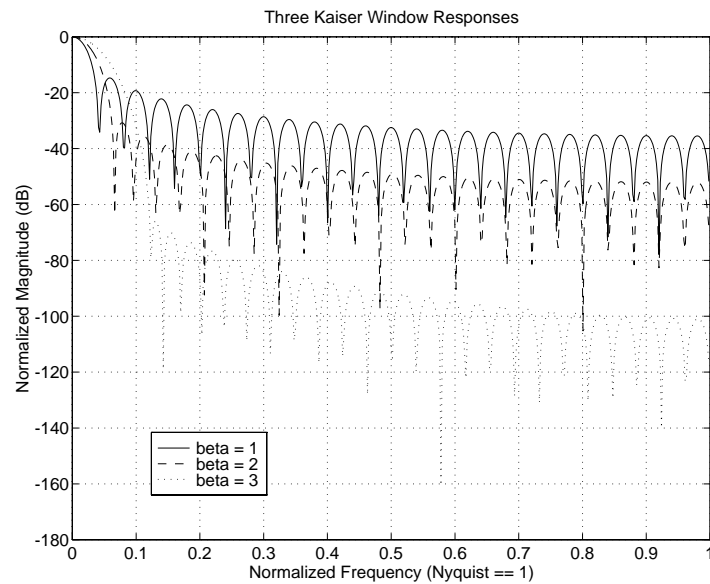
Kaiser Window

The *Kaiser window* is an approximation to the prolate-spheroidal window, for which the ratio of the mainlobe energy to the sidelobe energy is maximized. For a Kaiser window of a particular length, the parameter β controls the sidelobe height. For a given β , the sidelobe height is fixed with respect to window length.

The statement `kaiser(n, beta)` computes a length `n` Kaiser window with parameter `beta`.

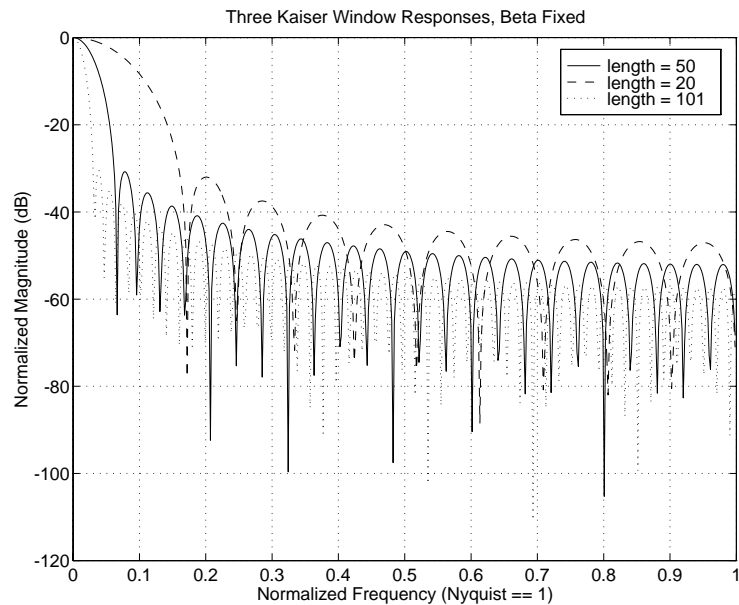
Examples of Kaiser windows with length 50 and various values for the `beta` parameter are

```
n = 50;
w1 = kaiser(n, 1);
w2 = kaiser(n, 4);
w3 = kaiser(n, 9);
[W1, f] = freqz(w1/sum(w1), 1, 512, 2);
[W2, f] = freqz(w2/sum(w2), 1, 512, 2);
[W3, f] = freqz(w3/sum(w3), 1, 512, 2);
plot(f, 20*log10(abs([W1 W2 W3])))
```



As β increases, the sidelobe height decreases and the mainlobe width increases. To see how the sidelobe height stays the same for a fixed β parameter as the length is varied, try

```
w1 = kaiser(50, 4);
w2 = kaiser(20, 4);
w3 = kaiser(101, 4);
[W1, f] = freqz(w1/sum(w1), 1, 512, 2);
[W2, f] = freqz(w2/sum(w2), 1, 512, 2);
[W3, f] = freqz(w3/sum(w3), 1, 512, 2);
plot(f, 20*log10(abs([W1 W2 W3])))
```



Kaiser Windows in FIR Design

There are two design formulas that can help you design FIR filters to meet a set of filter specifications using a Kaiser window. To achieve a sidelobe height of $-\alpha$ dB, the beta parameter is

$$\beta = \begin{cases} 0.1102(\alpha - 8.7), & \alpha > 50 \\ 0.5842(\alpha - 21)^{0.4} + 0.07886(\alpha - 21), & 50 \geq \alpha \geq 21 \\ 0, & \alpha < 21 \end{cases}$$

For a transition width of $\Delta\omega$ rad/sec, use the length

$$n = \frac{\alpha - 8}{2.285\Delta\omega} + 1$$

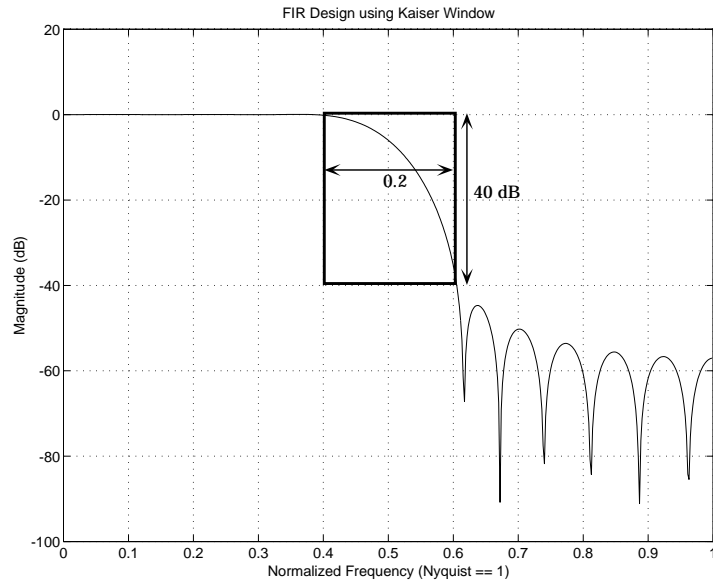
Filters designed using these heuristics will meet the specifications approximately, but you should verify this. To design a lowpass filter with cutoff frequency 0.5π rad/sec, transition width 0.2π rad/sec, and 40 dB of attenuation in the stopband, try

```
[n, wn, beta] = kaiserord([0.4 0.6]*pi, [1 0], [0.01 0.01], 2*pi);
h = fir1(n, wn, kaiser(n+1, beta), 'noscale');
```

The `kaiserord` function estimates the filter order, cutoff frequency, and Kaiser window beta parameter needed to meet a given set of frequency domain specifications.

The ripple in the passband is roughly the same as the ripple in the stopband. As you can see from the frequency response, this filter nearly meets the specifications:

```
[H, f] = freqz(h, 1, 512, 2);  
plot(f, 20*log10(abs(H))), grid
```

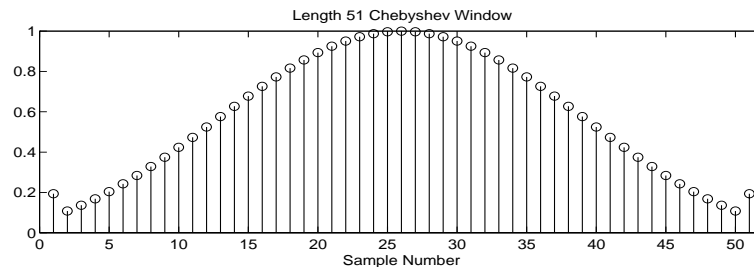


For details on `kaiserord`, see the reference description in Chapter 6.

Chebyshev Window

The Chebyshev window minimizes the mainlobe width, given a particular sidelobe height. It is characterized by an equiripple behavior, that is, its sidelobes all have the same height. The `chebwin` function, with length and sidelobe height parameters, computes a Chebyshev window:

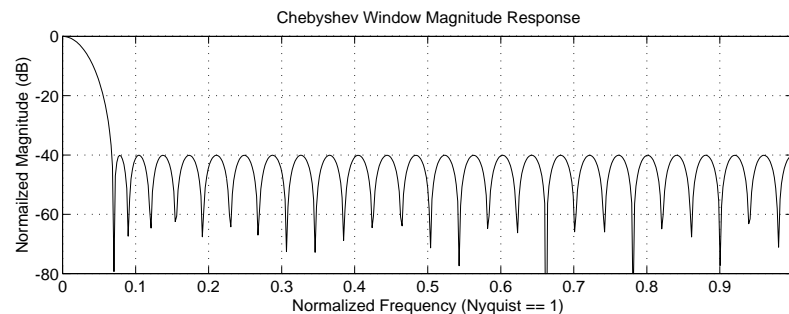
```
n = 51;
Rs = 40; % sidelobe height in decibels
w = chebwin(n, Rs);
stem(w)
```



As shown in the plot, the Chebyshev window has large spikes at its outer samples.

Plot the frequency response to see the equiripples at -40 dB:

```
[W, f] = freqz(w, 1, 512, 2);
plot(f, 20*log10(abs(W)/sum(w))), grid
```



For a detailed discussion of the characteristics and applications of the various window types, see Oppenheim and Schaffer [2], pgs. 444-462, and Parks and Burrus [3], pgs. 71-73.

Parametric Modeling

Parametric modeling techniques find the parameters for a mathematical model describing a signal, system, or process. These techniques use known information about the system to determine the model. Applications for parametric modeling include speech and music synthesis, data compression, high-resolution spectral estimation, communications, manufacturing, and simulation.

The toolbox parametric modeling functions operate with the rational transfer function model. Given appropriate information about an unknown system (impulse or frequency response data, or input and output sequences), these functions find the coefficients of a linear system that models the system.

One important application of the parametric modeling functions is in the design of filters that have a prescribed time or frequency response. These functions provide a data-oriented alternative to the IIR and FIR filter design functions discussed in Chapter 2.

Here is a summary of the parametric modeling functions in this toolbox. Note that the System Identification Toolbox provides a more extensive collection of parametric modeling functions.

Domain	Functions	Description
Time	lpc, levinson	Linear Predictive Coding. Generate all-pole recursive filter whose impulse response matches a given sequence.
	prony	Generate IIR filter whose impulse response matches a given sequence.
	stmcb	Find IIR filter whose output, given a specified input sequence, matches a given output sequence.
Frequency	invfreqz, invfreqs	Generate digital or analog filter coefficients given complex frequency response data.
	pburg	Generate IIR filter coefficients that model an input data sequence using the Levinson-Durbin algorithm (see Chapter 3).

Domain	Functions	Description
	<code>pyul ear</code>	Generate IIR filter coefficients that model an input data sequence using an estimate of the autocorrelation function (see Chapter 3).
	<code>yul ewal k</code>	Generate IIR filter that matches piecewise linear magnitude response by solving modified Yule-Walker equations (see Chapter 2).

Because `yul ewal k` is geared explicitly toward ARMA filter design, it is discussed in Chapter 2. `pburg` and `pyul ear` are discussed in Chapter 3 along with the other (nonparametric) spectral estimation methods.

Time-Domain Based Modeling

The `l pc`, `prony`, and `stmcb` functions find the coefficients of a digital rational transfer function that approximates a given time-domain impulse response. `l pc` is restricted to all-pole models, and `stmcb` can accept an input (besides an impulse) that causes the output response. The algorithms differ in complexity and accuracy of the resulting model.

Linear Prediction (AR Modeling)

Linear prediction models a given signal x as the impulse response of an all-pole filter. It assumes that each output sample of a signal, $x(k)$, is a linear combination of the past n outputs (that is, it can be “linearly predicted” from these outputs), and that the coefficients are constant from sample to sample:

$$x(k) = -a(2)x(k-1) - a(3)x(k-2) - \cdots - a(n+1)x(k-n)$$

An n th-order all-pole model of a signal x is

$$a = \text{l pc}(x, n)$$

`l pc` uses the autocorrelation method of all-pole modeling to find the linear prediction coefficients. This technique is also called the Yule-Walker AR method of spectral analysis. The filter generated is stable, but it might not model the process exactly even if the data sequence is truly an autoregressive (AR) process of the correct order. This is because the autocorrelation method implicitly windows the data, that is, it assumes that signal samples beyond the length of x are 0.

To illustrate `lpc`, create a sample signal that is the impulse response of an all-pole filter with additive white noise:

```
randn('seed', 0)
x = impz(1, [1 0.1 0.1 0.1 0.1], 10) + randn(10, 1)/10;
```

The coefficients for a fourth-order all-pole filter that models the system are

```
a = lpc(x, 4)

a =
    1.0000    0.0395    0.0338    0.0668    0.1264
```

`lpc` first calls `xcorr` to find a biased estimate of the correlation function of `x`, and then uses the Levinson-Durbin recursion, implemented in the `levinson` function, to find the model coefficients `a`. The Levinson-Durbin recursion is a fast algorithm for solving a system of symmetric Toeplitz linear equations. `lpc`'s entire algorithm for `n = 4` is

```
r = xcorr(x);
r(1:length(x)-1) = []; % remove corr. at negative lags
a = levinson(r, 4)

a =
    1.0000    0.0395    0.0338    0.0668    0.1264
```

You could form the linear prediction coefficients with other assumptions by passing a different correlation estimate to `levinson`, such as the unbiased correlation estimate:

```
r = xcorr(x, 'unbiased');
r(1:length(x)-1) = []; % remove corr. at negative lags
a = levinson(r, 4)

a =
    1.0000    0.0554    0.0462    0.0974    0.2115
```

Prony's Method (ARMA Modeling)

The `prony` function models a signal using a specified number of poles and zeros. Given a sequence `x` and numerator and denominator orders `nb` and `na`, respectively, the statement

```
[b, a] = prony(x, nb, na)
```

finds the numerator and denominator coefficients of an IIR filter whose impulse response approximates the sequence x .

`prony` implements the method described in Parks and Burrus [3] (pgs. 226-228). This method uses a variation of the covariance method of AR modeling to find the denominator coefficients a , and then finds the numerator coefficients b for which the resulting filter's impulse response matches exactly the first $n_b + 1$ samples of x . The filter is not necessarily stable, but it can potentially recover the coefficients exactly if the data sequence is truly an autoregressive moving average (ARMA) process of the correct order.

NOTE The functions `prony` and `stmcb` (described next) are more accurately described as ARX models in system identification terminology. ARMA modeling assumes noise only at the inputs, while ARX assumes an external input. `prony` and `stmcb` know the input signal: it is an impulse for `prony` and is arbitrary for `stmcb`.

A model for the test sequence x (from the earlier `lpc` example) using a third-order IIR filter is

```
[b, a] = prony(x, 3, 3)
```

```
b =
```

```
1.1165    -0.2181    -0.6084    0.5369
```

```
a =
```

```
1.0000    -0.1619    -0.4765    0.4940
```

The `impz` command shows how well this filter's impulse response matches the original sequence:

```
format long
[x impz(b, a, 10)]

ans =

    1.11649535105007    1.11649535105007
   -0.03731609173676   -0.03731609173676
   -0.08249198453223   -0.08249198453223
   -0.04583930972315   -0.04583930972315
   -0.14255125351637   -0.02829072973977
    0.20400424807471    0.01433198229497
    0.02685697779814    0.01148698991026
    0.18956307836948    0.02266475846451
    0.02717716288172    0.00206242734272
    0.08057060786906    0.00545783754743
```

Notice that the first four samples match exactly. For an example of exact recovery, recover the coefficients of a Butterworth filter from its impulse response:

```
[b, a] = butter(4, .2);
h = impz(b, a, 26);
[bb, aa] = prony(h, 4, 4);
```

Try this example; you'll see that `bb` and `aa` match the original filter coefficients to within a tolerance of 10^{-13} .

Steiglitz-McBride Method (ARMA Modeling)

`stmcb` determines the coefficients for the system $b(z)/a(z)$ given an approximate impulse response `x`, as well as the desired number of zeros and poles. This function identifies an unknown system based on both input and output

sequences that describe the system's behavior, or just the impulse response of the system. In its default mode, `stmcb` works like prony:

```
[b, a] = stmcb(x, 3, 3)

b =
    1.1165    -0.6213    -0.8365     1.3331

a =
    1.0000    -0.5401    -0.6109     1.1298
```

`stmcb` also finds systems that match given input and output sequences:

```
y = filter(1, [1 1], x); % Create an output signal.
[b, a] = stmcb(y, x, 0, 1)

b =
     1

a =
     1     1
```

In this example, `stmcb` correctly identifies the system used to create `y` from `x`.

The Steiglitz-McBride method is a fast iterative algorithm that solves for the numerator and denominator coefficients simultaneously in an attempt to minimize the signal error between the filter output and the given output signal. This algorithm usually converges rapidly, but might not converge if the model order is too large. As for prony, `stmcb`'s resulting filter is not necessarily stable due to its exact modeling approach.

`stmcb` provides control over several important algorithmic parameters; modify these parameters if you are having trouble modeling the data. To change the number of iterations from the default of five and provide an initial estimate for the denominator coefficients:

```
n = 10; % number of iterations
a = lpc(x, 3); % initial estimates for denominator
[b, a] = stmcb(x, 3, 3, n, a);
```

The function uses an all-pole model created with prony as an initial estimate when you do not provide one of your own.

To compare the functions `lpc`, `prony`, and `stmcb`, compute the signal error in each case:

```
a1 = lpc(x, 3);
[b2, a2] = prony(x, 3, 3);
[b3, a3] = stmcb(x, 3, 3);
[ x-impz(1, a1, 10)  x-impz(b2, a2, 10)  x-impz(b3, a3, 10) ]

ans =

    0.1165         0         0
   -0.0058         0   -0.0190
   -0.0535    0.0000    0.0818
    0.0151   -0.0000   -0.0176
   -0.1473   -0.1143   -0.0476
    0.2005    0.1897    0.0869
    0.0233    0.0154   -0.0103
    0.1901    0.1669   -0.0093
    0.0275    0.0251    0.0294
    0.0808    0.0751    0.0022

sum(ans.^2)

ans =

    0.1226    0.0834    0.0182
```

In comparing modeling capabilities for a given order IIR model, the last result shows that for this example, `stmcb` performs best, followed by `prony`, then `lpc`. This relative performance is typical of the modeling functions.

Frequency-Domain Based Modeling

The `invfreqs` and `invfreqz` functions implement the inverse operations of `freqs` and `freqz`; they find an analog or digital transfer function of a specified order that matches a given complex frequency response. Though the following examples demonstrate `invfreqz`, the discussion also applies to `invfreqs`.

To recover the original filter coefficients from the frequency response of a simple digital filter:

```
[b, a] = butter(4, .4)    % design Butterworth lowpass
b =
    0.0466    0.1863    0.2795    0.1863    0.0466
a =
    1.0000   -0.7821    0.6800   -0.1827    0.0301
[h, w] = freqz(b, a, 64);    % compute frequency resp.
[bb, aa] = invfreqz(h, w, 4, 4) % model: nb = 4, na = 4
bb =
    0.0466    0.1863    0.2795    0.1863    0.0466
aa =
    1.0000   -0.7821    0.6800   -0.1827    0.0301
```

The vector of frequencies w has the units in radians/second, and the frequencies need not be equally spaced. `invfreqz` finds a filter to fit the frequency data for any order filter; a third-order example is

```
[bb, aa] = invfreqz(h, w, 3, 3)    % find third-order IIR
bb =
    0.0464    0.1785    0.2446    0.1276
aa =
    1.0000   -0.9502    0.7382   -0.2006
```

Both `invfreqs` and `invfreqz` design filters with real coefficients; for a data point at positive frequency f , the functions fit the frequency response at both f and $-f$.

By default `invfreqz` uses an equation error method to identify the best model from the data. This finds b and a in

$$\min_{b,a} \sum_{k=1}^n wt(k) |h(k)A(w(k)) - B(w(k))|^2$$

by creating a system of linear equations and solving them with MATLAB's \ operator. Here $A(w(k))$ and $B(w(k))$ are the Fourier transforms of the polynomials a and b respectively at the frequency $w(k)$, and n is the number of frequency points (the length of h and w). $wt(k)$ weights the error relative to the error at different frequencies. The syntax

```
invfreqz(h, w, nb, na, wt)
```

includes a weighting vector. In this mode, the filter resulting from `invfreqz` is not guaranteed to be stable.

`invfreqz` provides a superior ("output-error") algorithm that solves the direct problem of minimizing the weighted sum of the squared error between the actual frequency response points and the desired response:

$$\min_{b,a} \sum_{k=1}^n wt(k) \left| h(k) - \frac{B(w(k))}{A(w(k))} \right|^2$$

To use this algorithm, specify a parameter for the iteration count after the weight vector parameter:

```
wt = ones(size(w)); % create unity weighting vector
[bbb, aaa] = invfreqz(h, w, 3, 3, wt, 30) % 30 iterations
```

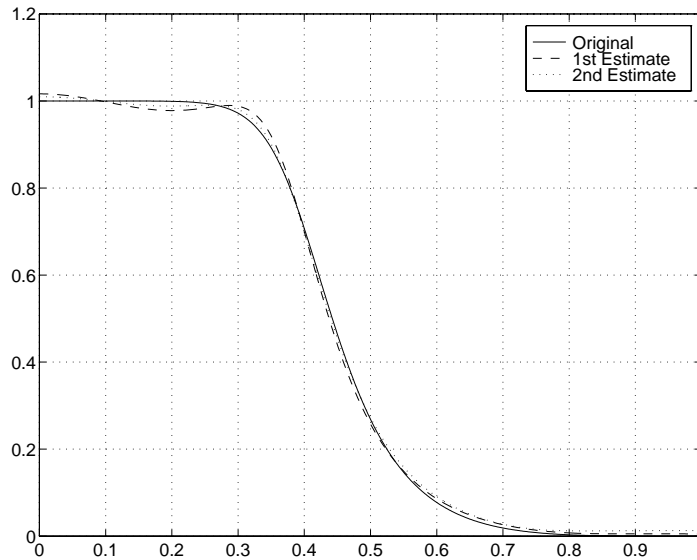
```
bbb =
```

```
    0.0464    0.1829    0.2572    0.1549
```

```
aaa =
```

```
    1.0000   -0.8664    0.6630   -0.1614
```

The resulting filter is always stable. Graphically compare the results of the first and second algorithms to the original Butterworth filter:



To verify the superiority of the fit numerically:

```
sum(abs(h-freqz(bb, aa, w)).^2) % total error, algorithm 1
```

```
ans =
```

```
0.0200
```

```
sum(abs(h-freqz(bbb, aaa, w)).^2) % total error, algorithm 2
```

```
ans =
```

```
0.0096
```

Resampling

The toolbox provides a number of functions that resample a signal at a higher or lower rate:

Operation	Function
Resample at new rate	resample
Decimation	decimate
Interpolation	interp
Apply FIR filter with resampling	upfirdn
Cubic spline interpolation	spline
Other 1-D interpolation	interp1

The `resample` function changes the sampling rate for a sequence to any rate that is a ratio of two integers. The basic syntax for `resample` is

$$y = \text{resample}(x, p, q)$$

where the function resamples the sequence x at p/q times the original sampling rate. The length of the result y is p/q times the length of x .

One resampling application is the conversion of digitized audio signals from one sampling rate to another, such as from 48 kHz (the Digital Audio Tape standard) to 44.1 kHz (the Compact Disc standard). In the next example, the sampling rates are different but the idea is the same.

The example file contains a length 4001 vector of speech sampled at 7418 Hz:

```
clear
load mtlb
whos
```

Name	Size	Bytes	Class
Fs	1x1	8	double array
mtlb	4001x1	32008	double array

Grand total is 4002 elements using 32016 bytes

```
Fs
Fs =
    7418
```

To play this speech signal on a workstation that can only play sound at 8192 Hz, use the `rat` function to find integers p and q that yield the correct resampling factor:

```
[p, q] = rat(8192/Fs, .0001)
p =
    127
q =
    115
```

Since $p/q \cdot F_s = 8192.05$ Hz, the tolerance of 0.0001 is acceptable; to resample the signal at very close to 8192 Hz:

```
y = resample(mtlb, p, q);
```

`resample` applies a lowpass filter to the input sequence to prevent aliasing during resampling. It designs this filter using the `fir1` function with a Kaiser window. The syntax

```
resample(x, p, q, l, beta)
```

controls the filter's length and the beta parameter of the Kaiser window. Alternatively, use the function `intfilt` to design an interpolation filter b and use it with

```
resample(x, p, q, b)
```

The `decimate` and `interp` functions do the same thing as `resample` with $p = 1$ and $q = 1$, respectively. These functions provide different anti-alias filtering options, and they incur a slight signal delay due to filtering. The `interp` function is significantly less efficient than the `resample` function with $q = 1$.

The toolbox also contains a function, `upfirdn`, that applies an FIR filter to an input sequence and outputs the filtered sequence at a different sample rate than its original rate. See “Multirate Filter Bank Implementation” on page 1-19 and the reference description of `upfirdn` in Chapter 6 for more details.

The standard MATLAB environment contains a function, `spline`, that works with irregularly spaced data. The MATLAB function `interp1` performs interpolation, or table lookup, using various methods including linear and cubic interpolation. See the online *MATLAB Function Reference* for information on `spline` and `interp1`.

Cepstrum Analysis

Cepstrum analysis is a nonlinear signal processing technique with a variety of applications in areas such as speech and image processing. The Signal Processing Toolbox provides three functions for cepstrum analysis:

Operation	Function
Complex cepstrum	cceps
Real cepstrum	rceps
Inverse complex cepstrum	i cceps

The complex cepstrum for a sequence x is calculated by finding the complex natural logarithm of the Fourier transform of x , then the inverse Fourier transform of the resulting sequence:

$$\hat{x} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left[X(e^{j\omega}) \right] e^{j\omega n} d\omega$$

The toolbox function `cceps` performs this operation, estimating the complex cepstrum for an input sequence. It returns a real sequence the same size as the input sequence,

```
xhat = cceps(x)
```

The complex cepstrum transformation is central to the theory and application of *homomorphic systems*, that is, systems that obey certain general rules of superposition. See Oppenheim and Schaffer [2] for a discussion of the complex cepstrum and homomorphic transformations, with details on speech processing applications.

Try using `cceps` in an echo detection application. First, create a 45 Hz sine wave sampled at 100 Hz:

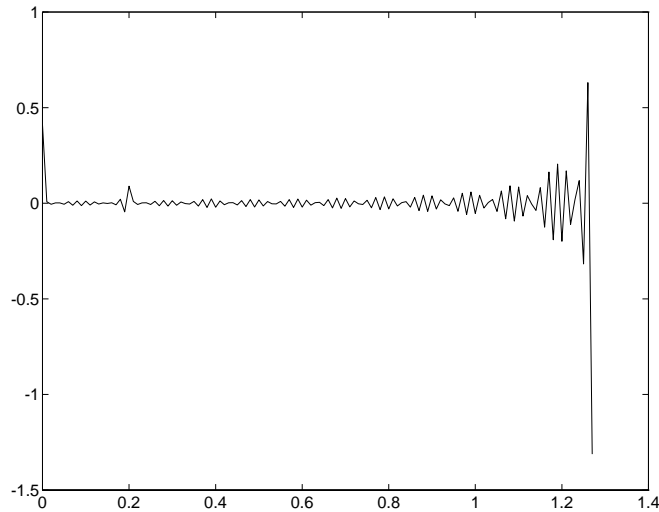
```
t = 0:0.01:1.27;
s1 = sin(2*pi*45*t);
```

Add an echo of the signal, with half the amplitude, 0.2 seconds after the beginning of the signal:

```
s2 = s1 + 0.5*[zeros(1,20) s1(1:108)];
```

The complex cepstrum of this new signal is

```
c = cceps(s2);
plot(t, c)
```



Note that the complex cepstrum shows a peak at 0.2 seconds, indicating the echo.

The *real cepstrum* of a signal x , sometimes called simply the cepstrum, is calculated by determining the natural logarithm of magnitude of the Fourier transform of x , then obtaining the inverse Fourier transform of the resulting sequence [2]:

$$c_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log |X(e^{j\omega})| e^{j\omega n} d\omega$$

The toolbox function `rceps` performs this operation, returning the real cepstrum for a sequence x . The returned sequence is a real-valued vector the same size as the input vector:

```
y = rceps(x)
```

By definition, you cannot reconstruct the original sequence from its real cepstrum transformation, as the real cepstrum is based only on the magnitude of the Fourier transform for the sequence (see [2]). The `rceps` function,

however, can reconstruct a minimum-phase version of the original sequence by applying a windowing function in the cepstral domain. To obtain both the real cepstrum and the minimum phase reconstruction for a sequence, use

```
[y, ym] = rceps(x)
```

where *y* is the real cepstrum and *ym* is the minimum phase reconstruction of *x*.

Inverse Complex Cepstrum

To invert the complex cepstrum, use the `icceps` function. Inversion is complicated by the fact that the `cceps` function performs a data dependent phase modification so that the unwrapped phase of its input is continuous at zero frequency. The phase modification is equivalent to an integer delay. This delay term is returned by `cceps` if you ask for a second output. For example:

```
x = 1:10;
[xh, nd] = cceps(x)

xh =
Columns 1 through 7
    2.2428   -0.0420   -0.0210    0.0045    0.0366    0.0788    0.1386
Columns 8 through 10
    0.2327    0.4114    0.9249

nd =
    1
```

To invert the complex cepstrum, use `icceps` with the original delay parameter:

```
icceps(xh, nd)

ans =
Columns 1 through 7
    1.0000    2.0000    3.0000    4.0000    5.0000    6.0000    7.0000
Columns 8 through 10
    8.0000    9.0000   10.0000
```

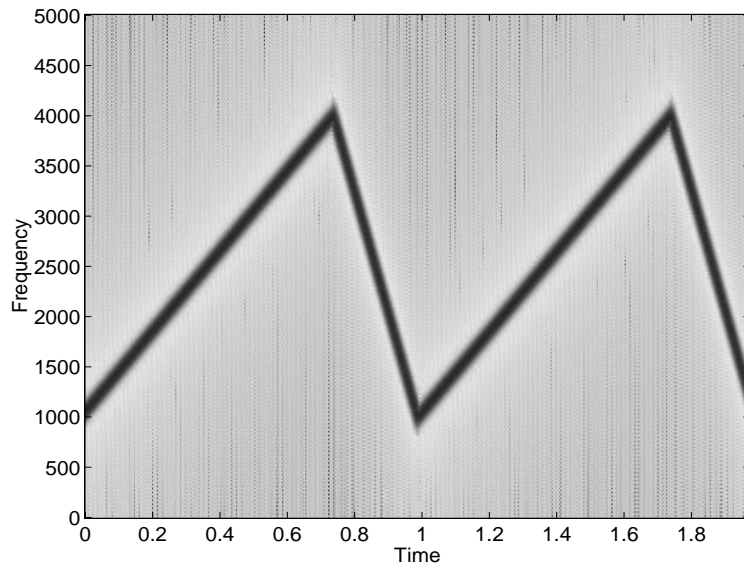
NOTE With any modification of the complex cepstrum, the original delay term may no longer be valid. Use the `icceps` function with care.

FFT-Based Time-Frequency Analysis

The Signal Processing Toolbox provides a function, `specgram`, that returns the time-dependent Fourier transform for a sequence, or displays this information as a spectrogram. The *time-dependent Fourier transform* is the discrete-time Fourier transform for a sequence, computed using a sliding window. This form of the Fourier transform, also known as the short-time Fourier transform (STFT), has numerous applications in speech, sonar, and radar processing. The *spectrogram* of a sequence is the magnitude of the time-dependent Fourier transform versus time.

To display the spectrogram of a linear FM signal:

```
Fs = 10000;  
t = 0: 1/Fs: 2;  
x = vco(sawtooth(2*pi*t, .75), [0.1 0.4]*Fs, Fs);  
specgram(x, 512, Fs, kaiser(256, 5), 220)
```



Note that the spectrogram display is an image, not a plot.

Median Filtering

The function `medfilt1` implements one-dimensional median filtering, a nonlinear technique that applies a sliding window to a sequence. The median filter replaces the center value in the window with the median value of all the points within the window [4]. In computing this median, `medfilt1` assumes zeros beyond the input points.

When the number of elements n in the window is even, `medfilt1` sorts the numbers, then takes the average of the $((n-1)/2)$ & $((n-1)/2 + 1)$ elements.

Two simple examples with fourth- and third-order median filters are

```
medfilt1([4 3 5 2 8 9 1], 4)
ans =
    1.500    3.500    3.500    4.000    6.500    5.000    4.500

medfilt1([4 3 5 2 8 9 1], 3)
ans =
     3     4     3     5     8     8     1
```

See the *Image Processing Toolbox User's Guide* for information on two-dimensional median filtering.

Communications Applications

The toolbox provides three functions for communications simulation:

Operation	Function
Modulation	modulate
Demodulation	demod
Voltage controlled oscillation	vco

Modulation varies the amplitude, phase, or frequency of a *carrier signal* with reference to a *message signal*. The `modulate` function modulates a message signal with a specified modulation method.

The basic syntax for the `modulate` function is

```
y = modulate(x, Fc, Fs, 'method', opt)
```

where:

- `x` is the message signal.
- `Fc` is the carrier frequency.
- `Fs` is the sampling frequency.
- `method` is a flag for the desired modulation method.
- `opt` is any additional argument that the method requires. (Not all modulation methods require an option argument.)

The table below summarizes the modulation methods provided; see Chapter 6 for complete details on each.

Method	Description
amdsb-sc or am	Amplitude modulation, double side-band, suppressed carrier
amdsb-tc	Amplitude modulation, double side-band, transmitted carrier
amssb	Amplitude modulation, single side-band
fm	Frequency modulation
pm	Phase modulation
ptm	Pulse time modulation
pwm	Pulse width modulation
qam	Quadrature amplitude modulation

If the input x is an array rather than a vector, `modulate` modulates each column of the array.

To obtain the time vector that `modulate` uses to compute the modulated signal, specify a second output parameter:

```
[y, t] = modulate(x, Fc, Fs, 'method', opt)
```

The `demod` function performs *demodulation*, that is, it obtains the original message signal from the modulated signal.

The syntax for `demod` is

```
x = demod(y, Fc, Fs, 'method', opt)
```

`demod` uses any of the methods shown for `modulate`, but the syntax for quadrature amplitude demodulation requires two output parameters:

```
[X1, X2] = demod(y, Fc, Fs, 'qam')
```

If the input y is an array, `demod` demodulates all columns.

Try modulating and demodulating a signal. A 50 Hz sine wave sampled at 1000 Hz is

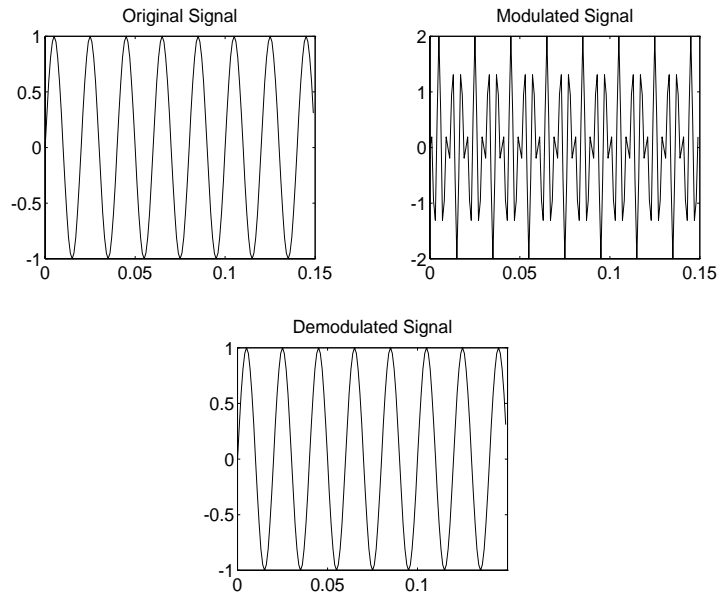
```
t = (0:1/1000:2);
x = sin(2*pi*50*t);
```

With a carrier frequency of 200 Hz, the modulated and demodulated versions of this signal are

```
y = modulate(x, 200, 1000, 'am');
z = demod(y, 200, 1000, 'am');
```

To plot portions of the original, modulated, and demodulated signal:

```
plot(t(1:150), x(1:150))
plot(t(1:150), y(1:150))
plot(t(1:150), z(1:150))
```



The voltage controlled oscillator function `vco` creates a signal that oscillates at a frequency determined by the input vector. The basic syntax for `vco` is

```
y = vco(x, Fc, Fs)
```

where `Fc` is the carrier frequency and `Fs` is the sampling frequency.

To scale the frequency modulation range:

$$y = \text{vco}(x, [F_{\min} \ F_{\max}], F_s)$$

In this case, `vco` scales the frequency modulation range so values of `x` on the interval $[-1 \ 1]$ map to oscillations of frequency on $[F_{\min} \ F_{\max}]$.

If the input `x` is an array, `vco` produces an array whose columns oscillate according to the columns of `x`.

See “FFT-Based Time-Frequency Analysis” on page 4-26 for an example using the `vco` function.

Deconvolution

Deconvolution, or polynomial division, is the inverse operation of convolution. Deconvolution is useful in recovering the input to a known filter, given the filtered output. This method is very sensitive to noise in the coefficients, however, so use caution in applying it.

The syntax for `deconv` is

```
[q, r] = deconv(b, a)
```

where `b` is the polynomial dividend, `a` is the divisor, `q` is the quotient, and `r` is the remainder.

To try `deconv`, first convolve two simple vectors `a` and `b` (see Chapter 1 for a description of the convolution function):

```
a = [1 2 3];  
b = [4 5 6];  
c = conv(a, b)  
  
c =  
    4    13    28    27    18
```

Now use `deconv` to deconvolve `b` from `c`:

```
[q, r] = deconv(c, a)  
  
q =  
    4     5     6  
  
r =  
    0     0     0     0     0
```

See the *System Identification Toolbox User's Guide* for advanced applications of signal deconvolution.

Specialized Transforms

In addition to the discrete Fourier transform (DFT) described in Chapter 1, the Signal Processing Toolbox and the MATLAB environment together provide the following transform functions:

- The chirp z -transform (CZT), useful in evaluating the z -transform along contours other than the unit circle. The chirp z -transform is also more efficient than the DFT algorithm for the computation of prime-length transforms, and it is useful in computing a subset of the DFT for a sequence.
- The discrete cosine transform (DCT), closely related to the DFT. The DCT's energy compaction properties are useful for applications such as signal coding.
- The Hilbert transform, which facilitates the formation of the analytic signal. The analytic signal is useful in the area of communications, particularly in bandpass signal processing.

Chirp z -Transform

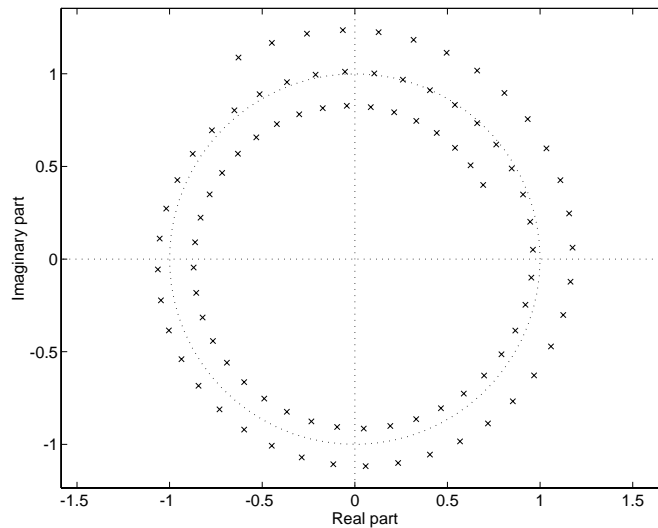
The chirp z -transform, or CZT, computes the z -transform along spiral contours in the z -plane for an input sequence. Unlike the DFT, the CZT is not constrained to operate along the unit circle, but can evaluate the z -transform along contours described by

$$z_l = AW^{-l}, \quad l = 0, \dots, M-1$$

where A is the complex starting point, W is a complex scalar describing the complex ratio between points on the contour, and M is the length of the transform.

One possible spiral is

```
A = 0.8*exp(j*pi/6);
W = 0.995*exp(-j*pi*.05);
M = 91;
z = A*(W.^(-(0:M-1)));
zplane([], z, 'x')
```



`czt(x, M, W, A)` computes the z -transform of x on these points.

An interesting and useful spiral set is m evenly spaced samples around the unit circle, parameterized by $A = 1$ and $W = \exp(-j\pi/M)$. The z -transform on this contour is simply the DFT, obtained by

```
y = czt(x)
```

`czt` is faster than the `fft` function for computing the DFT of sequences with certain odd lengths, particularly long prime-length sequences. (Try comparing the execution time for the `fft` and `czt` functions for a sequence of length 1013.)

Discrete Cosine Transform

The toolbox function `dct` computes the unitary discrete cosine transform, or DCT, for an input vector or matrix. Mathematically, the unitary DCT of an input sequence x is

$$y(k) = \sum_{n=1}^N w(n)x(n) \cos \frac{\pi(2n-1)(k-1)}{2N}, \quad k = 1, \dots, N$$

where

$$w(n) = \begin{cases} \frac{1}{\sqrt{N}}, & n = 1 \\ \sqrt{\frac{2}{N}}, & 2 \leq n \leq N \end{cases}$$

The DCT is closely related to the discrete Fourier transform; the DFT is actually one step in the computation of the DCT for a sequence. The DCT, however, has better *energy compaction* properties, with just a few of the transform coefficients representing the majority of the energy in the sequence. The energy compaction properties of the DCT make it useful in applications such as data communications.

The function `idct` computes the inverse DCT for an input sequence, reconstructing a signal from a complete or partial set of DCT coefficients. The inverse discrete cosine transform is

$$x(n) = \sum_{k=1}^N w(k)y(k) \cos \frac{\pi(2n-1)(k-1)}{2N}, \quad n = 1, \dots, N$$

where

$$w(k) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 1 \\ \sqrt{\frac{2}{N}}, & 2 \leq k \leq N \end{cases}$$

Because of the energy compaction mentioned above, it is possible to reconstruct a signal from only a fraction of its DCT coefficients. For example, generate a 10 Hz sinusoidal sequence, sampled at 1000 Hz:

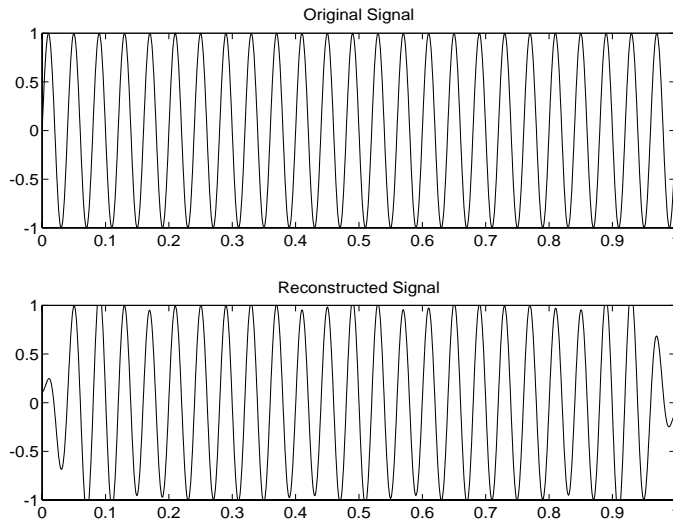
```
t = (0:1/999:1);
x = sin(2*pi*25*t);
```

Compute the DCT of this sequence and reconstruct the signal using only those components with value greater than 53 (12 of the original 1000 DCT coefficients):

```
y = dct(x); % compute DCT
y2 = find(abs(y) < 53); % use 12 coefs.
y(y2) = zeros(size(y2)); % zero out points < 53
z = idct(y); % reconstruct signal using inverse DCT
```

Plot the original and reconstructed sequences.

```
plot(t, x)
plot(t, z), axis([0 1 -1 1])
```



One measure of the accuracy of the reconstruction is

$$\text{norm}(x-z) / \text{norm}(x)$$

that is, the norm of the difference between the original and reconstructed signals, divided by the norm of the original signal. In this case, the relative error of reconstruction is 0.1778. The reconstructed signal retains approximately 82% of the energy in the original signal.

Hilbert Transform

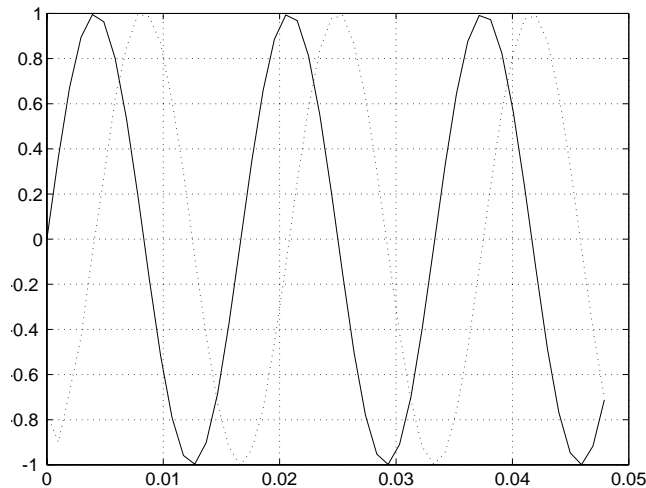
The toolbox function `hilbert` computes the Hilbert transform for a real input sequence `x` and returns a complex result of the same length:

```
y = hilbert(x)
```

where the real part of `y` is the original real data and the imaginary part is the actual Hilbert transform. `y` is sometimes called the *analytic signal*, in reference to the continuous-time analytic signal. A key property of the discrete-time analytic signal is that its z -transform is 0 on the lower half of the unit circle. Many applications of the analytic signal are related to this property; for example, the analytic signal is useful in avoiding aliasing effects for bandpass sampling operations. The magnitude of the analytic signal is the complex envelope of the original signal.

The Hilbert transform is related to the actual data by a 90° phase shift; sines become cosines and vice versa. To plot a portion of data (solid line) and its Hilbert transform (dotted line):

```
t = (0:1/1023:1);
x = sin(2*pi*60*t);
y = hilbert(x);
plot(t(1:50), real(y(1:50))), hold on
plot(t(1:50), imag(y(1:50)), ':'), hold off
```



The analytic signal is useful in calculating *instantaneous attributes* of a time series, the attributes of the series at any point in time. The instantaneous amplitude of the input sequence is the amplitude of the analytic signal. The instantaneous phase angle of the input sequence is the (unwrapped) angle of the analytic signal; the instantaneous frequency is the time rate of change of the instantaneous phase angle. You can calculate the instantaneous frequency using `diff`, as described in the online *MATLAB Function Reference*.

References

- 1 Kay, S.M. *Modern Spectral Estimation*. Englewood Cliffs, NJ: Prentice Hall, 1988.
- 2 Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989.
- 3 Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987.
- 4 Pratt, W.K. *Digital Image Processing*. New York: John Wiley & Sons, 1991.

Interactive Tools

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SPTool: An Interactive Signal Processing Environment

The Signal Processing Toolbox includes an interactive graphical user interface (GUI), called SPTool, for performing digital signal processing tasks. SPTool provides an easy-to-use interface to many of the most important toolbox functions. With it, you can use the mouse and on-screen controls to import, view, and measure digital signals; design, view, and implement digital filters; and analyze the frequency content of signals.

This chapter describes how to use the different components of SPTool. Where appropriate, we point you to other areas of the manual that describe how to perform similar tasks by calling functions from the command line or from M-files.

The section “Example: Generation of Bandlimited Noise” at the end of this chapter describes how to use this graphical environment for a complete filter design and analysis task.

Overview

SPTool is a graphical environment for analyzing and manipulating digital signals, filters, and spectra. It is the starting point for using the interactive signal processing environment. In SPTool, you can import signals, filters, and spectra either from the workspace or as MAT-files. Through SPTool, you access four additional GUI tools that provide an integrated environment for signal browsing, filter design, analysis, and implementation. The four components of the interactive signal processing environment include:

- The *Signal Browser*, which provides a graphical view of the signal objects currently selected in SPTool and enables you to interactively display, measure, and analyze these signals
- The *Filter Designer*, which enables you to create and edit lowpass, highpass, bandpass, and bandstop FIR and IIR digital filters of various lengths and types using the filter design functions of the Signal Processing Toolbox
- The *Filter Viewer*, which enables you to view various characteristics of a filter that you’ve imported or designed, including its magnitude and phase

responses, its group delay, its zero-pole plot, and its impulse and step responses

- The *Spectrum Viewer*, which enables you to interactively create, view, and modify spectra, and to perform graphical analysis of frequency domain data using a variety of common methods of spectral estimation

Using SPTool

SPTool is the data management tool for the interactive GUI environment of the Signal Processing Toolbox. Using SPTool you can

- Load a saved session
- Import a signal, filter, or spectrum
- Duplicate or clear a signal, filter, or spectrum
- Change the name of a signal, filter, or spectrum
- Change the sampling frequency of a signal or filter
- Activate the Signal Browser, Filter Viewer, Filter Designer, or Spectrum Viewer
- Save a session
- Use the **Window** menu to change to any open MATLAB Figure window

Opening SPTool

Open SPTool from the MATLAB command window by typing

```
sptool
```

and pressing **Enter** (**Return** on Macintosh).

Quick Start

Once SPTool is open, you can import data from the workspace or a file. You can then view it in the Signal Browser or generate its default spectrum in the Spectrum Viewer.

To get started right away, work through the following example. Then continue through this chapter to learn the details of using SPTool and its component tools.

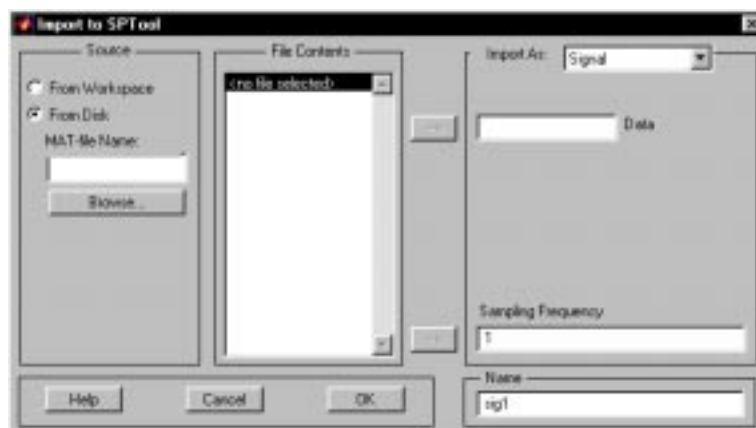
Or, you can skip the example, read through the rest of this section (from “Basic SPTool Functions” to “Using the Signal Browser: Interactive Signal Analysis”), and then work through the example.

Example: Importing Signal Data from a MAT-File

This example uses the sample file `mt1 b. mat`, which is in the `tool box/si gnal` directory.

- 1 Select **Import** from the **File** menu.

The Import window is displayed:



- 2 Make sure that **Signal** is displayed in the **Import As** pop-up menu.
- 3 Click on the **From Disk** radio button.

You can either enter a MAT-file name or click **Browse** to open the file dialog box and select a MAT-file.

- 4 To get started, type the file name `mt1 b` and press **Tab** or **Enter** (**Return** on Macintosh). Note that SPTool adds the `. mat` extension automatically.

The data from the file you selected is displayed in the **File Contents** list. In this example, `mt1 b` is the signal data and `Fs` is the sampling frequency.

Notice that `si g1` is displayed in the **Name** field. This is the default name of the imported signal.

- 5 Click on `mt1 b` to select it, and then click on the arrow at the left of the **Data** field. `mt1 b` is transferred to the **Data** field.

- 6 Click on **Fs** to select it, and then click on the arrow at the left of the **Sampling Frequency** field.

Fs is transferred to the **Sampling Frequency** field.

- 7 Click **OK**.

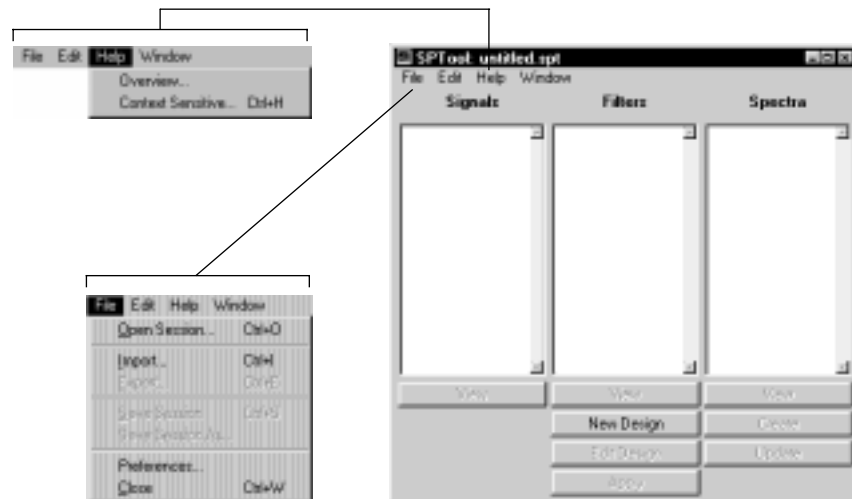
The signal has been imported into SPTool with the name **sig1**.

You can look at this signal in the Signal Browser by clicking on the **View** button under **Signals**.

You can look at the frequency content of the signal in the Spectrum Viewer by clicking on the **Create** button under **Spectra** and then clicking **Apply** in the Spectrum Viewer.

Basic SPTool Functions

When you first open SPTool, it contains no signals, filters, or spectra. You can import signals, filters, and spectra into SPTool, and you can also design filters using the Filter Designer and create spectra using the Spectrum Viewer. You can also save and export data from SPTool and customize many properties of the SPTool environment. The following figure shows the SPTool window and its **File** and **Help** menus, which are described below:



The only button that is enabled is **New Design**; you can always design a new filter. The rest of the buttons are enabled when an appropriate object is listed and selected.

File Menu

Open Session. Select **Open Session...** from the **File** menu to load a saved session file. An SPTool session is saved in a file with an . spt extension.

Import. Select **Import...** from the **File** menu to import a signal, filter, or spectrum into SPTool from either the workspace or from a file. You can import variables from any MAT-file into SPTool. See “Importing Signals, Filters, and Spectra” on page 5-8 and “Example: Importing Signal Data from a MAT-File” on page 5-5 for more information.

Export. Select **Export...** from the **File** menu to export signals, filters, and spectra to the MATLAB workspace as structure variables. See “Saving Signal Data” on page 5-53, “Saving Filter Data” on page 5-69, and “Saving Spectrum Data” on page 5-98 for complete information.

Save Session. Select **Save Session** and **Save Session As...** from the **File** menu to save the current session. **Save Session** overwrites the existing session file. **Save Session As...** saves the current session with a name you specify. An SPTool session is saved in a MAT-file with an . spt extension.

Preferences. Select **Preferences...** from the **File** menu to customize preferences for the behavior of all the Signal Processing GUI tools. See “Customizing Preferences” on page 5-21 for a complete discussion.

Close. Select **Close** from the **File** menu to close SPTool and all other active Signal Processing GUI tools. SPTool prompts you to save if you have not recently saved the current session.

When you close SPTool, all signal and filter customization and ruler information set in any of the GUI tools are lost. Settings you changed and saved using the **Preferences...** window in SPTool are saved and used the next time you open SPTool.

Help Menu

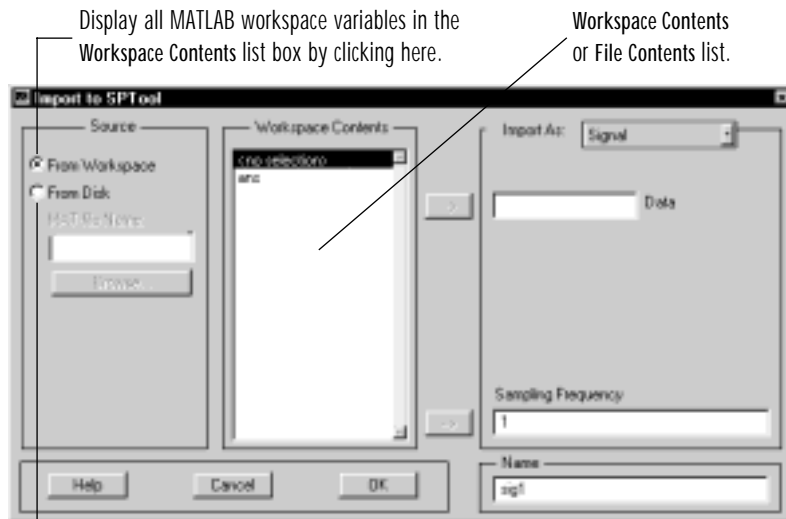
Overview... Select **Overview...** from the **File** menu to get general help on SPTool and the Signal Processing Toolbox GUI environment. This also gives you access to the MATLAB Help Desk.

Context Sensitive... Select **Context Sensitive...** from the **File** menu for help on a specific part of SPTool. When you click on **Context Sensitive...**, the mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in SPTool, including menu items, to find out what it is and how to use it.

Importing Signals, Filters, and Spectra

You can import a signal, filter, or spectrum into SPTool from either the workspace or from a file.

Click **Import...** from the **File** menu to open the Import window:



Loading Variables from the MATLAB Workspace

To import variables from the MATLAB workspace, first list the workspace variables in the **Workspace Contents** list. Then select the variables to be imported into SPTool:

- 1 Click the **From Workspace** radio button.

The contents of the MATLAB workspace are displayed in the **Workspace Contents** list.

- 2 You can now import one or more variables from the **Workspace Contents** list into SPTool. See “Importing Workspace Contents and File Contents” on page 5-9.

Loading Variables from Disk

To import variables from a MAT-file on disk, first list the file’s variables in the **File Contents** list. Then select the variables to be imported into SPTool:

- 1 Click the **From Disk** radio button.

You can either enter a MAT-file name in the **MAT-file Name** field or click **Browse** to open the file listing and select a MAT-file.

- 2 Type the exact name of the file you want to import into the **MAT-file Name** field and press **Tab** or **Enter** (**Return** on Macintosh).

or

Click **Browse**, and then find and select the file you want to import using the File Search window. Click **OK**.

The data from the file you selected is displayed in the **File Contents** list.

- 3 You can now import one or more variables from the **File Contents** list into SPTool (see below).

Importing Workspace Contents and File Contents

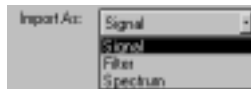
Once you’ve loaded the contents of the workspace or a file into the **Workspace Contents** or **File Contents** list, you can select one or more variables from the list to import into SPTool. You can import a variable as a signal parameter, a

filter parameter, or a spectrum parameter. You can also import a variable whose value represents a sampling frequency or other design parameter.

Depending on whether you're importing a signal, a filter, or a spectrum, you can customize different parameters before you import the data into SPTool. In each case, however, the general procedure for specifying a variable or a value is the same for all data types. In the following illustration, the selected variable is being imported as a signal. See "Importing a Signal" on page 5-12, "Importing a Filter" on page 5-12, and "Importing a Spectrum" on page 5-13 for details on customizing variables that are imported into SPTool.

- 1 Click on a variable name in the **Workspace Contents** list or the **File Contents** list to select it.

If the variable is not a saved data object from SPTool, select the appropriate data type (**Signal**, **Filter**, or **Spectrum**) from the **Import As** pop-up menu and type a name into the **Name** field.



If the variable is a saved data object from SPTool, its name is displayed in the **Name** field, and its type (**Signal**, **Filter**, or **Spectrum**) is automatically selected in the **Import As** pop-up menu.

- 2 Click on the arrow at the left of the **Data** field. The selected variable is transferred to the **Data** field.

NOTE You can also type a variable name into the **Data** field directly.

3 The default sampling frequency is 1. To change the sampling frequency of the variable you're importing, you can either:

- a Click on a variable in the **Workspace Contents** list or **File Contents** list whose value is a sampling frequency, and then click on the arrow at the left of the **Sampling Frequency** field.

The selected variable is transferred to the **Sampling Frequency** field.

or

- b Type a value or variable name in the **Sampling Frequency** field.

4 Click **OK**.

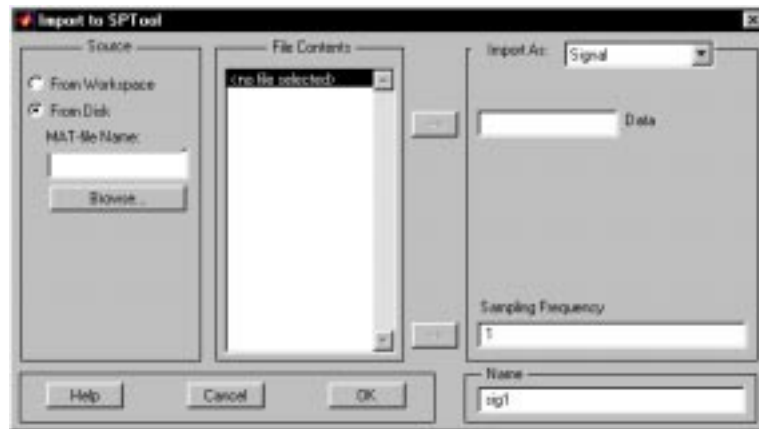
The signal is imported into SPTool with the specified name and sampling frequency.

5 To import another variable, select **Import...** again, click the **From Workspace** or **From File** radio button, and repeat steps 1 through 4 for each variable that you want to load into SPTool.

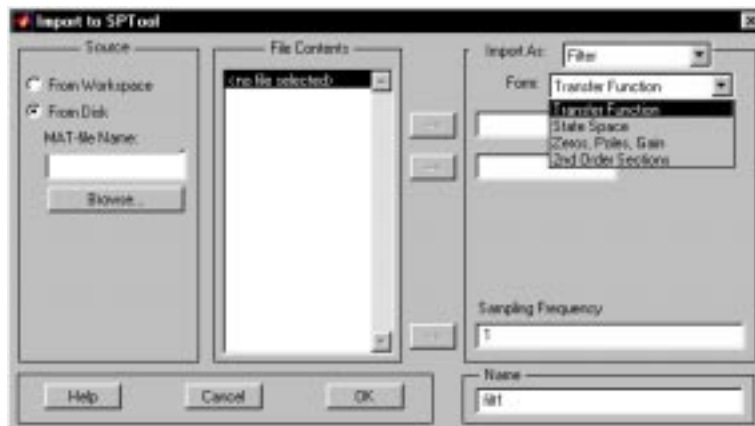
NOTE When you're importing from the workspace, you can specify either a variable or a value for each data field. When you're importing from a disk, you can only specify variables.

Importing a Signal. When you import a signal, you specify

- A variable name for the signal data (or the signal data values) in the **Data** field
- A variable or a value for the signal's sampling frequency in the **Sampling Frequency** field



Importing a Filter. When you import a filter, first select the appropriate filter form from the **Form** pop-up menu:

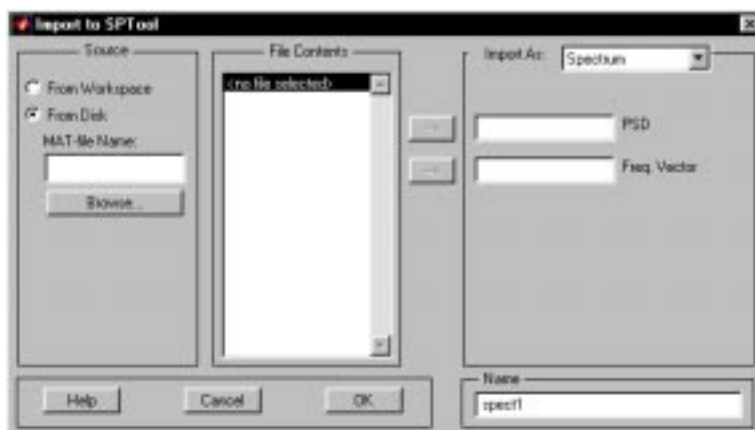


Each filter form requires different variables.

- For **Transfer Function** you specify:
 - A variable name or a value for the numerator in the **Numerator** field
 - A variable name or a value for the denominator in the **Denominator** field
- For **State-Space** you specify:
A variable name or a value for each matrix in the **A-Matrix**, **B-Matrix**, **C-Matrix**, and **D-Matrix** fields
- For **Zeros, Poles, Gain** you specify:
 - A variable name or a value for the zeros in the **Zeros** field
 - A variable name or a value for the poles in the **Poles** field
 - A variable name or a value for the gain in the **Gain** field
- For **Second-Order Sections** you specify:
A variable name or a value for the SOS matrix in the **SOS Matrix** field
- For every filter, you specify:
A variable name or a value for the filter's sampling frequency in the **Sampling Frequency** field

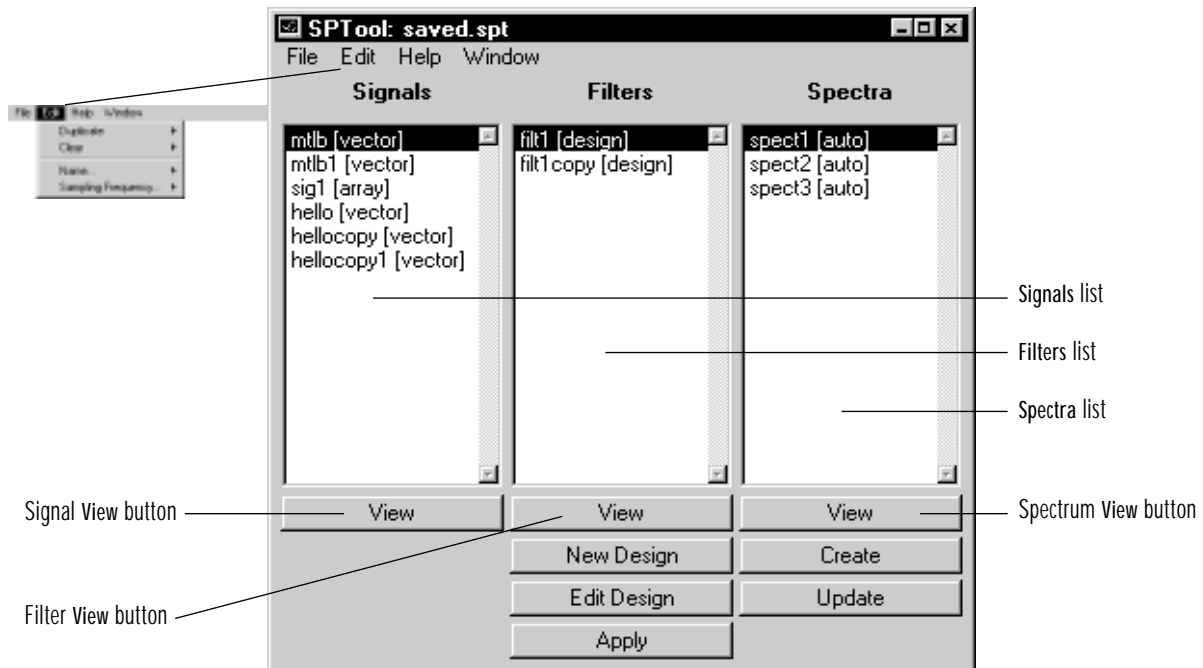
Importing a Spectrum. When you import a spectrum, you specify:

- A variable name or a value for the power spectral density (PSD) of the signal in the **PSD** field
- A variable name or a value for the frequency vector in the **Freq. Vector** field



Working with Signals, Filters, and Spectra

When a signal, filter, or spectrum is imported into SPTool or created in SPTool, it is displayed in the appropriate list box, as shown below. Using the **Edit** menu functions and SPTool buttons, you can edit various properties of the data in SPTool and invoke all of SPTool's digital signal processing functions.



Component Lists in SPTool

Each signal, filter, and spectrum in SPTool is displayed in the appropriate **Signals** list, **Filters** list, or **Spectra** list.

- Signals are displayed with the *signal type* [vector] or [array]:
 - A *vector signal* ([vector]) has one column of data.
 - An *array signal* ([array]) has more than one column of data.
- Filters are displayed with the *filter type* [design] or [imported]:
 - A *designed filter* ([design]) is a filter that was created using the Filter Designer. This type of filter can also be edited in the Filter Designer.
 - An *imported filter* ([imported]) is a filter that was imported from the MATLAB workspace or a file. It can be applied to a signal in SPTool, but it cannot be edited in the Filter Designer.
- Spectra are displayed with the *spectrum type* [auto]:

An *auto-spectrum* ([auto]) is a spectrum whose source is a single signal, as opposed to the cross-spectrum of two channels of data. `spectrum[auto]` is the only spectrum type in SPTool.

Selecting Data Objects in SPTool

Each signal, filter, and spectrum in SPTool is one data object. A data object is selected when it is highlighted. When you first import or create a data object, it is selected.

The **Signals** list shows all vector and array signals in the current SPTool session.

The **Filters** list shows all designed and imported filters in the current SPTool session.

The **Spectra** list shows all spectra in the current SPTool session.

You can select a single data object in a list, a range of data objects in a list, or multiple separate data objects in a list. You may also have data objects simultaneously selected in different lists.

- To select a single unselected data object, click on it. All other data objects in that list box become unselected.
- To add or remove a range of data objects, **Shift**-click on the data objects at the top and bottom of the section of the list that you want to add.
- To add a single data object to a selection or remove a single data object from a multiple selection, **Ctrl**-click (PC and UNIX) or **Command**-click (Macintosh) on the object. Instead of **Ctrl**-click, you can use the right mouse button.

Editing Data Objects in SPTool

The **Edit** menu entries are only available when there is at least one selected data object (signal, filter, or spectrum) in SPTool. Use the **Edit** menu to duplicate and clear objects in SPTool, and to edit object names and change sampling frequencies.

A signal, filter, or spectrum must be selected in order to be edited. When you click on an **Edit** menu entry, all *selected* data objects are displayed in a pop-up menu.

To edit an SPTool object:

- 1 Select a signal, filter, or spectrum.
- 2 Click the appropriate **Edit** menu function.

The pop-up menu shows the names of all selected data objects.

- 3 Drag to choose a specific signal, filter, or spectrum for editing.

Duplicate. Use **Duplicate** from the **Edit** menu to make a copy of the selected signal, filter, or spectrum in SPTool.

Click **Duplicate** and drag to choose the signal, filter, or spectrum you want to copy. When you select a data object to duplicate it, a new data object of the same type is automatically created. The new data object is named as a copy of the selected data object. It is placed at the bottom of the list and is selected. You can change its name using **Name...** from the **Edit** menu.

Clear. Use **Clear** from the **Edit** menu to delete the selected signal, filter, or spectrum from SPTool.

Click **Clear** and drag to choose the signal, filter, or spectrum you want to remove. The data object is deleted from the current SPTool session.

Name. Use **Name...** from the **Edit** menu to give the selected signal, filter, or spectrum a new, unique name.

- 1 Click **Name...** and drag to choose the signal, filter, or spectrum you want to rename.

The **Name Change** dialog box is displayed.

- 2 Type in the new name and click **OK**.

Sampling Frequency. Use **Sampling Frequency...** from the **Edit** menu to supply a sampling frequency for a selected signal or filter. The sampling frequency may be a number – such as 1, 0.001, 1/5000, or a valid MATLAB expression including workspace variables – such as F_s , $1/T_s$, or $\cos(.1 * \pi)$.

- 1 Click **Sampling Frequency...** and drag to choose the signal or filter you want to change.

The **Sampling Frequency...** dialog box is displayed.

- 2 Type in the value, variable name, or expression and click **OK**.

Viewing a Signal

Use the **Signal View** button to make the Signal Browser active and view one or more imported signals. The Signal Browser provides tools for graphical analysis of the selected signal(s).

Select one or more signals from the **Signals** list and click the **View** button in the signal panel. The Signal Browser displays the selected signal(s). See “Using the Signal Browser: Interactive Signal Analysis” on page 5-42 for a full description of Signal Browser functions and operations.

Viewing a Filter

Use the **View** button in the **Filters** panel to make the Filter Viewer active and view imported filters or filters designed/edited in the Filter Designer. The

Filter Viewer provides tools for analyzing filters; you can investigate the magnitude response, phase, group delay, zeros and poles, and impulse and step responses of the selected filters.

Select one or more filters from the **Filters** list and click the **View** button in the filter panel. The Filter Viewer displays the selected filters. See “Using the Filter Viewer: Interactive Filter Analysis” on page 5-74 for a full description of Filter Viewer functions and operations.

Designing a Filter

New Design. Use the Filter **New Design** button to make the Filter Designer active and design a filter. The Filter Designer lets you create FIR and IIR digital filters of various lengths and types using the filter design functions in the Signal Processing Toolbox.

Click the **New Design** button in the **Filter** panel. The Filter Designer displays a filter created with the default settings and assigns it a default name of `filt_n`, where *n* is a unique sequential identifying digit. Once the default filter is created, you can use it as is or edit it with the Filter Designer. You can rename it using **Name...** from the **Edit** menu.

Edit Design. Use the **Filter** panel’s **Edit Design** button to make the Filter Designer active and edit a filter. You can only edit filters whose filter type is [design]—that is, filters created in SPTool; you cannot edit filters imported into SPTool.

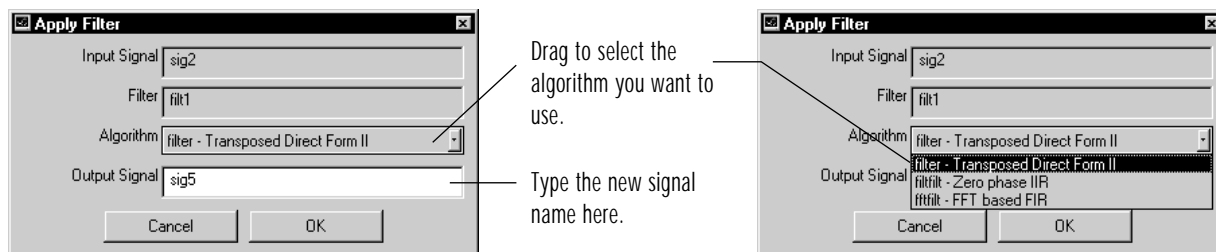
Select one or more filters from the **Filters** list and click the **Edit Design** button in the filter panel. The Filter Designer displays the first of the selected filters. The other selected filters can be edited, one at a time, by selecting them from the **Filter** pop-up menu. See “Using the Filter Designer: Interactive Filter Design” on page 5-55 for a full description of Filter Designer functions and operations.

Applying a Filter

Use the **Apply Filter** button to apply a filter to a selected signal. This creates a new signal.

- 1 Select one signal from the **Signals** list and one filter from the **Filters** list and click the **Apply Filter** button in the filter panel.

The **Apply Filter** dialog box is displayed:



- 2 Select the filtering algorithm from the **Algorithm** pop-up menu, type the name for the new signal in the **Output Signal** field, and click **OK**.

The selected filter is applied to the selected input signal and the new output signal is listed in the **Signals** list.

Creating a Spectrum

Use the **Create** button to activate the Spectrum Viewer and generate a default spectrum of a selected signal. Once you've generated a spectrum, you can view it in a variety of ways, measure it, and modify it in the Spectrum Viewer.

- 1 Select one signal from the **Signals** list and click the **Create** button in the **Spectra** panel.

The Spectrum Viewer is activated and a spectrum object with default parameters is created in the **Spectra** panel. No spectrum is computed or displayed yet. The newly created spectrum has a default name of `specn`, where `n` is a unique sequential identifying digit. Once the default spectrum

is created, you can use it as is or edit it in the Spectrum Viewer. You can rename it using **Name...** from the **Edit** menu.

- 2 Use the default parameters in the Spectrum Viewer, or modify the parameters as necessary.
- 3 Press **Apply** in the Spectrum Viewer to compute the spectrum. This button is enabled when the spectrum has just been created or when you have changed one or more parameters in the Spectrum Viewer.

The updated spectrum is displayed in the Spectrum Viewer. See “Using the Spectrum Viewer: Interactive PSD Analysis” on page 5-88 for a full description of Spectrum Viewer parameters and displays.

Viewing a Spectrum

Use the **View** button in the Spectrum panel to activate the Spectrum Viewer and display one or more selected spectra.

Select one or more spectra from the **Spectra** list and click **View** in the spectrum panel. The Spectrum Viewer displays the selected spectrum or spectra.

Updating a Spectrum

Use the **Update** button to update the selected spectrum so that it reflects the data in the currently selected signal.

- 1 Select one signal from the **Signals** list and one spectrum from the **Spectra** list and click **Update** in the **Spectra** panel.

The spectral data from the current spectrum is removed from the Spectrum Viewer. The Spectrum Viewer is activated. No spectrum is displayed yet.

- 2 Press **Apply** in the Spectrum Viewer to compute the spectrum and complete the update.

The spectrum is regenerated with the same parameters but using the data in the currently selected signal. This feature is useful when you have altered a signal (by filtering it, for example), and want to update the existing spectrum to reflect the change.

Customizing Preferences

Use **Preferences...** from the **File** menu to customize displays and certain parameters for SPTool and its four component tools. The new settings are saved on disk and are used when you restart MATLAB.

In the preferences panels, you can:

- Select colors and markers for rulers and set the initial ruler style
- Select color and line style sequence for displayed signals
- Configure axis labels, and enable/disable rulers, panner, and mouse zoom in the Signal Browser
- Configure axis parameters, and enable/disable rulers and mouse zoom in the Spectrum Viewer
- Configure filter and axis parameters and enable/disable mouse zoom in the Filter Viewer
- Configure tiling preferences in the Filter Viewer
- Specify FFT length, and enable/disable mouse zoom and grid in the Filter Designer
- Configure plug-ins

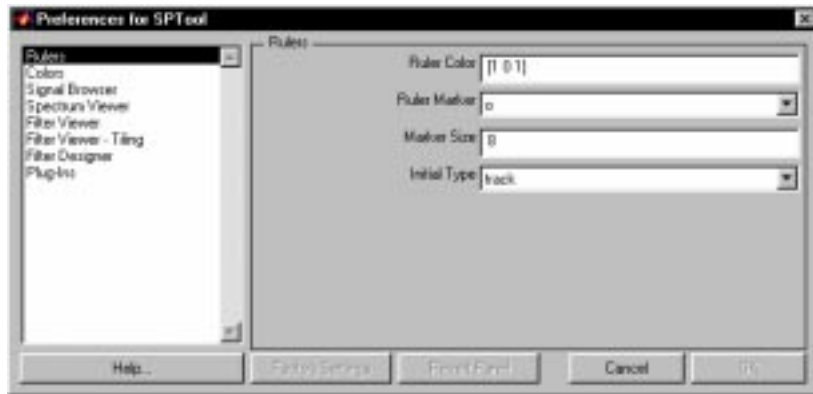
When you first select **Preferences...**, the **Preferences** dialog box is set to **Rulers**. You can change the settings for rulers, or click on any of the other settings categories to customize other settings.

Click once on a settings category to select it.

The following sections describe all of the settings you can modify. The illustrations show the default settings for each category. For additional information on preference settings, use the **Help...** button at the bottom of the **Preferences** dialog box.

Ruler Settings

The **Rulers** preferences apply to the rulers in the Signal Browser, Spectrum Viewer, and Filter Viewer.



Ruler Color. Specifies the color of the rulers. Color is specified as a string (e.g., 'r') or RGB triple (e.g., [1 0 0]).

Type in a new value to change the ruler color.

Ruler Marker. Specifies the marker used in the track and slope rulers.

Click in the pop-up menu and drag to select a different marker.

Marker Size. Specifies the size of the ruler marker in points. This can be any positive value.

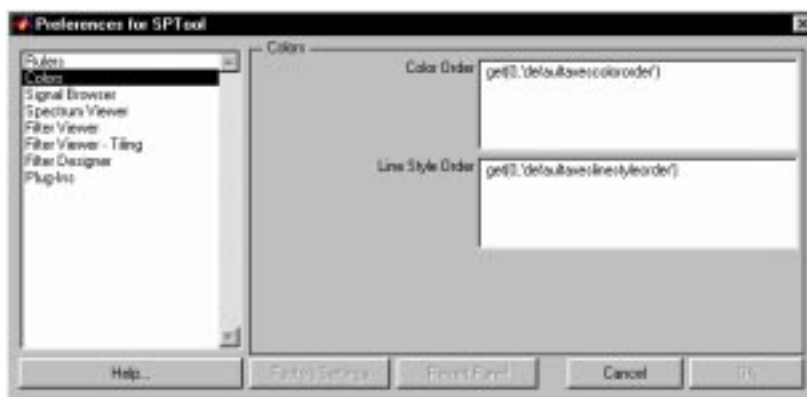
Type in a new value to change the marker size.

Initial Type. Specifies the type of ruler (horizontal, vertical, track, or slope) that is selected when you first open the Signal Browser, Spectrum Viewer, or Filter Viewer.

Click in the pop-up menu and drag to select a different ruler type.

Color Settings

The **Colors** preferences apply to signals displayed in the Signal Browser, Spectrum Viewer, and Filter Viewer.



Color Order. Specifies the color order to cycle through for data plotted in the Signal Browser, Spectrum Viewer, and Filter Viewer. The default is the axis color order.

Type in a new value or value string to change the color order. Color is specified as a string (e.g., 'r') or RGB triple (e.g., [1 0 0]), an n -by-3 matrix of n colors, or an n -by-1 cell array of such objects.

Line Style Order. Specifies the line styles to cycle through for data plotted in the Signal Browser, Spectrum Viewer, and Filter Viewer. The default is the axis line style order.

Type in a new string value (e.g., '--') or an array of strings (e.g., {'-', '--', ':'}) to change the line style order.

Signal Browser Settings

The **Signal Browser** preferences let you set optional *x*-axis and *y*-axis labels, enable and disable the display of the rulers and the panner, and toggle zoom persistence.



X Label, Y Label. Type in a string for the *x*-axis label and the *y*-axis label in the Signal Browser. The default is *Time* for the *x*-axis.

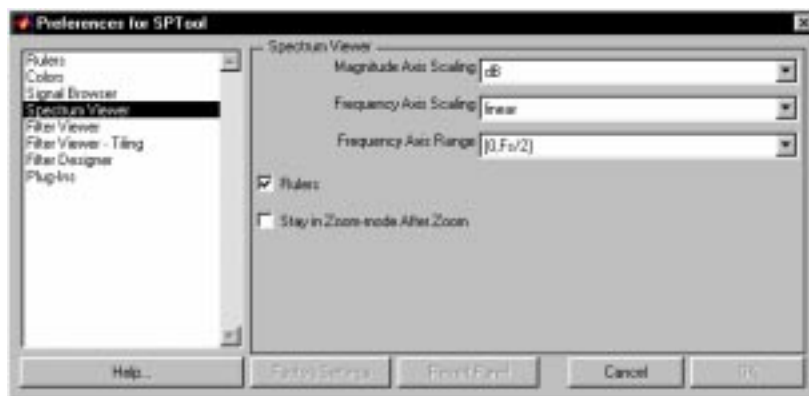
Rulers. Click in the check box to display (checked) or hide (unchecked) the ruler buttons and the ruler panel in the Signal Browser. See “Ruler Controls” on page 5-32 for details on using the rulers in the Signal Browser.

Panner. Click in the check box to display (checked) or hide (unchecked) the panner in the Signal Browser. See “Panner Display” on page 5-51 for details on using the panner in the Signal Browser.

Stay in Zoom-mode After Zoom. Click in the check box to enable (checked) or disable (unchecked) zoom persistence in the Signal Browser. See “Zoom Controls” on page 5-30 for details on zoom controls in the Signal Browser.

Spectrum Viewer Settings

The **Spectrum Viewer** preferences let you set axis parameters, enable and disable the display of the rulers, and toggle zoom persistence.



Magnitude Axis Scaling. Specifies the scaling units for the magnitude (y -) axis in the Spectrum Viewer. Scaling units can be **dB** or **linear**.

Click in the pop-up menu and drag to select a different scaling.

Frequency Axis Scaling. Specifies the scaling units for the frequency (x -) axis in the Spectrum Viewer. Scaling units can be **linear** or **log**.

Click in the pop-up menu and drag to select a different scaling.

Frequency Axis Range. Specifies the numerical range for the frequency (x -) axis in the Spectrum Viewer. Scaling options are **[0,Fs/2]**, **[0,Fs]**, or **[-Fs/2,Fs/2]**.

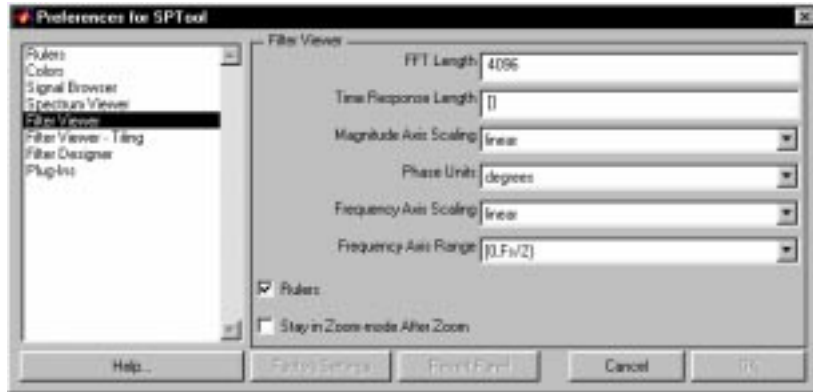
Click in the pop-up menu and drag to select a different frequency range.

Rulers. Click in the check box to display (checked) or hide (unchecked) the ruler buttons and the ruler panel in the Spectrum Viewer. See “Ruler Controls” on page 5-32 for details on using the rulers in the Spectrum Viewer.

Stay in Zoom-mode After Zoom. Click in the check box to enable (checked) or disable (unchecked) zoom persistence in the Spectrum Viewer. See “Zoom Controls” on page 5-30 for details on zoom controls in the Spectrum Viewer.

Filter Viewer Settings

The **Filter Viewer** preferences let you set key filter plot configuration parameters and toggle zoom persistence.



FFT Length. Specifies the number of points used for the magnitude, phase, and group delay plots.

Type in a new value to change the FFT length.

Time Response Length. Specifies the time response length for the impulse and step response plots. An empty value [] indicates that a response length will automatically be determined using the `impz` function.

Type in a new value to change the time response length.

Magnitude Axis Scaling. Specifies the scaling units for the magnitude (y -) axis in the Filter Viewer. Scaling units can be **linear**, **log**, or **decibels**.

Click in the pop-up menu and drag to select a different scaling.

Phase Units. Specifies the phase units for the phase response plot. Phase units can be **degrees** or **radians**.

Click in the pop-up menu and drag to select a different phase unit.

Frequency Axis Scaling. Specifies the scaling units for the frequency (x -) axis in the Filter Viewer. Scaling units can be **linear** or **log**.

Click in the pop-up menu and drag to select a different scaling.

Frequency Axis Range. Specifies the numerical range for the frequency (x -) axis in the Filter Viewer. Scaling options are **[0,Fs/2]**, **[0,Fs]**, or **[-Fs/2,Fs/2]**.

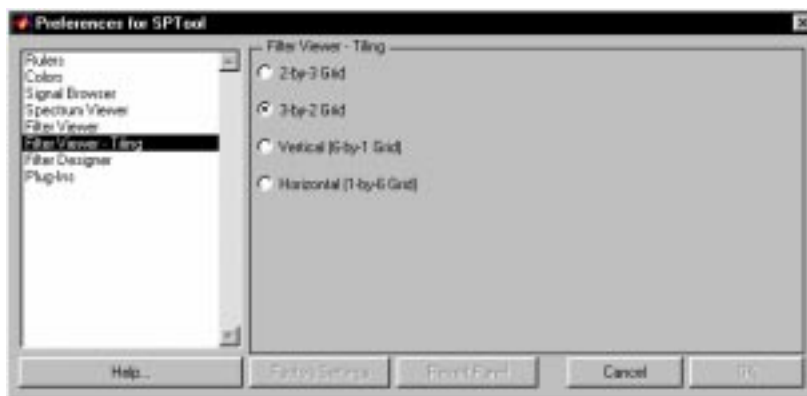
Click in the pop-up menu and drag to select a different frequency range.

Rulers. Click in the check box to display (checked) or hide (unchecked) the ruler buttons and the ruler panel in the Filter Viewer. See “Ruler Controls” on page 5-32 for details on using the rulers in the Filter Viewer.

Stay in Zoom-mode After Zoom. Click in the check box to enable (checked) or disable (unchecked) zoom persistence in the Filter Viewer. See “Zoom Controls” on page 5-30 for details on zoom controls in the Filter Viewer.

Filter Viewer Tiling Settings

The **Filter Viewer-Tiling** preferences let you change the way the Filter Viewer displays the analysis plots.



Click the radio button to select how the plots are tiled in the display area. Options are **2-by-3 Grid**, **3-by-2 Grid**, **Vertical (6-by-1 Grid)**, and **Horizontal (1-by-6 Grid)**.

This specifies how the plots are arranged when all six plot options are turned on. When fewer options are turned on, the plots are displayed as symmetrically as possible.

Filter Designer Settings

The **Filter Designer** preferences let you set key filter configuration and plot parameters.



FFT Length. Specifies the number of points used to calculate a filter's frequency response.

Type in a new value or variable name to change the FFT length.

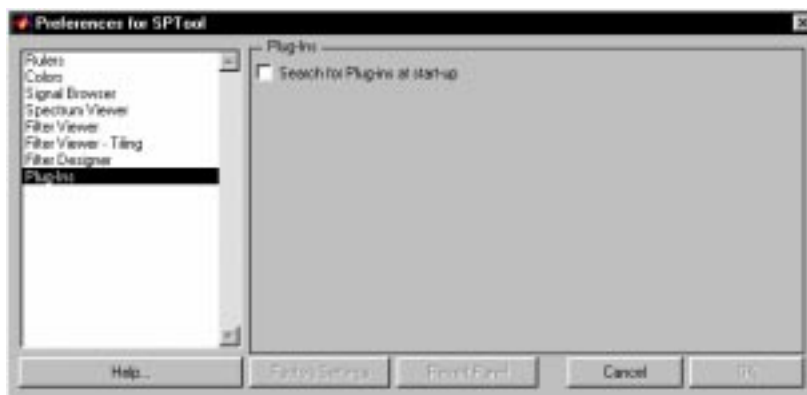
Display grid lines. Turns plot grid lines on (checked) or off (unchecked).

Auto Design – initial value. Specifies the default setting for the **Auto Design** check box in the Filter Designer. When the Filter Designer is first launched, the **Auto Design** check box will have the same setting (checked or unchecked) as the **Auto Design – initial value** check box.

Stay in Zoom-mode After Zoom. Turns persistent zooming on and off, as described in “Zoom Controls” on page 5-30.

Plug-Ins Settings

The **Plug-Ins** preferences let you search for plug-ins when SPTool is started up.



Search for Plug-Ins at start-up. Enables (checked) or disables (unchecked) searching for installed plug-ins.

A *plug-in* is an extension to SPTool. Plug-ins include customized add-on panels and new buttons in the panels in SPTool, new spectral methods in the Spectrum Viewer, and new SPTool preferences. You can also plug one or more toolboxes into SPTool.

You only need to use this setting when you have installed extensions or have other toolboxes plugged into SPTool.

To use SPTool with extensions, check **Search for Plug-ins at start-up**, close SPTool, and restart it.

Saving and Discarding Changes to Preferences Settings

The buttons at the bottom of the **Preferences...** panels let you save or discard any changes you have made, or return to the default settings:



Factory Settings. Restores the preferences in the current panel to their original settings; that is, the settings at the time the Signal Processing Toolbox was first installed.

Revert Panel. Cancels changes in the current panel only. Settings in the current panel revert to the previous settings.

Cancel. Cancels changes in all preferences categories and closes the **Preferences** window. Settings in all panels revert to their previous state.

OK. Applies changes in all preferences panels and closes the **Preferences** window. Settings in all panels are saved in a MAT-file called `si gprefs. mat`.

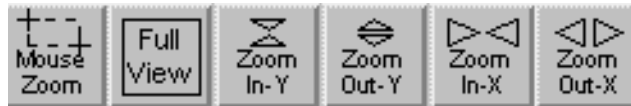
If `si gprefs. mat` does not exist, either on the current MATLAB path or in the current directory, you are prompted for a location to save the file. The saved settings are used the next time you open SPTool.

Controls for Viewing and Measuring

The GUI tools share common controls for viewing and measuring signals. These controls are described in this section. Not all tools use all of the viewing and measuring controls; specific details about the tools and procedures for viewing and measuring are described in the section on each tool.

Zoom Controls

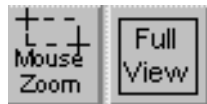
The GUI tools share a common set of zoom control buttons. The Signal Browser and Spectrum Viewer use the same set of common zoom control buttons:



The Filter Designer has one additional viewing button, the **Pass Band** button:



The Filter Viewer has a subset of the zoom control buttons:



Each button works the same way in every GUI tool where it occurs.

In normal use, you click a button once to zoom in or out of the signal display.

Zoom In-X, Zoom Out-X, Zoom In-Y, and Zoom Out-Y. Click once to perform one zoom operation (in or out) on the x - or y -axis. Each zoom operation changes the axes limits by a factor of two on the specified axis, about the center of the displayed signal. You can click repeatedly on one or more buttons to continue to change the scale in one or both axes.

When you zoom in the x -axis (horizontal scaling), the Y limits (vertical scaling) of the main axes are not changed. Similarly, when you zoom in the y -axis, the X limits of the main axes are not changed.

Full View. Click once to restore the displayed signal to its full sample size in both axes.

Mouse Zoom. Click once to activate zoom mode. The cursor changes to a crosshair. You can either zoom in without specifying a zoom window, or you can use a zoom rectangle to select a specific zoom window. In either case, the x - and y -axis are automatically adjusted to display the selected signal.

- To zoom in without specifying a zoom window, click on the plot. The position of the crosshair is the center of a zoom operation that halves both the x - and y -axis limits.
- To use a zoom rectangle, click where you want the rectangle to begin, drag the mouse diagonally to where you want it to end, and release the mouse button.
- To get out of mouse zoom mode without zooming in or out, click on the **Mouse Zoom** button again.

Zoom Persistence. Mouse zooming can either be *one-time* or *persistent*:

- One-time zooming is activated when you click the **Mouse Zoom** button. It automatically turns itself off after you click in the display area and the zoom operation occurs. This is the default for all the tools.
- Persistent zooming is also activated by clicking the **Mouse Zoom** button. It does not turn off after you click in the display area and a zoom operation occurs; you can continue to click and zoom without resetting the **Mouse Zoom** button.

You can change whether zooming is one-time or persistent by selecting **Preferences...** from the **File** menu and toggling **Stay in Zoom-mode After**

Zoom in the preferences panels for the Signal Browser, the Spectrum Viewer, the Filter Designer, and the Filter Viewer.

When **Stay in Zoom-mode After Zoom** is selected, zooming is persistent. To turn off mouse zooming when **Stay in Zoom-mode After Zoom** is selected, click the **Mouse Zoom** button.

Passband Zoom (Filter Designer). Click once to zoom in on the passband of the response.

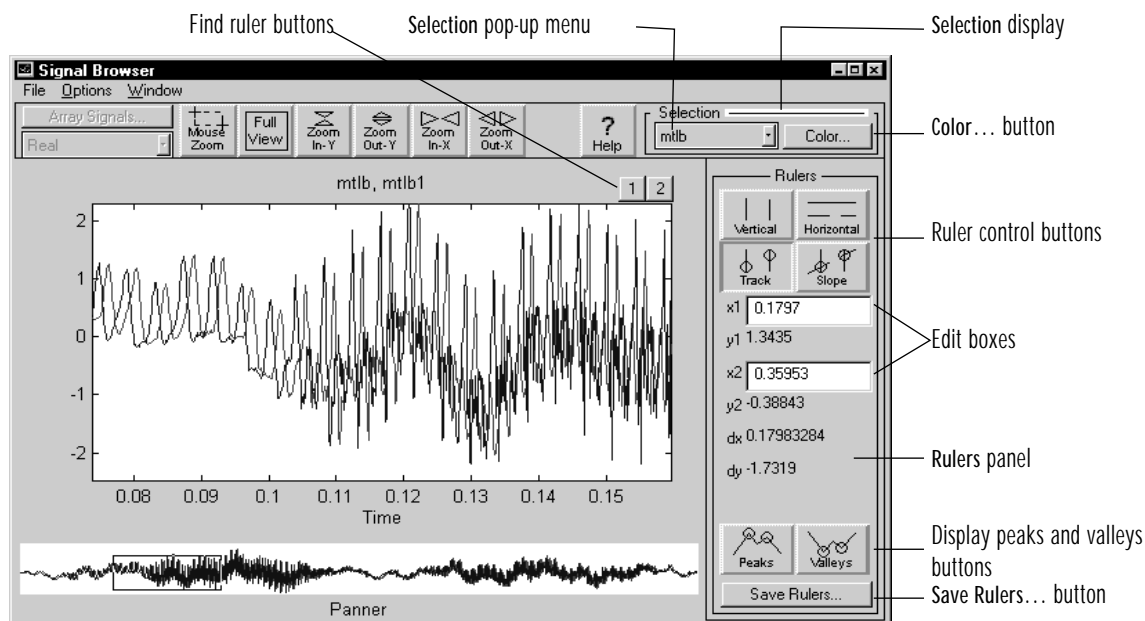
Both the x - and y -limits of the main axes are changed so that the passband fills the main axes.

There is no stopband zoom button. To zoom the stopband, use standard **Mouse Zoom**, centering the crosshair on the area of the stopband you want to view. If you are in passband zoom, first press **Full View** to return to the standard view.

Ruler Controls

The Signal Browser, Filter Viewer, and Spectrum Viewer share a common set of ruler controls. Use the rulers to make measurements on the signals or spectra in the main axes (display) area. The ruler controls give you a variety of ways to read and control the values of the rulers in the main axes. With the rulers you can measure such information as the vertical and horizontal distance between features in a signal or spectrum, the dimensions of peaks and valleys, and point and slope information.

In the following discussion, the Signal Browser is shown. The ruler controls include the **Selection** controls at the top right of the window and the buttons and edit boxes in the **Rulers** panel. The controls in the Filter Viewer and Spectrum Viewer work the same way. In the Filter Viewer, the rulers only appear on one subplot at a time. You can choose which subplot the rulers appear on by selecting the subplot from the pop-up menu at the top of the **Rulers** panel. If a subplot is not currently visible when you select its name from the pop-up menu, the Filter Viewer creates the subplot and places the rulers in it.



Selecting a line to measure. When there is only one signal displayed, the displayed signal is automatically selected and is measured when you use the rulers. When there is more than one signal displayed, only one signal (line) may be selected and measured at a time.

When a signal (line) is selected, you can use the ruler controls (**Vertical**, **Horizontal**, **Track**, or **Slope**) and the **Peaks** and/or **Valleys** controls on the selected line. The label of the selected signal (line) is displayed in the **Selection** pop-up menu.

There are two ways to select a signal (line):

- Click on the **Selection** pop-up menu and drag to select the line to measure. All signals that are currently selected in SPTool are listed. Vector signals in the Signal Browser, spectra in the Spectrum Viewer, and filters in the Filter Viewer are listed as single variables; in the Signal Browser, each column of a two-dimensional signal matrix is listed as a separate variable.
- Move the mouse pointer over any point in the line you want to select and click on it.

The label of the signal, including the column number if the line is one column of a matrix, is displayed in the **Selection** pop-up menu.

The line selection display changes to the color and pattern of the selected signal, spectrum, or filter.

Line Selection Pop-Up Menu. Use to select a line (vector signal, array column, filter, or spectrum) to measure.

Click the **Selection** pop-up menu and drag to select the line.

Line Selection Display. The line color and style of the selected signal are displayed.



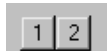
Color... Button. Use to edit the line style or display color of the selected line.

Click on the **Color...** button at the top right of the window to display the **Edit Line** pop-up menu, which is shown on the left. The label of the selected line is displayed in the **Label** field.

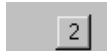
- Click the **Line Style** pop-up menu and drag to select a line style, as shown on the left.
- Click a radio button to select a color. If you select **Other**, you can type a color value in the **Enter colorspec** box; the color value can be a string (e.g., 'r') or an RGB triple (e.g., [1 0 0]).
- Click **OK** to apply the line style and color you selected.

Find Ruler Buttons. Use the find ruler buttons to bring one or both rulers into the viewing area of the main axes. When both rulers are within the signal display (main axes) area, the find ruler buttons, at the top right of the main axes area, are not displayed.

If the rulers are not within the signal display area, both **Find Ruler** buttons are displayed, as shown on the right:



If one ruler is within the signal display area, the button for the other ruler is displayed, as shown on the right:

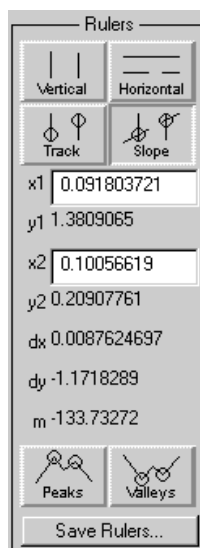


Click a **Find Ruler** button to bring the specified ruler into the display area.

When a ruler is visible, you can click on it and drag it to make a measurement on the selected signal. See “Making Signal Measurements” on page 5-37 for details on manipulating the rulers and measuring the selected signal.

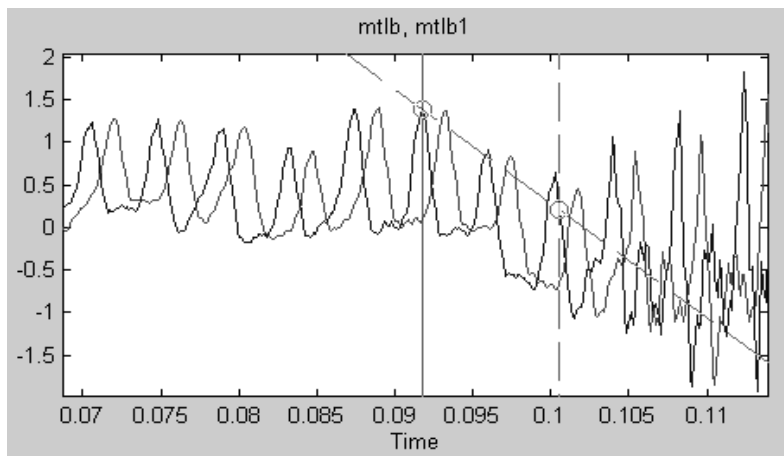
Ruler Control Buttons. Use the ruler control buttons to select the type of measurement you want to make: **Vertical**, **Horizontal**, **Track**, or **Slope**. The default setting is **Track**.

Click a ruler control button to select it. The **Rulers** panel changes depending on which ruler control is selected. See “Making Signal Measurements” on page 5-37 for details on the four kinds of measurements you can make and the parameters for each one.



Rulers Panel and Edit Boxes. The **Rulers** panel changes depending on which ruler control is selected: **Vertical**, **Horizontal**, **Track**, or **Slope**. It shows the parameters for the selected ruler control. Depending on which ruler control is selected, the following fields are displayed: **x1**, **y1**, **x2**, **y2**, **dx**, **dy**, **m**. The picture on the left shows the **Rulers** panel when **Slope** is selected.

When you click on a ruler control button, rulers are displayed superimposed on the signal(s) in the main axes display area. The rulers are either vertical (for **Vertical**, **Track**, and **Slope**) or horizontal (for **Horizontal**). For **Track** and **Slope**, ruler markers are also displayed. The rulers and ruler markers are associated with the currently selected signal. The following picture shows the rulers and ruler markers that are displayed when **Slope** is selected.



To position a ruler, you can click and drag on it. When you drag a ruler, the parameters in the **Rulers** panel change to reflect the measurements on the selected signal.

You can also position a ruler by specifying parameters in the edit boxes in the **Rulers** panel. The parameters are either the **x1** and **x2** values or the **y1** and **y2** values, depending on which ruler control is selected.

Type the value or variable for the ruler parameter in the **x1** and **x2** boxes or the **y1** and **y2** boxes. See “Making Signal Measurements” on page 5-37 for details on manipulating the rulers and the parameters you can measure with each one.

Peaks and Valleys. Use these buttons to show or hide the local maxima and/or local minima of the currently selected signal, filter response, or spectrum. Only peaks or valleys, or both peaks and valleys may be displayed.

- Click **Peaks** to toggle showing (down) or hiding (up) the maxima of the signal.
- Click **Valleys** to toggle showing (down) or hiding (up) the minima of the signal.

In track and slope mode (see “Making Signal Measurements” on page 5-37), the rulers are constrained to the peaks or valleys. In horizontal and vertical mode, the peaks and valleys are only visual and do not affect the behavior of the rulers.

Save Rulers... Button. Once you’ve set up and made a certain set of measurements, you may find it useful to save them for future reference. Use the **Save Rulers...** button to save a structure in the MATLAB workspace with the fields `x1`, `y1`, `x2`, `y2`, `dx`, `dy`, `m`, `peaks`, and `valleys`. Undefined values are set to NaN.

- 1 Click **Save Rulers...** to save the current measurements as a variable in the workspace.

The Save Rulers dialog box is displayed.


- 2 Type a variable name in the edit field and click **OK**.

Making Signal Measurements

Use the rulers to make measurements on a selected line, which is a vector or a column of a matrix in the Signal Browser, a filter response in the Filter Viewer, or a spectrum in the Spectrum Viewer. To make a measurement:

- 1 Select a line as described in “Selecting a line to measure” on page 5-33.
- 2 Apply a ruler to the display as described in “Ruler Control Buttons” on page 5-35.
- 3 Position a ruler where you want it in the main axes area by clicking and dragging it:

- a Move the mouse over the ruler (1 or 2) that you want to drag.

The hand cursor is displayed when you're over a ruler, with the ruler number inside it: 

- b Click and drag the ruler to where you want it on the signal.

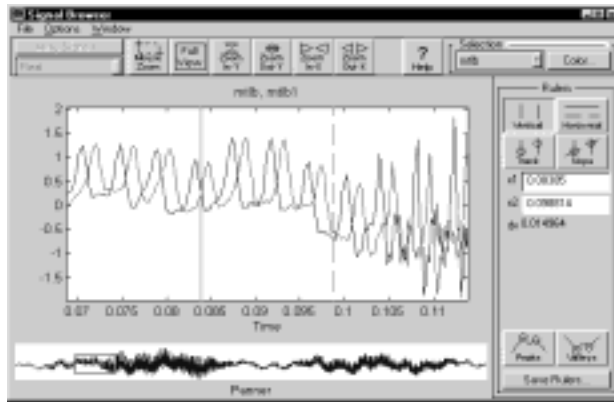
Depending on which ruler control is selected, you can drag the ruler to the right and left (**Vertical**, **Track**, and **Slope**) or up and down (**Horizontal**).

As you drag a ruler, the **Rulers** panel shows the current position of both rulers. Depending on which ruler control is selected, the following fields are displayed: **x1**, **y1**, **x2**, **y2**, **dx**, **dy**, **m**.

You can also position a ruler by typing its **x1** and **x2** or **y1** and **y2** values in the **Rulers** panel, as described on page 5-35.

Ruler Controls: Vertical. There are two vertical rulers, called ruler 1 and ruler 2. When vertical rulers are in use, the measurements displayed in the **Rulers**

panel are **x_1** (the position of ruler 1 on the x -axis), **x_2** (the position of ruler 2 on the x -axis), and **dx** (the value of $x_2 - x_1$).



Click **Vertical** to put the rulers in vertical mode.

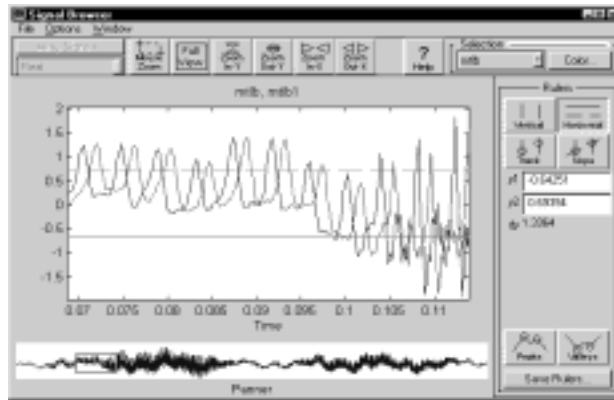
In vertical mode, you may change the x -values of the rulers (that is, their horizontal position). As the **x_1** and **x_2** values change, the value of **dx** changes automatically.

Change the **x_1** and **x_2** values by either:

- Dragging the rulers to the left and the right with the mouse
- or
- Entering their values in the **x_1** and **x_2** edit boxes in the **Rulers** panel

Ruler Controls: Horizontal. There are two horizontal rulers, called ruler 1 and ruler 2. When horizontal rulers are in use, the measurements displayed in the

Rulers panel are **y1** (the position of ruler 1 on the *y*-axis), **y2** (the position of ruler 2 on the *y*-axis), and **dy** (the value of **y2-y1**).



Click **Horizontal** to put the rulers in horizontal mode.

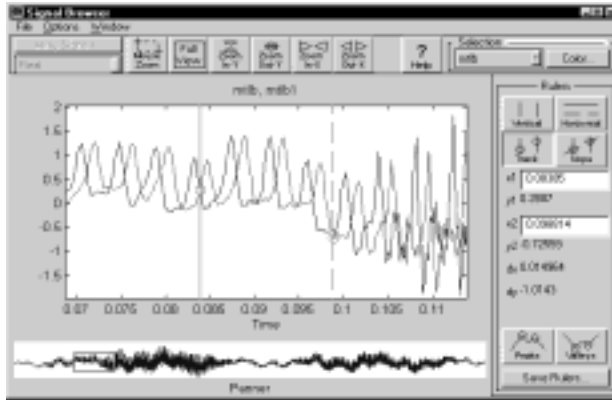
In horizontal mode, you may change the *y*-values of the rulers (that is, their vertical position). As the **y1** and **y2** values change, the value of **dy** changes automatically.

Change the **y1** and **y2** values by either:

- Dragging the rulers up and down with the mouse
- or
- Entering their values in the **y1** and **y2** edit boxes in the **Rulers** panel

Ruler Controls: Track. There are two vertical rulers, called ruler 1 and ruler 2, with a marker on each that shows the *y*-values of the signal at the *x*-values of the rulers. When track rulers are in use, the measurements displayed in the **Rulers** panel are **x1** (the position of ruler 1 on the *x*-axis), **y1** (the position of

ruler 1 on the y -axis), x_2 (the position of ruler 2 on the x -axis), y_2 (the position of ruler 2 on the y -axis), dx (the value of $x_2 - x_1$), and dy (the value of $y_2 - y_1$).



Click **Track** to put the rulers in track mode.

You can change the track marker in the **Rulers** preferences panel; see “Ruler Settings” on page 5-22.

In track mode, you may change the x -values of the rulers (that is, their horizontal position). As the x_1 and x_2 values change, the values of y_1 , y_2 , dx , and dy change automatically.

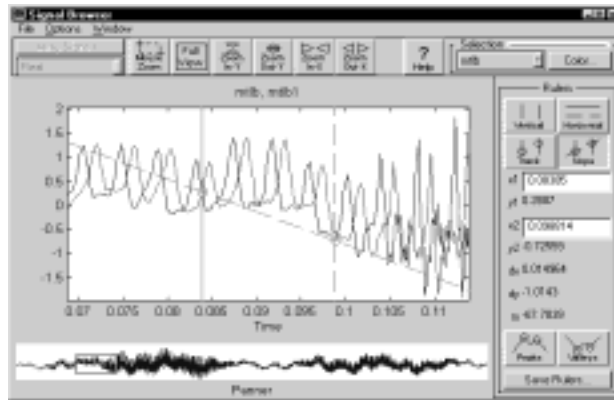
Change the x_1 and x_2 values by either:

- Dragging the rulers to the left and the right with the mouse
- or
- Entering their values in the x_1 and x_2 edit boxes in the **Rulers** panel

Ruler Controls: Slope. There are two vertical rulers, called ruler 1 and ruler 2, with the slope line passing through the y -axis intersections of the two vertical rulers and the signal. The rulers also track the signal with markers on each ruler that shows the y -values of the signal at the x -values of the rulers. The line connecting (x_1, y_1) and (x_2, y_2) is included in the main plot, so you can approximate derivatives and slopes of the signal.

When slope rulers are in use, the measurements displayed in the **Rulers** panel are x_1 (the position of ruler 1 on the x -axis), y_1 (the position of ruler 1 on the y -axis), x_2 (the position of ruler 2 on the x -axis), y_2 (the position of ruler 2 on

the y -axis), \mathbf{dx} (the value of $\mathbf{x2} - \mathbf{x1}$), \mathbf{dy} (the value of $\mathbf{y2} - \mathbf{y1}$), and \mathbf{m} (equal to $\mathbf{dy/dx}$, the slope of the line between $\mathbf{x1}$ and $\mathbf{x2}$).



Click **Slope** to put the rulers in slope mode.

In slope mode, you may change the x -values of the rulers (that is, their horizontal position). As the $\mathbf{x1}$ and $\mathbf{x2}$ values change, the values of \mathbf{dy} and \mathbf{m} change automatically.

Change the $\mathbf{x1}$ and $\mathbf{x2}$ values by either:

- Dragging the rulers to the left and the right with the mouse
or
- Entering their values in the $\mathbf{x1}$ and $\mathbf{x2}$ edit boxes in the **Rulers** panel

Using the Signal Browser: Interactive Signal Analysis

The Signal Browser tool is an interactive signal exploration environment. It provides a graphical view of the signal object(s) currently selected in the **Signals** column of SPTool.

Using the Signal Browser you can:

- View and compare vector or array signals
- Zoom in on a range of signal data to examine it more closely
- Measure a variety of characteristics of signal data
- Play signal data on audio hardware

Opening the Signal Browser

To open or activate the Signal Browser from SPTool:

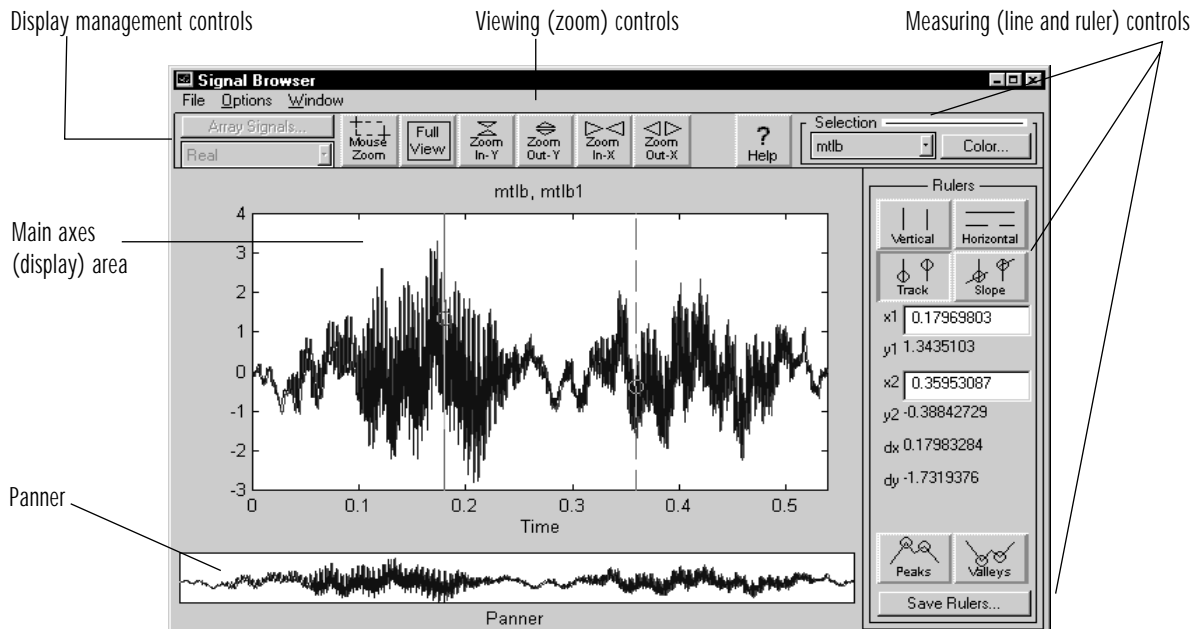
- 1 Click on one or more signals in the **Signals** list of SPTool.
- 2 Press **View** in the **Signals** panel of SPTool.

The Signal Browser is activated and the selected signal(s) are loaded into the Signal Browser and displayed.

Basic Signal Browser Functions

The Signal Browser has the following components:

- A main axes (display) area for viewing signals graphically
- Display management controls: **Array Signals...** and the complex signal display pop-up menu
- Zoom controls for getting a closer look at signal features
- Rulers and line display controls for making signal measurements and comparisons
- A panner for seeing what part of the signal is currently being displayed, and quickly moving the view to other features of the signal
- A menu option for playing a selected signal through audio equipment



Menus

File Menu. Use **Close** from the **File** menu to close the Signal Browser. All signal selection and ruler information will be lost. Settings you changed and saved using the **Preferences...** window in SPTool are saved and used the next time you open a Signal Browser.



Options Menu. Use **Play** from the **Options** menu to play the selected signal.

Play only works when you have sound capabilities on your computer. If your computer does not have sound capabilities, this menu choice does nothing.

The entire selected signal is played at either F_s (the sampling frequency of the signal) or at the default platform sampling frequency if F_s is less than 25 Hz. The real part and the imaginary part of a complex signal are played in separate channels.

Window Menu. Use the window menu to select a currently open MATLAB Figure window.

Zoom Controls

The available zoom controls in the Signal Browser are **Mouse Zoom**, **Full View**, **Zoom In-Y**, **Zoom Out-Y**, **Zoom In-X**, and **Zoom Out-X**. See “Zoom Controls” on page 5-30 for details on using the zoom controls in the Signal Browser.

Zoom persistence is off by default in the Signal Browser; use the Signal Browser settings panel in **Preferences...** to toggle zoom persistence on and off. See “Signal Browser Settings” on page 5-24.

Ruler and Line Display Controls

Using the rulers and line display controls, you can measure a variety of characteristics of signals in the Signal Browser. See “Ruler Controls” on page 5-32 for details on using rulers and modifying line displays in the Signal Browser.

The rulers are displayed by default in the Signal Browser; you can turn off the ruler display in the **Signal Browser** settings panel in the **Preferences** dialog box. See “Signal Browser Settings” on page 5-24.

Help Button

To use context-sensitive help, click on the **Help** button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in the Signal Browser, including menu items, to find out what it is and how to use it.

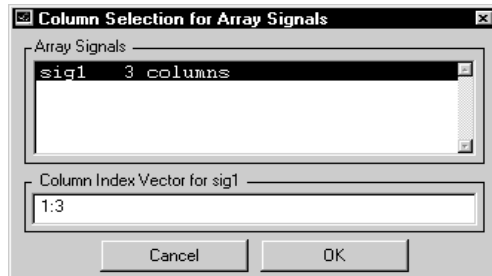
Display Management Controls

Array Signals... Button. Use this to enter a column index vector for a selected array signal. All array signals start out with only the first column displayed.

The **Array Signals...** button is enabled when at least one array signal is selected in SPTool.

1 Click on **Array Signals...**

The **Column Selection for Array Signals** dialog box is displayed:



All array signals that are selected in SPTool are shown in the list.

2 Select a signal from the list.

3 Type a column index vector for the selected signal.

Valid index vectors are of the form 1 or 1:3 or [1 3 5].

Complex Signal Display. Use to specify whether the Signal Browser plots the real part, the imaginary part, the magnitude, or the angle of a complex signal.

This menu is enabled when at least one of the signal variables selected in SPTool is complex. The Complex Display mode affects all of the variables in the current selection, even those that are strictly real.

Click and drag to select the plotting mode.

Main Axes Display Area

The **Signals** list in SPTool shows all signals in the current SPTool session. One or more signals may be selected. The signal data of all selected signals are displayed graphically in the main axes display area of the Signal Browser.

When there is only one signal displayed, its properties are reflected in the display management controls and its measurements are displayed in the ruler display panel. When more than one signal is displayed, select the line you want to focus on.

When a signal is selected, you can use the ruler controls on the selected line (see “Making Signal Measurements” on page 5-37), you can choose how to display the signal (see “Display Management Controls” on page 5-44), and you can play the signal (see “Options Menu” on page 5-43). The label of the selected signal (line) is displayed in the **Selection** pop-up menu.

There are three ways to select a signal (line) in the Signal Browser:

- Click on the **Selection** pop-up menu and drag to select the line to measure.
or
- Move the mouse pointer over any point in the line in the main axes display and click on it.
or
- Move the mouse pointer over any point in the line in the panner and click on it.

See “Selecting a line to measure” on page 5-33 for details.

Axes Labels. By default, the x -axis in the Signal Browser is labeled Time. You can change the x -axis label and add a y -axis label using the **Signal Browser** settings panel in **Preferences....** See “Signal Browser Settings” on page 5-24.

Click-and-Drag Panning. You can use the mouse to pan around the main axes display:

Click on a line in the main axes, hold down the mouse button, and drag the mouse.

Click-and-drag panning is not enabled in mouse zoom mode.

Panner

The panner gives a panoramic view of the signal(s) displayed in the main axes. The panner always displays the entire signal sample. When you zoom in on the main axes, a patch in the panner shows the section of the plot that is currently in view in the main axes. Click-and-drag the patch in the panner window to pan dynamically across the signal data in the main axes.

You can also select a line by clicking on it in the panner; the selected line is highlighted in both the panner and in the main axes display area.

See “Panner Display” on page 5-51 for more details.

The panner is displayed by default in the Signal Browser; you can turn off the panner in the Signal Browser settings panel in **Preferences....** See “Signal Browser Settings” on page 5-24.

Making Signal Measurements

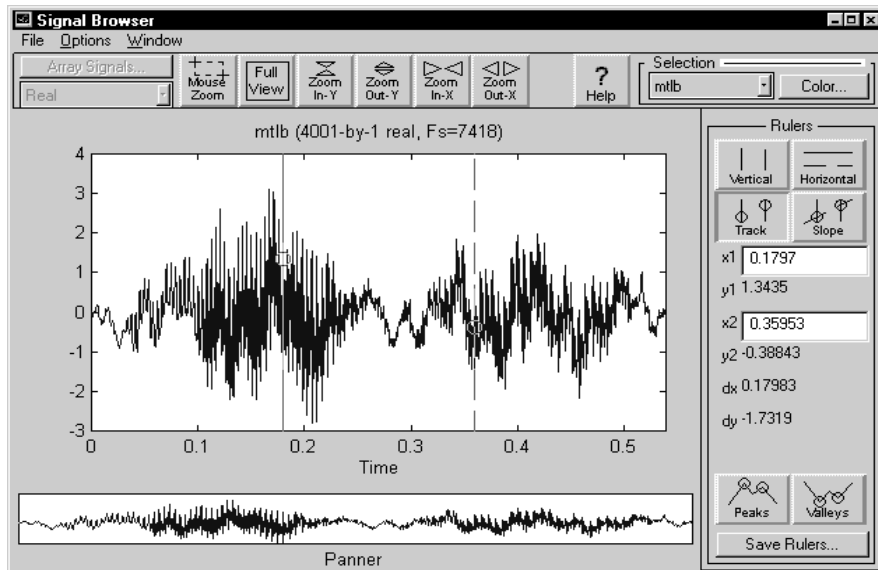
Use the rulers to make a variety of measurements on the selected signal. See “Making Signal Measurements” on page 5-37 for details.

Viewing and Exploring Signals

You can open or activate the Signal Browser in SPTool by selecting one or more signals and pressing **View** in the Signal panel. The selected signals are loaded into the Signal Browser. See “Viewing a Signal” on page 5-17 for details.

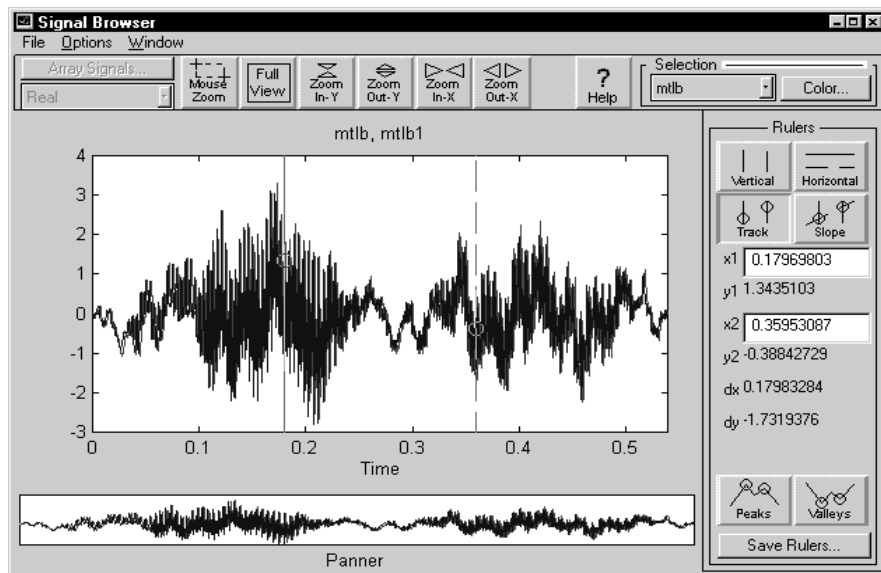
Selecting and Displaying a Signal

When the Signal Browser is activated, all selected signals are displayed in the main axes display area and in the panner. The elements of the selected signals are plotted versus an equally spaced time vector in both the main axes display and the panner:



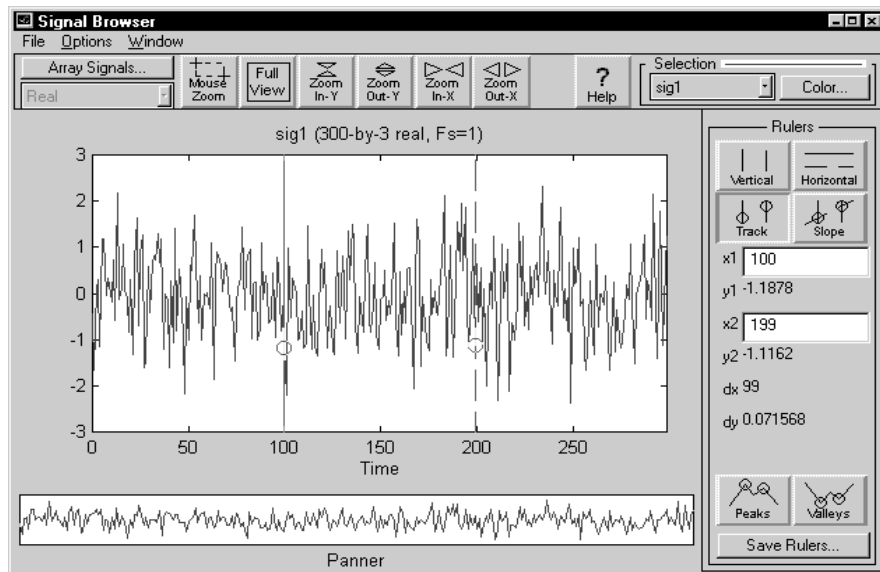
When the variable is a vector, one signal is displayed, as in the example above. It is automatically selected and its name, size, type, and sampling frequency are displayed above the main axes display; the name is also highlighted in the **Selection** pop-up menu.

When more than one signal is selected, each signal is displayed in a different color in both the main axes display and the panner:



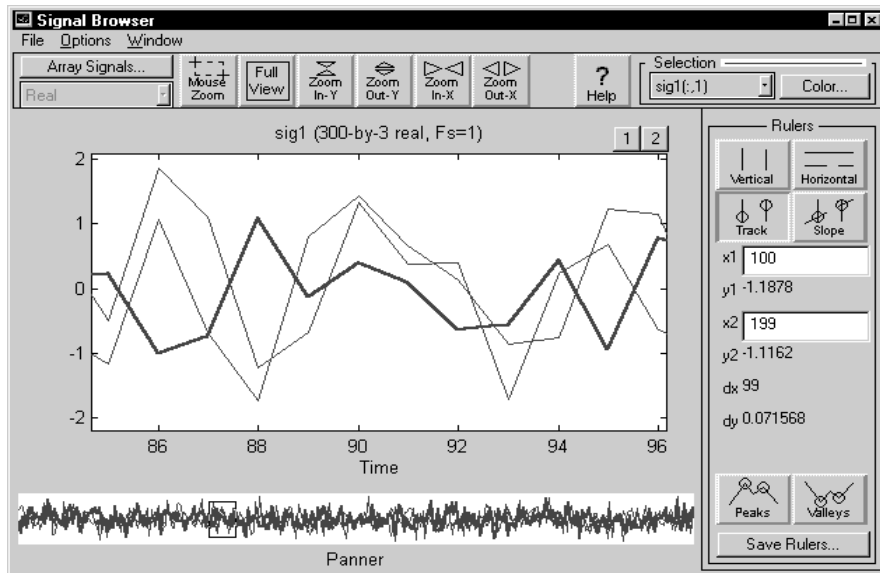
The names of all signals are displayed above the main axes display. The first signal in the list is automatically selected in both the main axes display and the panner, its name is highlighted in the **Selection** pop-up menu, and its color is shown in the **Selection** display.

When the signal is an array, only the first column is initially displayed in both the main axes and the panner:



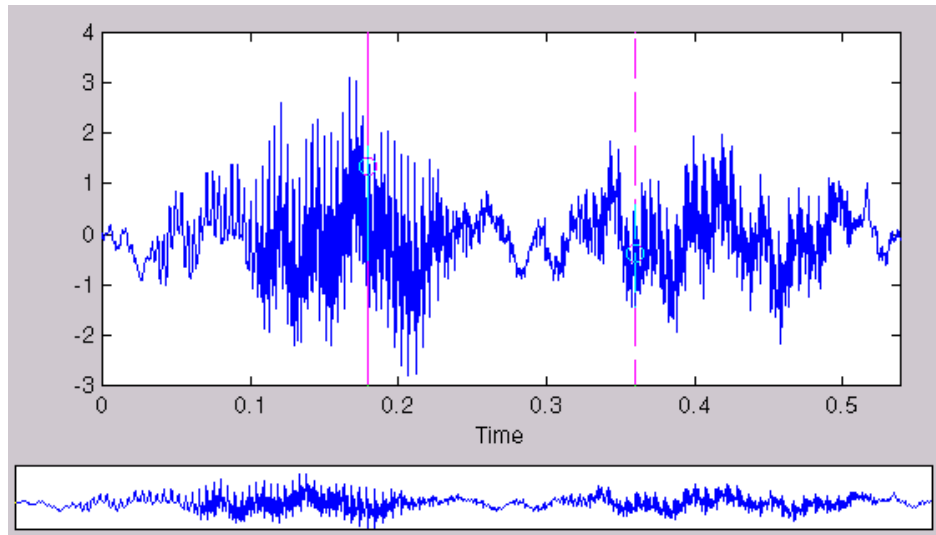
To display a different array column, or more than one column of the array, click the **Array Signals...** button and specify the column vectors to be displayed (see “Array Signals... Button” on page 5-44). All displayed columns of an array are shown in the same color; the selected column is emphasized with a heavier line

in both the main axes and the panner, and its label is displayed in the **Selection** pop-up menu:

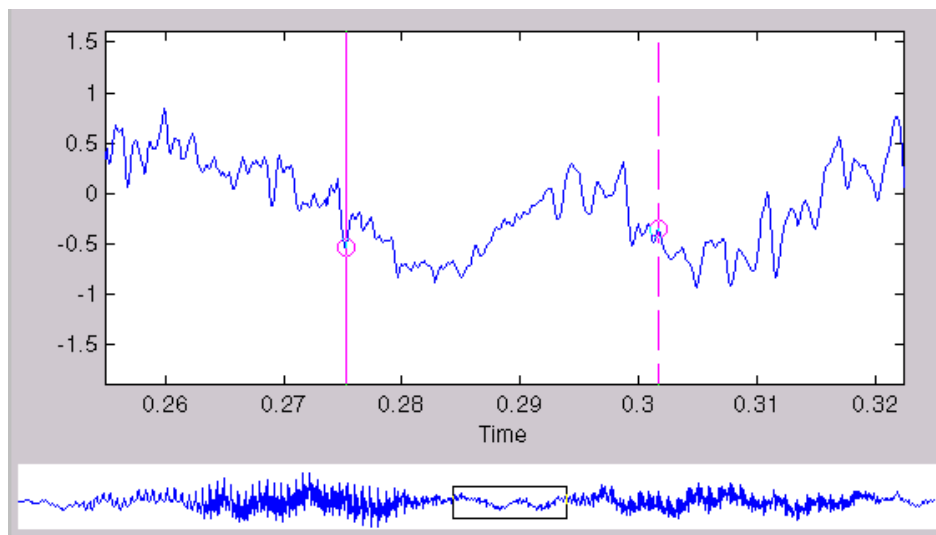


Panner Display

The panner displays the entire signal sample at all times:



When the signal in the main axes is zoomed, the part of the signal that is shown in the main axes is shown in the panner with a window around it:



Each time you zoom, the panner is updated to frame the region of data displayed in the main axes.

Click-and-drag on the panner window to move it. As the window moves over the signal in the panner, the signal in the main axes area is panned.

Manipulating Displays

Changing Signal Displays. The signals are displayed in the default line colors and default line styles. You can change the defaults using the **Color Order** and **Line Style Order** fields in the **Colors** settings panel (see “Color Settings” on page 5-23).

Changing the Sample Interval. You can change the sample interval by selecting **Sampling Frequency...** from the **Edit** menu in SPTool. See “Editing Data Objects in SPTool” on page 5-16.

Displaying Complex Signals. You can change how complex numbers are displayed by selecting **Real**, **Imaginary**, **Magnitude**, or **Angle** from the pop-up menu. See “Array Signals... Button” on page 5-44.

Changing Signal Browser Displays. Using the **Signal Browser** settings panel in **Preferences...**, you can set optional *x*-axis and *y*-axis labels, enable and disable the display of the rulers and the panner, and toggle zoom persistence. See “Signal Browser Settings” on page 5-24.

Working with Signals

Once a signal is displayed, you can browse it in a variety of ways:

- You can zoom in on a specific area of the display (see “Zoom Controls” on page 5-30).
- You can mark off a segment of the display with the rulers (see “Ruler Controls” on page 5-32) and save ruler settings (see “Save Rulers... Button” on page 5-36).
- You can select a segment of the display with the panner (see “Panner Display” on page 5-51).
- You can make certain measurements on the displayed signals (see “Making Signal Measurements” on page 5-37).
- When there is more than one signal in the display, you can select which one you want to measure (see “Selecting a line to measure” on page 5-33).

You can use the other GUI tools to manipulate signals in a variety of ways:

- You can interactively design and analyze filters to be applied to signals (see “Using the Filter Designer: Interactive Filter Design” on page 5-55 and “Using the Filter Viewer: Interactive Filter Analysis” on page 5-74).
- You can create a spectrum for a signal and interactively analyze its spectral density with a variety of estimation methods (see “Using the Spectrum Viewer: Interactive PSD Analysis” on page 5-88).

You access the Filter Designer, Filter Viewer, and Spectrum Viewer tools from SPTool. You can access SPTool from the Signal Browser by:

- Clicking on an active SPTool window
or
- Activating SPTool using the **Window** menu in the Signal Browser

Saving Signal Data

After creating a signal in SPTool (by applying a filter to an imported signal, for example), you can export the signal information to the workspace or to disk using **Export...** from the **File** menu in SPTool. The signal information is stored in a structure that you can access to retrieve the signal data and sample frequency. The signal structure also contains a number of fields that are used internally by SPTool.

To see the fields of the signal structure, try exporting a signal to the workspace:

- 1 Import a signal into SPTool if there are none currently loaded (see “Importing Signals, Filters, and Spectra” on page 5-8). Label the imported signal `si g1`.
- 2 Export the signal. Select **Export...** from the **File** menu.
- 3 In the **Export** dialog box, select `si g1` and press the **Export to Workspace** button.
- 4 Type `who` at the MATLAB command line to look at the variables in the workspace. The variable called `si g1` is the signal structure you exported from SPTool.
- 5 Type `si g1` at the command line to list the fields of the signal structure.

The `data` and `Fs` fields of the signal structure contain the information that defines the signal. The other fields are used internally by SPTool, and are subject to change in future releases.

- The `data` field is a vector or array containing the signal's data.
- The `Fs` field contains the sample frequency of the signal in Hertz.

Using the Filter Designer: Interactive Filter Design

The Filter Designer provides an interactive graphical environment for the design of digital IIR and FIR filters based on specifications that you enter on a magnitude plot. Using the Filter Designer you can design IIR and FIR filters of various lengths and types, with standard frequency band configurations (highpass, lowpass, bandpass, or bandstop filters).

The Filter Designer provides access to many of the IIR filter design functions discussed in Chapter 2, including the `butter`, `cheby1`, `cheby2`, and `ellip` functions. In addition, the Filter Designer provides access to the `remez`, `firls`, and `kaiser` functions for the design of FIR filters with highpass, lowpass, bandpass, or bandstop configurations.

Using the Filter Designer you can:

- Design IIR filters with standard band configurations, using the Butterworth, Chebyshev type I, Chebyshev type II, and elliptic design options
- Design FIR filters with standard band configurations, using the equiripple, least squares, and Kaiser window design options
- Specify the filter's sampling frequency and the passband and stopband edge frequencies
- Specify the desired amount of ripple in the filter's passband, and attenuation in the stopband
- Redesign a filter by manually adjusting indicators in the magnitude plot
- Use an automatically computed filter order or set a custom filter order
- Overlay a spectrum on the filter's magnitude response plot

When you have designed a filter to your specifications, you can apply the filter to a selected signal using the **Apply** button in SPTool (see “Applying a Filter” on page 5-19).

NOTE For information on using filter design functions from the command line or from M-files, see Chapter 2.

Opening the Filter Designer

You can open or activate the Filter Designer from SPTool by pressing the **New Design** or **Edit Design** buttons.

- To create a filter, press **New Design** in SPTool. A default filter is generated and displayed in the Filter Designer. You can view the filter in a variety of ways in the Filter Viewer and modify it in the Filter Designer.
- To edit a filter, select it in SPTool and press **Edit Design**. You can modify the filter in a variety of ways in the Filter Designer.

See “Designing a Filter” on page 5-18 for complete details.

Basic Filter Designer Functions

The Filter Designer has the following components:

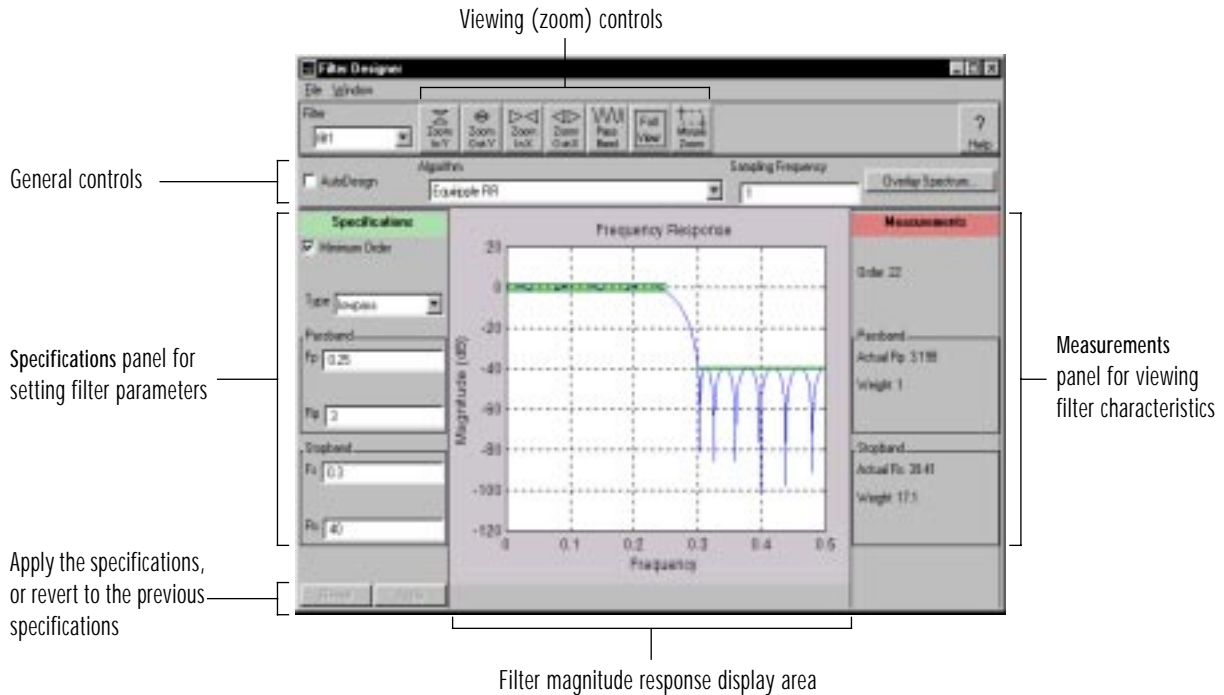
- A pop-up menu for selecting different filter designs
- A magnitude plot (display) area for viewing filters graphically
- A **Specifications** panel for viewing and modifying the design parameters of the current filter
- A **Measurements** panel for viewing the characteristics of the current filter
- Specification lines for graphically adjusting the response parameters of a filter
- Measurement lines for measuring the response parameters of a filter
- Zoom controls for getting a closer look at the magnitude response features

The following sections describe the different components of the Filter Designer’s interface, and how you can use them together to create and edit filters.

Menus

File Menu. Use **Close** from the **File** menu to close the Filter Designer. Settings you changed and saved using the **Preferences...** window in SPTool are saved and used the next time you open the Filter Designer.

Window Menu. Use the **Window** menu to select a MATLAB Figure window.



Filter Pop-Up Menu

The **Filter** pop-up menu displays all of the filters currently selected in SPTool. Select a filter in the menu to make it the current filter in the Filter Designer.

Zoom Controls

The available zoom controls in the Filter Designer are **Zoom In-Y**, **Zoom Out-Y**, **Zoom In-X**, **Zoom Out-X**, **Pass Band**, **Full View**, and **Mouse Zoom**. See “Zoom Controls” on page 5-30 for details on using the zoom controls.

Zoom persistence is off by default in the Filter Designer; use the Filter Designer settings panel in the **Preferences** dialog to toggle zoom persistence on and off. See “Filter Designer Settings” on page 5-28.

Help Button

To use context-sensitive help, click on the **Help** button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on any

object in the Filter Designer, including menu items, to find out what it is and how to use it.

General Controls

Beneath the zoom controls are several general controls for filter design and display.

Algorithm. Use the **Algorithm** pop-up menu to select a design for the current filter. When you select a design from the **Algorithm** pop-up menu, the magnitude response plot, **Specifications** panel, and **Measurements** panel all update to reflect the new design parameters.

Auto Design. When the **Auto Design** check box is checked, the filter's magnitude response is redrawn whenever a filter specification is changed, either by entering a value in the **Specifications** panel or by dragging a specification line on the plot. When the box is not checked, the new response is computed and redrawn only when the **Apply** button is pressed or the specification line is released.

Auto Design is initially off by default; use the **Filter Designer** settings panel in the **Preferences** dialog to change this default setting. See "Filter Designer Settings" on page 5-28.

Sampling Frequency. The **Sampling Frequency** field allows you to specify the filter's sampling frequency in Hertz. To change the sampling frequency, type a value in the box and press **Enter** (**Return** on Macintosh). This is the same as changing the sampling frequency by selecting **Sampling Frequency...** from the SPTool **Edit** menu (see "Editing Data Objects in SPTool" on page 5-16). The frequency axis of the magnitude response plot is updated to reflect the new sampling frequency.

Overlay Spectrum. The Filter Designer allows you to overlay a signal spectrum on the filter's magnitude response plot. Press the **Overlay Spectrum...** button to display a list of the current spectra in SPTool. Select a spectrum from the list and press **OK** to overlay it on the current magnitude response plot. Note that the spectrum is plotted on the existing frequency axis, which is scaled to the filter's sampling frequency.



Filter Specifications Panel

When you design a new filter, the Filter Designer initially contains the specifications and magnitude response plot for an order 22, lowpass, equiripple filter, as shown in the panel at the left.

Use the **Type** pop-up menu in the **Specifications** panel to select a band configuration. Use the edit boxes below it in the panel to change the band edge frequencies and the amount of ripple in the passband and attenuation in the stopband. Check the **Minimum Order** box to let the Filter Designer automatically determine the lowest filter order that achieves the current specifications.

The design parameters that are available in the **Specifications** panel depend on the filter design selected in the **Algorithm** pop-up menu, the band configuration selected in the **Type** pop-up menu, and the state of the **Minimum Order** check box.



Specifications Parameters—Automatic Order Selection. When the **Minimum Order** box is checked, all of the filter designs except **Least Squares FIR** display the same set of parameters in the **Specifications** panel. (The order for the **Least Squares FIR** design cannot be automatically computed). For lowpass and highpass band configurations, these parameters include the passband edge frequency F_p , the stopband edge frequency F_s , the passband ripple R_p , and the stopband attenuation R_s . For bandpass and bandstop configurations, the parameters include the lower and upper passband edge frequencies, F_{p1} and F_{p2} , the lower and upper stopband edge frequencies, F_{s1} and F_{s2} , the passband ripple R_p , and the stopband attenuation R_s . Frequency values are in Hertz, and ripple and attenuation values are in dB.

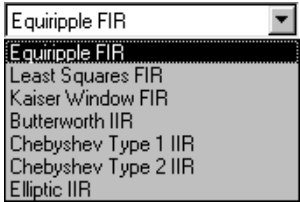


Specifications Parameters—Manual Order Selection. When the **Minimum Order** box is not checked, an **Order** parameter for setting the filter order appears below it, and each filter design displays a unique set of parameters in the **Specifications** panel. These are shown in the table below, where W_p and W_s are the weights for the passband and stopband, β is the Kaiser window β parameter, F_c is the cutoff frequency, and F_{3db} is the 3 dB frequency.

	Lowpass	Highpass	Bandpass	Bandstop
Equiripple FIR	Fp, Fs, Wp, Ws	Fp, Fs, Wp, Ws	Fp1, Fp2, Fs1, Fs2, Wp, Ws	Fp1, Fp2, Fs1, Fs2, Wp, Ws
Least Squares FIR	Fp, Fs, Wp, Ws	Fp, Fs, Wp, Ws	Fp1, Fp2, Fs1, Fs2, Wp, Ws	Fp1, Fp2, Fs1, Fs2, Wp, Ws
Kaiser Window FIR	Fc, Beta	Fc, Beta	Fc1, Fc2, Beta	Fc1, Fc2, Beta
Butterworth IIR	F3db	F3db	F3db 1, F3db 2	F3db 1, F3db 2
Chebyshev Type I IIR	Fp, Rp	Fp, Rp	Fp1, Fp2, Rp	Fp1, Fp2, Rp
Chebyshev Type II IIR	Fs, Rs	Fs, Rs	Fs1, Fs2, Rs	Fs1, Fs2, Rs
Elliptic IIR	Fp, Rp, Rs	Fp, Rp, Rs	Fp1, Fp2, Rp, Rs	Fp1, Fp2, Rp, Rs

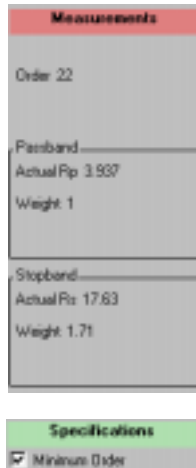
In general, follow these steps to design a new filter using the **Specifications** panel parameters:

- 1 Click-and-drag to select an appropriate filter design from the **Algorithm** pop-up menu.
- 2 Click-and-drag to select the band configuration from the **Type** pop-up menu.
- 3 Type in the **Specifications** panel's editable fields to change the values for band edge frequencies, ripple, attenuation, etc.
- 4 Press **Enter** (**Return** on Macintosh) after typing in an editable text box to enter the value. If **Auto Design** is not checked, press the **Apply** button below the **Specifications** panel to update the magnitude plot.
- 5 To specify a filter order, click **Minimize Order** to deselect the check box and disable automatic filter order selection. Then type a value for the **Order** parameter. If **Auto Design** is not checked, press **Apply**.



Edit a new filter or an existing filter in the same way. Note that when **Auto Design** is disabled and you change a filter's parameter values, the magnitude response plot changes from solid lines to dashed lines. This indicates that the plot no longer reflects the current specifications. When you press the **Apply** button, the new response is computed and the lines revert to the solid style. When **Auto Design** is enabled, the plot updates whenever you change a filter specification.

NOTE You can only edit filters that were designed in SPTool.



Filter Measurements Panel

When you design a filter, the **Measurements** panel (shown at left) displays the values of filter parameters that *do not* appear in the **Specifications** panel. For example, when the Filter Designer provides an **Fs** parameter in the **Specifications** panel, it displays the **Actual Fs** value in the **Measurements** panel. Similarly, when the **Minimum Order** option is selected in the **Specifications** panel, the computed filter order is displayed in the **Order** field of the **Measurements** panel. The values in the **Measurements** panel are updated whenever the magnitude plot is redrawn.

Measurement Parameters – Automatic Order Selection. The parameter combinations that appear in the **Measurements** panel are shown in the following two tables. The first table lists the parameters that appear when **Minimum Order** is checked (automatic order selection). The stopband edge frequency parameters listed (F_s , F_{s1} , F_{s2}) are the *actual* edge frequencies for the design, rather than the *desired* frequencies entered in the **Specifications** panel.

	Lowpass	Highpass	Bandpass	Bandstop
Equiripple FIR	Order, Rp, Rs, Wp, Ws	Order, Rp, Rs, Wp, Ws	Order, Rp, Rs, Wp, Ws	Order, Rp, Rs, Wp, Ws
Least Squares FIR	Order cannot be automatically computed.			
Kaiser Window FIR	Order, Fc, Beta, Rp, Rs	Order, Fc, Beta, Rp, Rs	Order, Fc1, Fc2, Beta, Rp, Rs	Order, Fc1, Fc2, Beta, Rp, Rs
Butterworth IIR	Order, Rp, F3db	Order, Rp, F3db	Order, Rp, F3db 1, F3db 2	Order, Rp, F3db 1, F3db 2
Chebyshev Type I IIR	Order, F_s	Order, F_s	Order, F_{s1} , F_{s2}	Order, F_{s1} , F_{s2}
Chebyshev Type II IIR	Order, F_s	Order, F_s	Order, F_{s1} , F_{s2}	Order, F_{s1} , F_{s2}
Elliptic IIR	Order, F_s	Order, F_s	Order, F_{s1} , F_{s2}	Order, F_{s1} , F_{s2}



Measurement Parameters – Manual Order Selection. The next table shows the parameter sets that appear in the **Measurements** panel when **Minimum Order** is not selected. The measurements that can be interactively changed by dragging the red measurement lines on the response plot are shown in italics.

	Lowpass	Highpass	Bandpass	Bandstop
Equiripple FIR	Rp, Rs	Rp, Rs	Rp, Rs	Rp, Rs
Least Squares FIR	Rp, Rs	Rp, Rs	Rp, Rs	Rp, Rs
Kaiser Window FIR	<i>Fp, Fs, Rp, Rs</i>	<i>Fp, Fs, Rp, Rs</i>	<i>Fp1, Fp2, Fs1, Fs2, Rp, Rs</i>	<i>Fp1, Fp2, Fs1, Fs2, Rp, Rs</i>
Butterworth IIR	<i>Fp, Fs, Rp, Rs</i>	<i>Fp, Fs, Rp, Rs</i>	<i>Fp1, Fp2, Fs1, Fs2, Rp, Rs</i>	<i>Fp1, Fp2, Fs1, Fs2, Rp, Rs</i>
Chebyshev Type I IIR	<i>Fs, Rs</i>	<i>Fs, Rs</i>	<i>Fs1, Fs2, Rs</i>	<i>Fs1, Fs2, Rs</i>
Chebyshev Type II IIR	<i>Fp, Rp</i>	<i>Fp, Rp</i>	<i>Fp1, Fp2, Rp</i>	<i>Fp1, Fp2, Rp</i>
Elliptic IIR	F_s	F_s	F_{s1} , F_{s2}	F_{s1} , F_{s2}

Magnitude Plot (Display) Area

The response of the filter selected in the **Filter** pop-up menu is displayed graphically in the magnitude response plot area of the Filter Designer, and its characteristics are shown in the **Specifications** and **Measurements** panels.


You can zoom in on the displayed filter in the magnitude plot (see “Zoom Controls” on page 5-30) and you can drag the specification lines to visually redesign the displayed filter (see the next section).

Specification Lines

You can redesign a filter by manipulating the green specification lines in the magnitude plot. Using the specification lines, you can change the passband ripple, stopband attenuation, and edge frequencies of a filter.

- Set passband ripple or stopband attenuation by clicking on a green line and dragging it up or down. The Rp and Rs values displayed in the **Specifications** panel change as you drag. If **Auto Design** is checked, the response plot also updates as you drag the lines. If **Auto Design** is disabled, the response plot updates when you release the mouse.

NOTE With IIR filters you can only drag the lower passband bar and the stopband bar. The upper passband bar is fixed at 0.

- Set edge frequencies either by clicking on the edge of a *horizontal* green line (the mouse pointer changes to ) and dragging the edge to a new frequency, or by clicking anywhere on a *vertical* green line (if the Filter Designer provides one) and dragging it horizontally to a new frequency. The Fp and Fs values displayed in the **Specifications** panel change as you drag. If **Auto Design** is checked, the response plot also updates as you drag the lines. If **Auto Design** is disabled, the response plot updates when you release the mouse.

See “Redesigning a Filter Using the Magnitude Plot” on page 5-68 for details.

Measurement Lines

A number of the filter designs provide rulers on the response plot that allow you to measure response magnitude levels. These measurement lines, which appear in red on the plot, are available for the Kaiser window, Butterworth,

Chebyshev type I, and Chebyshev type II filters when the **Minimum Order** check box is *not* selected. As you drag a measurement line, the corresponding values in the **Measurements** panel change to reflect the measurement line's current position.

Designing Finite Impulse Response (FIR) Filters

The Filter Designer provides three options for basic FIR filter design. These options allow you to create FIR filters with standard band configurations (lowpass, highpass, bandpass, or bandstop configurations only). The three options for FIR filter design in the **Algorithm** pop-up menu are:

- **Equiripple FIR**, which accesses the toolbox function `remez` to create an equiripple FIR filter.
- **Least Squares FIR**, which accesses the toolbox function `fi rl s` to create an FIR filter using the least square design method.
- **Kaiser Window FIR**, which accesses the `fi r1` function to create an FIR filter using a Kaiser window.

Example: FIR Filter Design, Standard Band Configuration

In the following example, use the Kaiser window filter design option:

- 1 Select **Kaiser Window FIR** as the filter design from the **Algorithm** pop-up menu.
- 2 Select **bandpass** from the **Type** pop-up menu as the configuration.
- 3 Set the filter's sampling frequency to 2000 Hz by entering this value in the **Sampling Frequency** text box.
- 4 Click **Apply** to redraw the response with these settings.

NOTE This must be done before you change the following parameters.

- 5 Check the **Minimum Order** check box to enable automatic order selection.

- 6 Set **Fp1** to 290 and **Fp2** to 525.

These fields respectively define the lower and upper passband edge frequencies in Hertz.

- 7 Set **Fs1** to 200 and **Fs2** to 625.

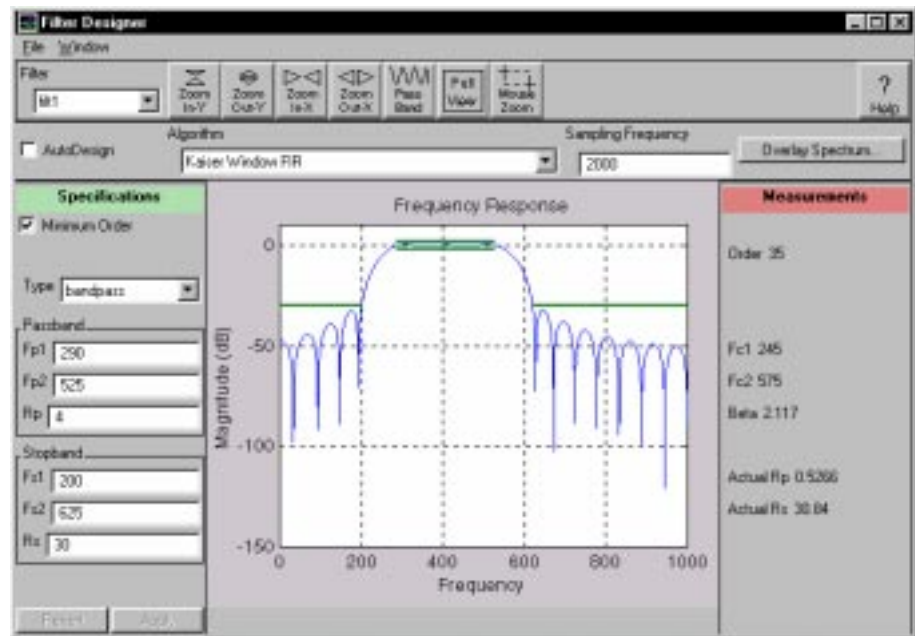
These fields respectively define the lower and upper stopband edge frequencies in Hertz.

- 8 Set **Rp** (passband ripple) to 4 and **Rs** (stopband attenuation) to 30.

Rp and **Rs** are specified in dB.

- 9 Press the **Apply** button.

The Filter Designer calls `fir1` to create the filter using a Kaiser window. The Filter Designer updates the magnitude plot to show the new filter's magnitude response:



Filter Design Options

When the **Minimum Order** option is disabled, you can specify parameters that define characteristics unique to certain filter types:

- For equiripple and least squares filters: the weights for error minimization
- For Kaiser window filters: the cutoff frequency and β parameter of the Kaiser window

Order Selection for FIR Filter Design

As described earlier, the FIR filter design options available through the Filter Designer call the toolbox functions `remez`, `fir1s`, and `fir1`. In calculating filter order, the Filter Designer uses the same guidelines as the toolbox functions:

- The **Equiripple FIR** design option calls the `remezord` order estimation function to determine a filter order that meets a set of specifications. In some cases, `remezord` underestimates the filter order n . If the filter does not appear to meet the given specifications using **Minimum Order** order selection, deselect **Minimum Order** and manually specify a slightly larger order ($n+1$ or $n+2$).
- The **Least Squares FIR** design option calls the toolbox function `fir1s`. Because the toolbox does not provide an order estimation function for use with `fir1s`, you cannot use the **Minimum Order** option with the **Least Squares FIR** method.
- The **Kaiser Window FIR** design option calls `kaiserord`, the order estimation function, which sometimes underestimates the filter order n . If the filter does not appear to meet the given specifications using **Minimum Order** order selection, deselect **Minimum Order** and manually specify a slightly larger order ($n+1$ or $n+2$).

All of the FIR filter design options in the Filter Designer require an even filter order for the highpass and bandstop configurations. For more information on order selection with the FIR filter design options, see the reference descriptions of `remez`, `remezord`, `kaiserord`, `fir1s`, and `fir1` in Chapter 6.

Designing Infinite Impulse Response (IIR) Filters

The Filter Designer lets you design a number of classical IIR filters, including Butterworth, Chebyshev type I, Chebyshev type II, and elliptic filters.

Example: Classical IIR Filter Design

In the following example, design a simple Chebyshev type I filter:

- 1 Select **Chebyshev Type I IIR** as the filter design from the **Algorithm** pop-up menu.
- 2 Select **highpass** from the **Type** menu as the configuration.
- 3 Set the filter's sample frequency to 2000 Hz by entering this value in the **Sampling Frequency** text box.
- 4 Click **Apply** to redraw the response with these settings.

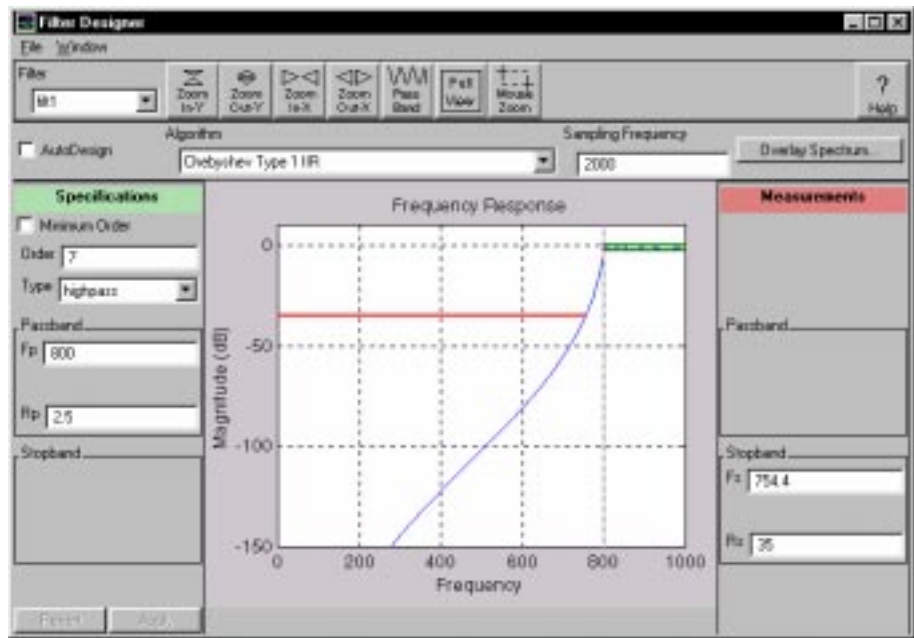
NOTE This must be done before you change the following parameters.

- 5 Check the **Minimum Order** check box.
- 6 Set **Fp** (passband edge frequency) to 800 and **Fs** (stopband edge frequency) to 700.

Fp and **Fs** are specified in Hertz.
- 7 Set **Rp** (passband ripple) to 2.5 and **Rs** (stopband attenuation) to 35.

Rp and **Rs** are specified in dB.
- 8 Press **Apply** to draw the magnitude response.
- 9 Deselect **Minimum Order** and specify a filter order of 7. Press **Apply** to redraw the magnitude response.

The Filter Designer calls the appropriate filter design function to create the filter:



To zoom in on plot details, use the zoom control buttons as described in “Zoom Controls” on page 5-30.

Filter Design Options

When the **Minimum Order** option is disabled, you can specify parameters that define characteristics unique to certain filter types:

- For Butterworth filters: the 3 dB frequencies
- For Chebyshev type I filters: the passband edge frequencies
- For Chebyshev type II filters: the stopband edge frequencies
- For elliptic filters: the passband edge frequencies

In the following example, redesign the Chebyshev type I filter from the previous example as a Butterworth filter, using a 3 dB frequency of 800 Hz:

- 1 Select the **Butterworth IIR** filter design from the **Algorithm** menu.

The magnitude response plot changes to reflect the new design.

- 2 Type a value of 800 in the **F3db** text field.

- 3 Press **Apply** to update the response plot.

Order Selection for IIR Filter Design

The IIR filter design options available through the Filter Designer call the toolbox filter design functions. In calculating the order for a given filter, the Filter Designer uses the corresponding order estimation function if the **Minimum Order** check box is selected.

For details on order selection with the IIR filter design options, see the reference descriptions of `buttord`, `cheb1ord`, `cheb2ord`, and `ellipord` in Chapter 6.

Redesigning a Filter Using the Magnitude Plot

After designing a filter in the Filter Designer, you can redesign it by dragging the specification lines in the magnitude plot. Use the specification lines to change passband ripple, stopband attenuation, and edge frequencies (see “Specification Lines” on page 5-62 for details). In the following example, create a Chebyshev filter and modify it by dragging the specification lines:

- 1 Create a Chebyshev type I highpass filter with a sample frequency of 2000 Hz. Set the following parameters:

Fp = 800
Fs = 700
Rp = 2.5
Rs = 35

- 2 Check **Minimum Order** so the Filter Designer can calculate the lowest filter order that produces the desired characteristics.
- 3 Press **Apply** to update the response plot.

- 4 Position the cursor over the green line specifying the stopband.

The cursor changes to the up/down drag indicator.

- 5 Drag the line until the **Rs** (stopband attenuation) field reads 100.

Note that the **Order** value in the **Measurements** panel changes because a higher filter order is needed to meet the new specifications.

Saving Filter Data

After designing a filter in the Filter Designer, you can export the filter information to the workspace or to disk using **Export...** from the **File** menu in SPTool. The filter information is stored in a structure that you can access to retrieve the coefficients and design parameters of the filter you created. The filter structure also contains a number of fields that are used internally by SPTool.

To see the fields of the filter structure, first export a filter to the workspace:

- 1 Create a new filter by pressing **New Design** in SPTool. The new filter is called `filt1`.
- 2 Select **Export...** from the **File** menu.
- 3 In the **Export from SPTool** dialog box, select `filt1` and press the **Export to Workspace** button.
- 4 Type `who` at the MATLAB command line to look at the variables in the workspace. The variable called `filt1` is the filter structure you exported from SPTool.
- 5 At the command line type `filt1` to list the fields of the filter structure.

The `tf`, `Fs`, and `specs` fields of the filter structure contain the information that describes the filter. These fields are discussed below. The other fields in the structure are used internally by SPTool, and are subject to change in future releases.

tf. The **tf** field is a structure containing the transfer function representation of the filter:

- **tf.num** contains the numerator coefficients, and
- **tf.den** contains the denominator coefficients

both in descending powers of z :

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}}$$

where b is a vector containing the coefficients from the **tf.num** field, a is a vector containing the coefficients from the **tf.den** field, nb is the numerator order, and na is the denominator order. You can change the filter representation from the default transfer function to another form by using the **tf2ss** or **tf2zp** functions.

Fs. The **Fs** field contains the sampling frequency of the filter in Hertz.

specs. The **specs** field is a structure containing information about the filter design. The first field, **specs.currentModule**, contains a string representing the design selected for the filter in the Filter Designer's **Algorithm** pop-up menu. The possible contents of the **currentModule** field, and the corresponding designs, are shown below.

currentModule	Algorithm
fdbutter	Butterworth IIR
fdcheby1	Chebyshev Type I IIR
fdcheby2	Chebyshev Type II IIR
fdellip	Elliptic IIR
fdfirls	Least Squares FIR
fdkaiser	Kaiser Window FIR
fdremez	Equiripple FIR

Following the `specs.currentModule` field, there may be up to seven additional fields, with labels such as `specs.fdreamez`, `specs.fdfirls`, etc. The design specifications for the most recently exported filter are contained in the field whose label matches the `currentModule` string. For example, if the `specs` structure is

```
currentModule: 'fdkaiser'
fdreamez: [1x1 struct]
fdfirls: [1x1 struct]
fdkaiser: [1x1 struct]
```

the filter specifications are contained in the `fdkaiser` field, which is itself a data structure.

The specifications include the parameter values from the **Specifications** panel of the Filter Designer, such as band edges and filter order. For example, the Kaiser window filter above has the following specifications stored in `specs.fdkaiser`:

```
setOrderFlag: 0
type: 1
f: [0 0.1000 0.1500 1]
Rp: 3
Rs: 20
Wn: 0.1250
order: 34
Beta: 0
wind: [35x1 double]
```

Since certain filter parameters are unique to a particular design, this structure has a different set of fields for each filter design. For example, the `Beta` field above only appears in the `specs` structure if the design is a Kaiser window FIR filter.

The table below lists the possible specifications fields that can appear in the export structure, and describes their contents.

Parameter	Description
Beta	Kaiser window β parameter.
f	Contains a vector of band-edge frequencies, normalized to 1 (i.e., 1 = Nyquist).
Fpass	Passband cutoff frequencies. Scalar for lowpass and highpass designs, two-element vector for bandpass and bandstop designs.
Fstop	Stopband cutoff frequencies. Scalar for lowpass and highpass designs, two-element vector for bandpass and bandstop designs.
m	The response magnitudes corresponding to the band-edge frequencies in f.
order	Filter order.
Rp	Passband ripple (dB)
Rs	Stopband attenuation (dB)
setOrderFlag	Contains 1 if the filter order was specified manually (i.e., the Minimum Order box in the Specifications panel was not checked). Contains 0 if the filter order was computed automatically.
type	Contains 1 for lowpass, 2 for highpass, 3 for bandpass, or 4 for bandstop.
w3db	3 dB frequency for Butterworth IIR designs.
wi nd	Vector of Kaiser window coefficients.
Wn	Cutoff frequency for the Kaiser window FIR filter when setOrderFlag = 1.
wt	Vector of weights, one weight per frequency band.

Viewing Frequency Response Plots

It is often useful to view a filter's frequency response, impulse response, and step response during the filter design process. You can use the Filter Viewer to view frequency-domain information about filters in the Filter Designer:

- 1 Activate SPTool from the **Window** menu.
- 2 Make sure the filters you want to analyze are selected in the **Filters** list.
- 3 Click **View** in the Filter panel.

The Filter Viewer is activated with the selected filters displayed.

- 4 To edit one of the filters you're viewing, you can reactivate the Filter Designer from the **Window** menu in the Filter Viewer.
- 5 When you want to review a filter's characteristics after you've edited it, reactivate the Filter Viewer from the **Window** menu in the Filter Designer.

When the Filter Viewer is open at the same time that the Filter Designer is open, they both display the same filter. You can move back and forth between the Filter Designer and the Filter Viewer until the filter design is finished.

You can apply the filter to a signal by activating SPTool, selecting the filter in the **Filters** list, and the signal to apply it to from the **Signals** list, and pressing **Apply**. See "Applying a Filter" on page 5-19 for details.

See "Using the Filter Viewer: Interactive Filter Analysis" on page 5-74 for more information on the Filter Viewer.

Using the Filter Viewer: Interactive Filter Analysis

An important aspect of filter design is filter analysis, which encompasses both frequency and time-domain analysis of a filter. The Filter Viewer is a GUI-based frequency analysis tool that provides an interactive environment for the graphical display of digital filter characteristics.

The Filter Viewer can display six different characteristics subplots of a selected filter. Any combination of the six subplots may be displayed.

Using the Filter Viewer you can:

- View magnitude-response plots for one or more filters
- View phase-response plots for one or more filters
- View group-delay plots for one or more filters
- View zero-pole plots for one or more filters
- View impulse-response plots for one or more filters
- View step-response plots for one or more filters
- Zoom in to explore filter response details
- Modify selected plot parameters and display characteristics
- Measure a variety of characteristics of the filter response

For information on frequency analysis using toolbox functions from the command line or from M-files, see “Frequency Response” in Chapter 1 of this manual.

Opening the Filter Viewer

Open or activate the Filter Viewer from SPTool:

- 1 Select one or more filters from the **Filters** list in SPTool.
- 2 Press **View** in the **Filters** panel in SPTool.

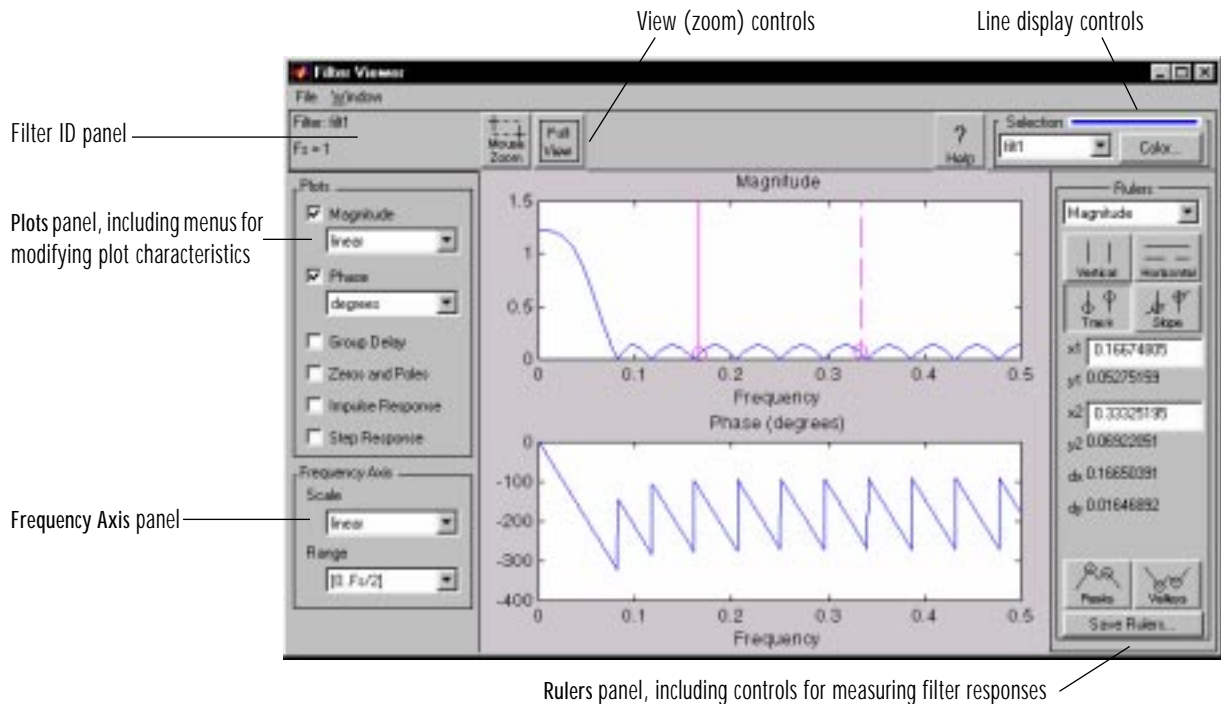
The Filter Viewer is activated and the selected filters are loaded into the Filter Viewer and displayed.

Basic Filter Viewer Functions

The Filter Viewer has the following components:

- A **Plots** panel for selecting which subplots display in the main plots window
- A **Rulers** panel and line display controls for making signal measurements and comparisons
- A **Frequency Axis** panel for specifying x -axis scaling in the main plots window
- A filter identification panel that displays information about the currently selected filter(s)
- A main plots (display) area for viewing one or more frequency-domain plots for the selected filter(s)
- Zoom controls for getting a closer look at filter response characteristics

When you first open or activate the Filter Viewer, it displays the default plot configuration for the selected filter(s):



The filters' magnitude and phase plots are displayed. The frequency axis of the plots is set to **linear**, and the frequency axis range is set to **[0,Fs/2]**.

You can choose to display one or any combination of the six available subplots by using the check boxes in the **Plots** panel, and you can modify many of the plot display characteristics using the pop-up menus in the **Plots** panel and the **Frequency Axis** panel.

Menus

File Menu. Use **Close** from the **File** menu to close the Filter Viewer. Settings you changed and saved using the **Preferences...** window in SPTool are saved and used the next time you open a Filter Viewer.

Window Menu. Use the **Window** menu to select a currently open MATLAB Figure window.

Filter Identification Panel

This panel displays the variable names and the highest sampling frequency of the currently selected filters. To change names or sampling frequencies, use **Name...** or **Sampling Frequency...** from the **Edit** menu in SPTool.

Plots Panel

The check boxes in this panel select the subplots to display in the main plots area. Any combination of subplots may be displayed.

To display a subplot, check the box at the left of the plot description.

There are six available subplots:

- **Magnitude:** displays the magnitude of the frequency response of the currently selected filter(s).
- **Phase:** displays the phase of the frequency response.
- **Group Delay:** displays the negative of the derivative of the phase response.
- **Zeros and Poles:** displays the poles and zeros of the filter transfer function(s) and their proximity to the unit circle.
- **Impulse Response:** displays the response of the currently selected filter(s) to a discrete-time unit-height impulse at $t=0$.
- **Step Response:** displays the response of the currently selected filter(s) to a discrete-time unit-height step function.

You can customize the display characteristics of the magnitude and phase subplots using the **Magnitude** and **Phase** pop-up menus. The options include:

- **Magnitude:** Scaling for the magnitude plot may be **linear**, **log**, or **decibels**.
- **Phase:** Phase units may be **degrees** or **radians**.

You can also change the magnitude and phase display characteristics for the Filter Viewer using the Filter settings panel of the **Preferences** dialog in SPTool.

Frequency Axis Settings

You can change frequency axis scaling and range parameters for plots in the Filter Viewer.

Click on the option in the **Frequency Axis** panel you want to edit and drag to select a value. The options include:

- **Scale:** Scaling for the frequency axis may be **linear** or **log**.
- **Range:** The range for the frequency axis may be **[0,Fs/2]**, **[0,Fs]**, or **[-Fs/2,Fs/2]**, where **Fs** represents the filter's sampling frequency.

The frequency range cannot be negative if **Scale** is set to **log**.

You can also change the frequency axis display characteristics for the Filter Viewer using the **Filter Viewer** settings panel of the **Preferences** dialog in SPTool.

Zoom Controls

The available zoom controls in the Filter Viewer are **Mouse Zoom** and **Full View**. You can zoom independently in each displayed subplot.

By default, persistent zooming is disabled in the Filter Viewer. You can turn persistent zooming on from the **Filter Viewer** settings panel of the **Preferences** dialog in SPTool.

See “Zoom Controls” on page 5-30 for details on using the zoom controls in the Filter Viewer.

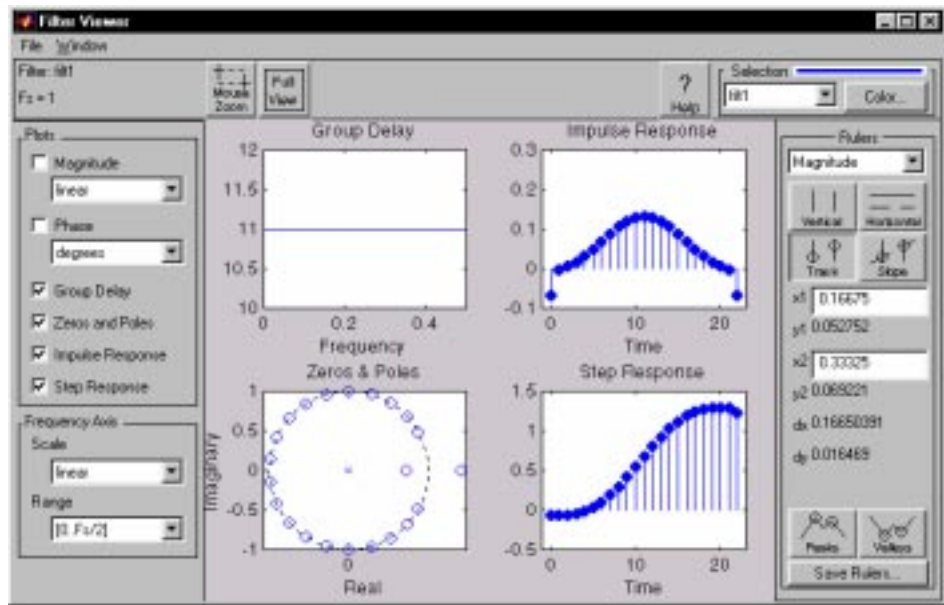
Help Button

To use context-sensitive help, click on the **Help** button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in the Filter Viewer, including menu items, to find out what it is and how to use it.

Main Plots Area

One or more of the six filter response subplots may be displayed graphically in the main plots area of the Filter Viewer. You can specify how the subplots are arranged by selecting **Filter Viewer Tiling** from the **Preferences** dialog in SPTool. The options are **2-by-3 Grid**, **3-by-2 Grid**, **Vertical (6-by-1 Grid)**, and **Horizontal (1-by-6 Grid)**.

The following figure shows the Filter Viewer when four subplots are turned on and the 2-by-3 grid option is selected:



You can experiment to find the tiling option that works best for each specific combination and number of subplots.

You can zoom in on a subplot by clicking on **Mouse Zoom** and then clicking on or dragging over a selected area of the subplot. By default, mouse zooming in the Filter Viewer is not persistent; after you click once, the zoom mode is turned off. You can make zooming persistent by checking **Stay in Zoom-mode after Zoom** in the SPTool **Preferences** dialog box. This allows you to click repeatedly in a subplot to continue to zoom in on a particular feature of the display.

After you zoom in on a subplot, you can click and drag to pan around the subplot:

- 1 Click on **Mouse Zoom** to turn on mouse zoom mode.
- 2 Click on a feature of a subplot to zoom in on it.
- 3 If persistent zooming is enabled, click on **Mouse Zoom** again to turn off mouse zoom mode.
- 4 Click again in the same subplot, hold down the mouse button until the hand cursor is displayed, and drag the mouse to pan around the subplot.

Viewing Filter Plots

This section has a brief description and picture of each of the six filter response plots available in the Filter Viewer. A sequence of connected examples shows you how to display each plot on its own; you can also display any combination of plots, as needed.

Each plot in the example sequence displays the response of an order 22 equiripple lowpass filter with a sampling frequency of 1 Hz.

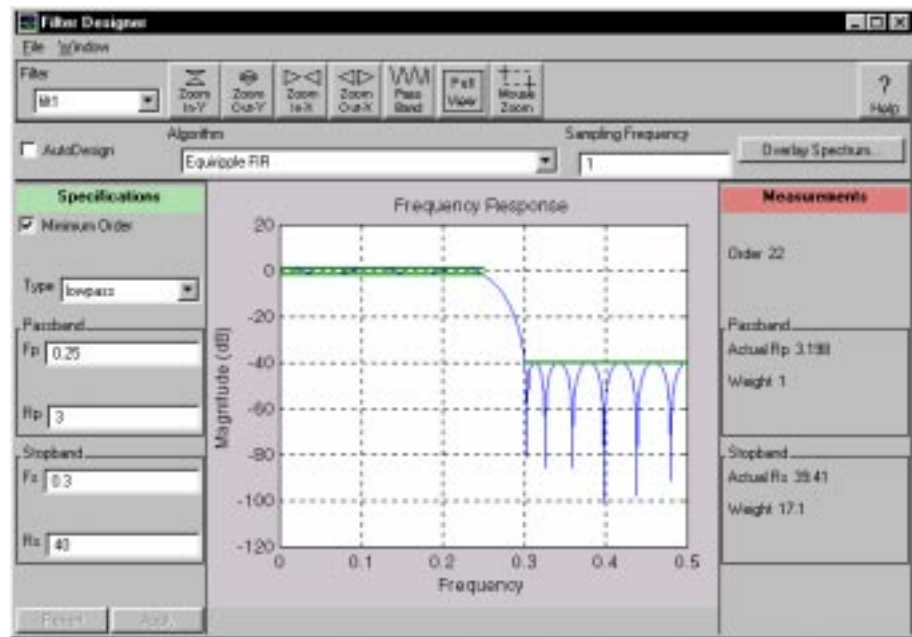
Regardless of how many or what combination of plots is displayed, you can zoom in on and pan each subplot independently.

Viewing Magnitude Response

A magnitude response plot is generally the simplest way to obtain a high-level view of a filter's shape and fit to specifications. In the following example, use the Filter Designer to create a standard default filter and then view its magnitude response plot in the Filter Viewer:

- 1 From SPTool, click **Create**.

The Filter Designer is activated and a standard default filter is created and displayed:



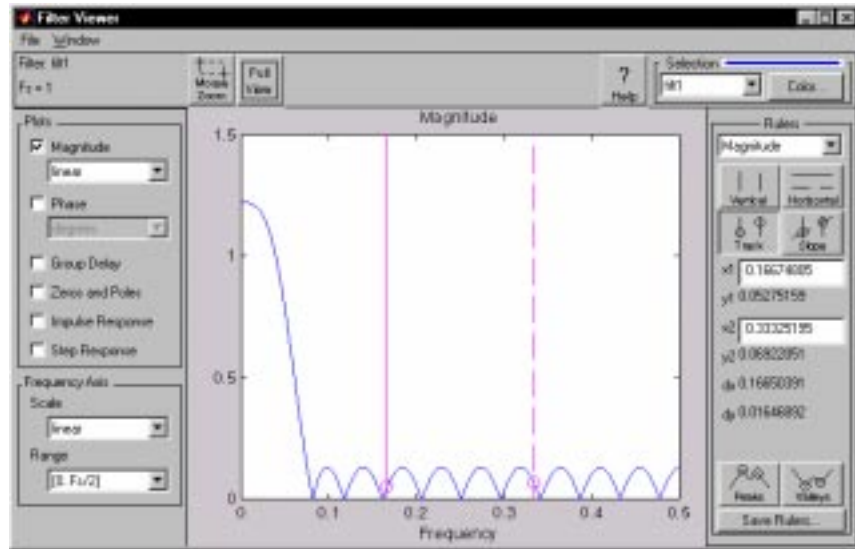
This is an order 22 equiripple lowpass filter with a sampling frequency of 1 Hz.

- 2 Use the **Window** menu in the Filter Designer to activate SPTool.
- 3 Click **View** from the **Filters** panel in SPTool to activate the Filter Viewer.

The Filter Viewer is displayed with a magnitude response plot and a phase response plot.

- 4 Click the check box next to the **Phase** option to turn off the phase plot.

The magnitude plot for the default filter is displayed:



By default, this plot uses the default scaling (**linear**) for both axes and the default range for the frequency axis.

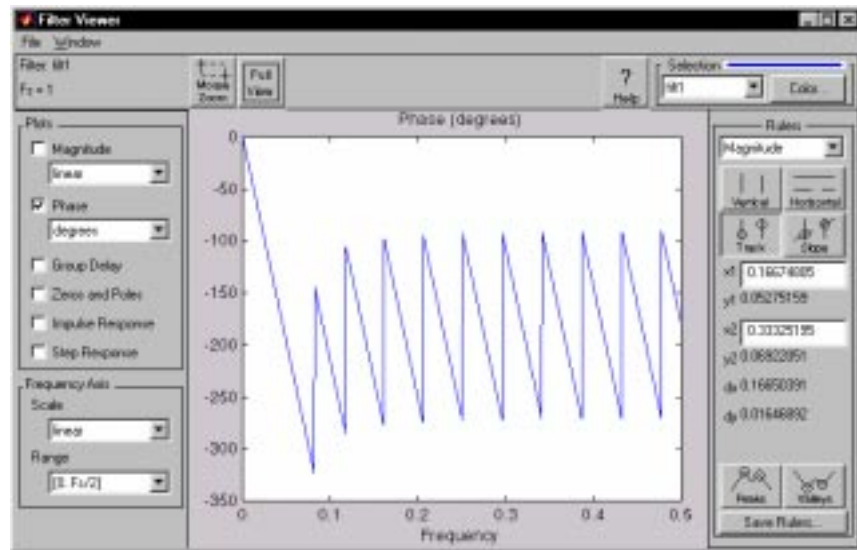
You can change the following display characteristics of the magnitude plot:

- Use the **Magnitude** pop-up menu to choose between **linear**, **log**, or **decibels** scaling of the *y*-axis.
- Use the **Scale** pop-up menu to choose between **linear** and **log** scaling of the *x*-axis.
- Use the **Range** pop-up menu to choose between the following ranges for the *x*-axis: **[0,Fs/2]**, **[0,Fs]**, or **[-Fs/2,Fs/2]**, where **Fs** represents the filter's sampling frequency.

Viewing Phase Response

In addition to displaying magnitude response, the Filter Viewer can calculate and plot the filter's phase response. *Phase response* is the angular component of a filter's frequency response. To display only a phase response plot for the current filter:

- 1 Click the check box next to the **Magnitude** option to turn off the magnitude plot.
- 2 Click the check box next to the **Phase** option to turn on the phase plot and update the display:



By default, this plot uses the default phase (**degrees**) and the default scaling and range for the frequency axis.

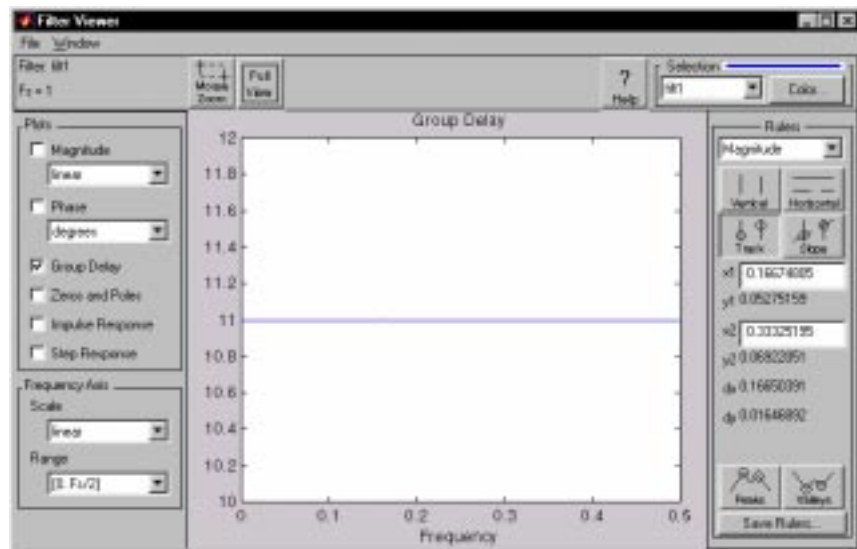
You can change the following display characteristics of the phase plot:

- Use the **Phase** pop-up menu to choose between displaying phase in **degrees** or **radians**.
- Use the **Scale** pop-up menu to choose between **linear** and **log** scaling of the x -axis.
- Use the **Range** pop-up menu to choose between the following ranges for the x -axis: $[0, F_s/2]$, $[0, F_s]$, or $[-F_s/2, F_s/2]$, where F_s represents the filter's sampling frequency.

Viewing Group Delay

Group delay is a measure of the average delay of a filter as a function of frequency. To display only a group delay plot for the currently selected filter(s):

- 1 Click the check box next to the **Phase** option to turn off the phase plot.
- 2 Click the check box next to the **Group Delay** option to turn on the group delay plot and update the display:



By default, this plot uses the default scaling and range for the frequency axis.

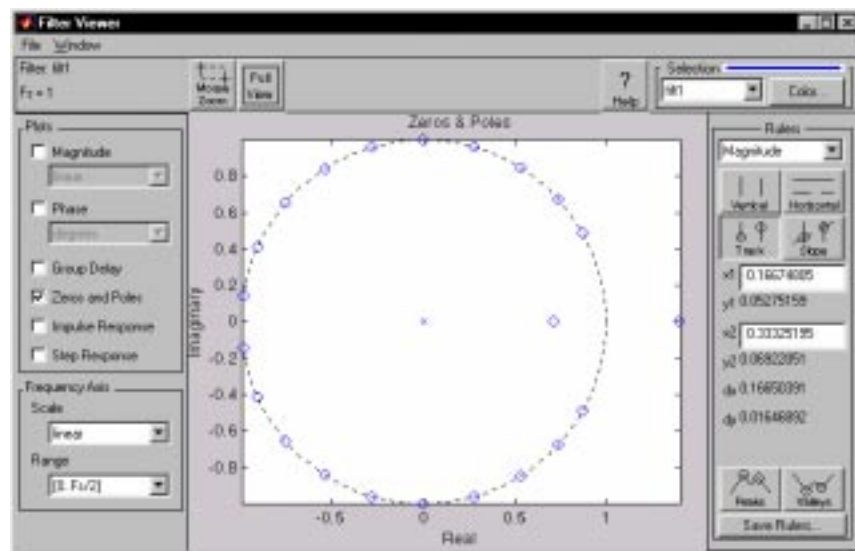
You can change the following display characteristics of the group delay plot:

- Use the **Scale** pop-up menu to choose between **linear** and **log** scaling of the x -axis.
- Use the **Range** pop-up menu to choose between the following ranges for the x -axis: $[0, F_s/2]$, $[0, F_s]$, or $[-F_s/2, F_s/2]$, where F_s represents the highest sampling frequency of the currently selected filters.

Viewing a Zero-Pole Plot

The *zero-pole plot* displays the poles and zeros of the transfer function and their proximity to the unit circle. An x represents a pole of the transfer function; a o represents a zero of the transfer function. To display only a zero-pole plot for the currently selected filter(s):

- 1 Click the check box next to the **Group Delay** option to turn off the group delay plot.
- 2 Click the check box next to the **Zeros and Poles** option to turn on the zero-pole plot and update the display:

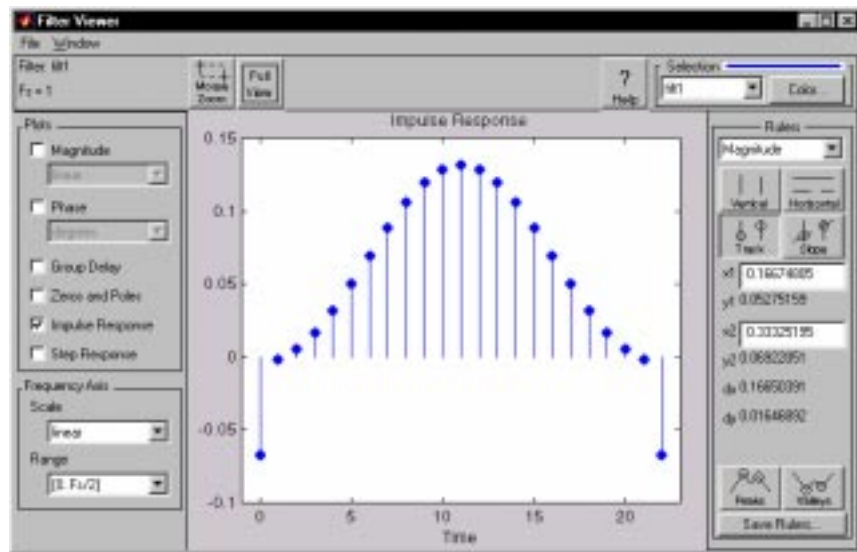


Viewing Impulse Response

The *impulse response plot* displays the response of the current filter(s) to a discrete-time unit-height impulse at $t=0$.

To display only an impulse response plot for the currently selected filter(s):

- 1 Click the check box next to the **Zeros and Poles** option to turn off the zero-pole plot.
- 2 Click the check box next to the **Impulse Response** option to turn on the impulse response plot and update the display:



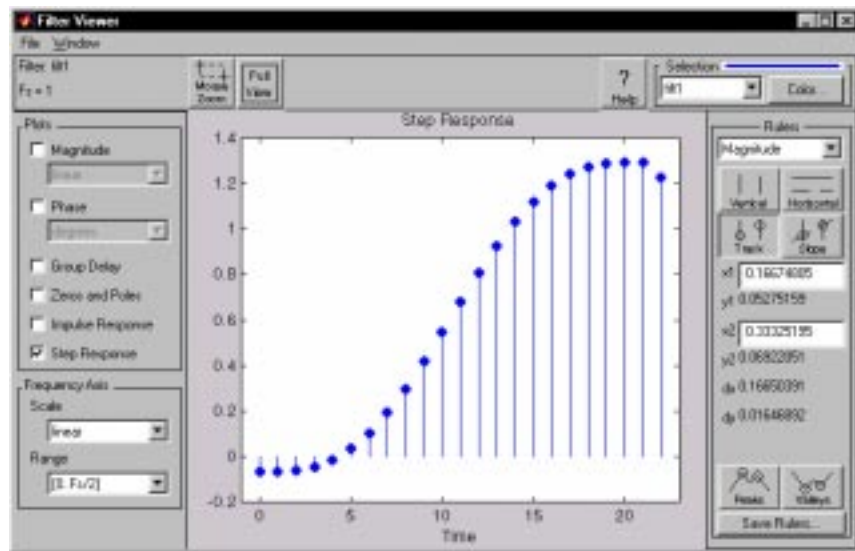
You can change the following display characteristics of the impulse response plot:

Edit the **Time Response Length** field in the **Filter Viewer** preferences panel to set the number of samples used to display the impulse response.

Viewing Step Response

The *step response plot* displays the response of the current filter(s) to a discrete-time unit-height step function. To display only a step response plot for the currently selected filter(s):

- 1 Click the check box next to the **Impulse Response** option to turn off the impulse response plot.
- 2 Click the check box next to the **Step Response** option to turn on the step response plot and update the display:



You can change the following display characteristics of the step response plot:

Edit the **Time Response Length** field in the **Filter Viewer** preferences panel to set the number of samples used to display the step response.

Using the Spectrum Viewer: Interactive PSD Analysis

The Spectrum Viewer provides an interactive environment for the estimation of power spectral density for one data channel. It allows you to view and modify spectra created in SPTool.

Using the Spectrum Viewer, you can control the spectral computation parameters, including FFT length, window type, and sampling frequency. Using the Spectrum Viewer you can:

- View and compare spectral density plots
- Use different estimation methods, including Burg, FFT, MTM, MUSIC, Welch, and Yule-Walker AR
- Modify spectrum parameters such as FFT length and window type

For information on spectral analysis using toolbox functions from the command line or from M-files, see Chapter 3 of this manual.

Opening the Spectrum Viewer

You can open or activate the Spectrum Viewer from SPTool by pressing one of the following buttons: **Create**, **View**, and **Update**. See “Creating a Spectrum” on page 5-19, “Viewing a Spectrum” on page 5-20, and “Updating a Spectrum” on page 5-20 for complete details.

Here is a brief summary of each method of activating the Spectrum Viewer:

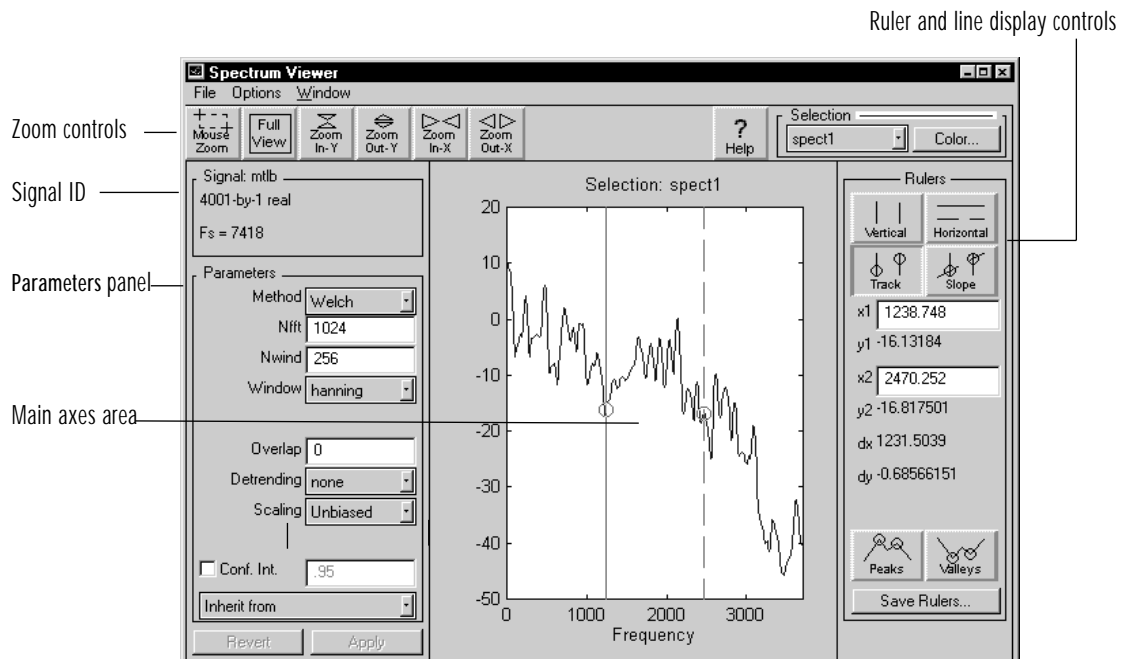
- To create a spectrum, select a signal in SPTool and press **Create**. Press **Apply** in the Spectrum Viewer.
A default spectrum of the selected signal is generated and displayed. You can view it in a variety of ways, measure it, and modify it in the Spectrum Viewer.
- To view a spectrum, select one or more spectra in SPTool and press **View** in the **Spectra** panel.
- To update a spectrum, select exactly one signal and one spectrum in SPTool and press **Update**. Press **Apply** in the Spectrum Viewer.

The spectrum is updated to reflect the data in the currently selected signal.

Basic Spectrum Viewer Functions

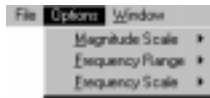
The Spectrum Viewer window has the following components:

- A **Parameters** panel for viewing and modifying the parameters or method of the current spectrum
- A signal identification panel that displays information about the signal linked to the current spectrum
- A main axes (display) area for viewing spectra graphically
- Zoom controls for getting a closer look at spectral features
- Rulers and line-display controls for making spectral measurements and comparisons
- Spectrum management controls: **Inherit from...**, **Revert**, and **Apply**
- Menu options for modifying plot display characteristics



Menus

File Menu. Use **Close** from the **File** menu to close the Spectrum Viewer. All spectrum selection and ruler information will be lost. Settings you changed using the **Preferences** window in SPTool are saved and used the next time you open the Spectrum Viewer.



Options Menu. Use these options to change scaling and range parameters for plots in the Spectrum Viewer.

Click on the option you want to edit and drag to select a value.

The options include:

- **Magnitude Scale:** Scaling for the magnitude plot may be **dB** or **linear**.
- **Frequency Range:** The range for the frequency axis may be **[0, Fs/2]**, **[0, Fs]**, or **[-Fs/2, Fs/2]**, where **Fs** is the sampling frequency. If multiple spectra are displayed, the value of **Fs** is the maximum of all the sampling frequencies. **Fs** is not defined for the case of a spectrum whose signal is <None>, that is, a spectrum whose associated signal has been deleted from SPTool. In this case, a value twice the highest frequency in the spectrum's frequency vector is chosen.

The frequency range cannot be negative if **Frequency Scale** is set to **log**.

- **Frequency Scale:** Scaling for the frequency axis may be **linear** or **log**.

Window Menu. Use the window menu to select a currently open MATLAB Figure window.

Signal ID Panel

This panel displays information about the signal linked to the currently selected spectrum. The information includes the signal's name, size, data type (real or complex), and sampling frequency. To change any of these signal properties, use SPTool.

To associate a completely new signal with a displayed spectrum, select the signal in SPTool and click **Update** in the **Spectra** panel.

Spectrum Management Buttons

Inherit from... Choose a spectrum from this menu to let the active spectrum inherit its parameters (not including the associated signal).

Click on **Inherit from...** and drag to select the spectrum from which you want to inherit parameters.

Revert. Restores the properties of the current spectrum to what they were the last time **Apply** was pressed.

Apply. Compute and display the active spectrum using the parameters set in the **Parameters** panel.

Zoom Controls

The available zoom controls in the Spectrum Viewer are **Mouse Zoom**, **Full View**, **Zoom In-Y**, **Zoom Out-Y**, **Zoom In-X**, and **Zoom Out-X**. See “Zoom Controls” on page 5-30 for details on using the zoom controls in the Spectrum Viewer.

Ruler and Line Display Controls

Using the rulers and line-display controls, you can measure a variety of characteristics of spectra in the Spectrum Viewer. See “Ruler Controls” on page 5-32 for details on using rulers and modifying line displays in the Spectrum Viewer.

Help Button

To use context-sensitive help, click on the **Help** button. The mouse pointer becomes an arrow with a question mark symbol. You can then click on anything in the Spectrum Viewer, including menu items, to find out what it is and how to use it.

Main Axes Display Area

The **Spectra** list in SPTool shows all spectra in the current SPTool session. One or more spectra may be selected. The spectral data of all selected spectra are displayed graphically in the main axes display area of the Spectrum Viewer.

NOTE If a spectrum is not displayed, or if it is displayed with the wrong signal information, press **Apply** to recompute the spectral data.

When there is only one spectrum displayed, its properties are displayed in the **Parameters** panel and its measurements are displayed in the **Rulers** panel. When more than one spectrum is displayed, select the line you want to focus on.

When a spectrum is selected, you can use the ruler controls on the selected line (see “Making Signal Measurements” on page 5-37) and you can modify its parameters (see below). The label of the selected spectrum (line) is displayed in the **Selection** pop-up menu.

There are two ways to select a spectrum (line):

- Click on the **Selection** pop-up menu and drag to select the line to measure or
- Move the mouse pointer over any point in the line you want to select and click on it

See “Selecting a line to measure” on page 5-33 for details.

Click-and-Drag Panning. You can use the mouse to pan around the main axes display:

Click on a line in the main axes, hold down the mouse button, and drag the mouse.

Click-and-drag panning is not enabled in mouse zoom mode.

Making Spectrum Measurements

Use the rulers to make a variety of measurements on the selected spectrum. See “Making Signal Measurements” on page 5-37 for details.

Viewing Spectral Density Plots

Spectral density estimation is a technique that finds the approximate frequency content of a signal. The Spectrum Viewer calculates single-channel power spectral density (PSD). When you first generate a spectrum, the Spectrum Viewer shows a default power spectral density function of the input data. By default, the Spectrum Viewer uses the Burg method of PSD computation with an order of 10 and an FFT length of 1024.

You can change plot properties and computation parameters for a displayed spectrum, and you can set confidence intervals.

Controlling and Manipulating Plots

Changing Plot Properties

You can control the axes units and scaling properties that affect the Spectrum Viewer's plots.

Use the **Options** menu to select:

- Linear or decibel scaling for the magnitude axis
- Linear or logarithmic scaling for the frequency axis
- The frequency range to view

See “Options Menu” on page 5-90 for details.

You can also zoom in on any of the Spectrum Viewer's plots. See “Zoom Controls” on page 5-30 for details.

You can set other scaling properties in the **Parameters** panel, depending on the PSD method computation parameters you choose.

Choosing Computation Parameters

The Spectrum Viewer lets you control the PSD computation parameters of the selected spectrum. Different parameters are available, depending on which method of PSD computation you choose. Set these parameters from the **Parameters** panel, as illustrated in the following steps:

1 Click on the **Method** pop-up menu and drag to select one of the following methods:

- Burg
- FFT
- MTM
- MUSIC
- Welch
- Yule AR

Appropriate parameter selections are displayed for each method you choose.

2 Modify the appropriate parameters.

- When a parameter is in a pop-up menu, click on the parameter label and drag to select a value from the menu.
- When a parameter is in an edit box, type the value or variable into the box.

You can also modify the parameters by using **Inherit from** to copy the parameters of another spectrum in SPTool. See “Inherit from...” on page 5-91 for details.

3 If you change your mind, you can discard changes you make by clicking **Revert**.

4 To apply the modified parameters, click **Apply**.

The new parameters are applied to the selected spectrum; the Spectrum Viewer recalculates the spectral density function and displays the modified spectrum.

Computation Methods and Parameters

You can choose from seven PSD computation methods. Each method has its own set of parameters.

This section shows the **Parameters** panel for each of the PSD computation methods. For detailed definitions and values for each parameter, use context-sensitive help (see “Help Button” on page 5-91).

Burg. For the Burg method, you can specify the following parameters:

- **Order**
Type in a value.
- **Nfft**
Type in a value.

FFT. For the FFT method, you can specify the following parameter:

- **Nfft**
Type in a value.

MTM. For the MTM method, you can specify the following parameters:

- **NW**
Type in a value.
- **Nfft**
Type in a value.
- **Weights**
Select one of the following from the pop-up menu:

- **adapt**
- **unity**
- **eigen**

- **Conf. Int.**

Check to compute a confidence interval and type in a value (see “Setting Confidence Intervals” on page 5-98).

MUSIC. For the MUSIC method, you can specify the following parameters:

- **Signal Dim.**
Type in a value.
- **Threshold**
Type in a value.
- **Nfft**
Type in a value.
- **Nwind**
Type in a value.
- **Window**
Select a window from the pop-up menu.
- **Overlap**
Type in a value.
- **Corr. Matrix**
Check if selected signal is a correlation matrix.
- **Eigenvector Weights**
Check to select eigenvector weights.



Welch. For Welch's method, you can specify the following parameters:

- **NFFT**

Type in a value.

- **Nwind**

Type in a value.

- **Window**

Select a window from the pop-up menu.

- **Overlap**

Type in a value.

- **Detrending**

Select one of the following from the pop-up menu:

- **none**
- **linear**
- **mean**

- **Scaling**

Select one of the following from the pop-up menu:

- **Unbiased**
- **Peaks**
- **by Fs**

- **Conf. Int.**

Check to compute a confidence interval and type in a value (see "Setting Confidence Intervals" on page 5-98).



Yule AR. For the Yule AR method, you can specify the following parameters:

- **Order**

Type in a value.

- **Nfft**

Type in a value.

- **Corr. Matrix**

Check if selected signal is a correlation matrix.



Setting Confidence Intervals

By default, the Spectrum Viewer does not compute confidence intervals for spectral density. You can enable the computation of confidence intervals for the Welch and MTM methods by following these steps:

- 1 Click the **Conf. Int.** check box so that it is selected.
- 2 Type a value for the confidence level in the **Conf. Int.** edit box.

This value must be a scalar between 0 and 1.

- 3 Click **Apply**.

NOTE Confidence intervals are only reliable for nonoverlapping sections.

Saving Spectrum Data

After creating a spectrum in SPTool, you can export spectrum information to the workspace or to disk using **Export...** from the **File** menu in SPTool. The spectrum information is stored in a structure that you can access to retrieve the spectral power and frequency data. The spectrum structure also contains a number of fields that are used internally by SPTool.

To see the fields of the spectrum structure, try exporting a spectrum to the workspace:

- 1 Create a new spectrum if none are currently loaded. Label the spectrum spect 1.
- 2 In SPTool, select **Export...** from the **File** menu.
- 3 In the **Export** dialog box, select spect 1 and press the **Export to Workspace** button.
- 4 Type who at the MATLAB command line to look at the variables in the workspace. The variable called spect 1 is the spectrum structure you exported from SPTool.
- 5 Type spect 1 to list the fields of the spectrum structure.

The following structure fields describe the spectrum:

Field	Description
P	The spectral power vector.
f	The spectral frequency vector.
conf i d	A structure containing the confidence intervals data: <ul style="list-style-type: none">• The conf i d. l e v e l field contains the chosen confidence level.• The conf i d. P c field contains the spectral power data for the confidence intervals.• The conf i d. e n a b l e field contains a 1 if confidence levels are enabled for the spectrum.
si g n a l Label	The name of the signal from which the spectrum was generated.
Fs	The associated signal's sample rate.

The other fields are used internally by SPTool, and are subject to change in future releases.

Example: Generation of Bandlimited Noise

This section provides a complete example of using the GUI-based interactive tools to design and implement an FIR digital filter, apply it to a signal, and display signals and spectra. The steps include:

- Importing and naming a signal using SPTool
- Designing a filter using the Filter Designer
- In SPTool, applying the filter to the signal to create another signal
- Viewing the time domain information of the original and filtered signals using the Signal Browser
- Comparing the spectra of both signals using the Spectrum Viewer

Create, Import, and Name a Signal

You can import an existing signal into SPTool, or you can create a new signal and edit and name it in SPTool. In this step, you'll create a new signal at the command line and then import it into SPTool.

- 1 At the command line, create a random signal by typing:

```
x = randn(5000, 1);
```

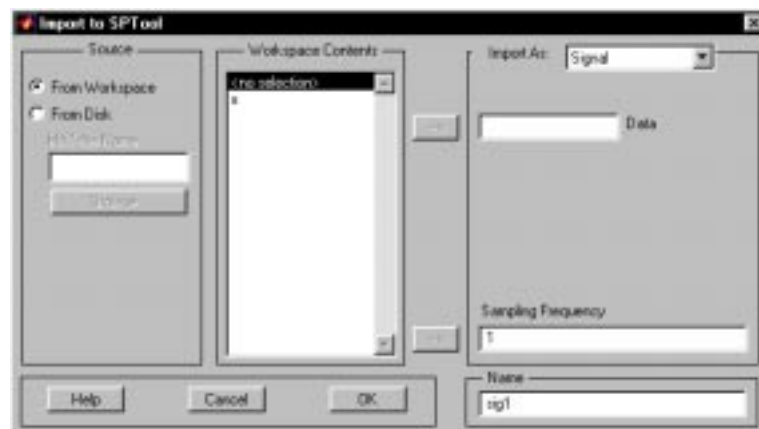
- 2 Activate SPTool by typing:

```
sptool
```

The SPTool window is displayed.

- 3 Select **Import...** from the **File** menu:

The **Import to SPTool** window is displayed:



Notice that the variable **x** is displayed in the **Workspace Contents** list. (If it is not, click the **From Workspace** radio button to display the contents of the workspace.)

- 4 Name the signal and import it into SPTool:
 - a Make sure that **Signal** is selected in the **Import As** pop-up menu.
 - b Click in the **Data** field and type **x**.

You can also move the variable **x** into the **Data** field by clicking on **x** in the **Workspace Contents** list and then clicking on the arrow to the left of the **Data** field.

- c Click in the **Sampling Frequency** field and type 5000.
- d Name the signal by clicking in the **Name** field and typing `noi se`.
- e Click **OK**.

The SPTool window is reactivated, and the signal `noi se[vector]` is selected in the **Signals** list.

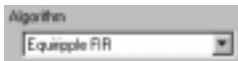
Design a Filter

You can import an existing filter into SPTool, or you can design and edit a new filter using the Filter Designer. In this step, you'll create a default filter and customize it in the Filter Designer.

- 1 Click **New Design** in SPTool to activate the Filter Designer and generate a default filter.

The Filter Designer window is displayed with the default filter `filt1`.

- 2 Change the filter sampling frequency to 5000 by entering this value in the **Sampling Frequency** text box in the Filter Designer.

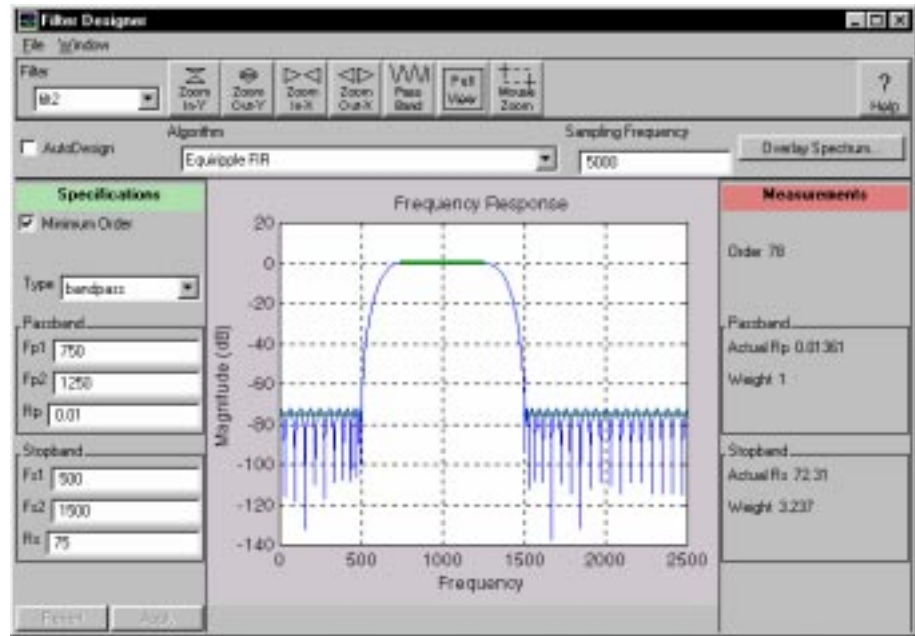


- 3 Specify the filter parameters shown at left:



- a Make sure **Equiripple FIR** is selected in the **Algorithm** pop-up menu.
- b Select **bandpass** from the **Type** pop-up menu.
- c Set the passband edge frequencies by entering 750 for **Fp1** and 1250 for **Fp2**.
- d Set the stopband edge frequencies by entering 500 for **Fs1** and 1500 for **Fs2**.
- e Type `.01` into the **Rp** field and `75` into the **Rs** field.
Rp sets the maximum passband ripple and **Rs** sets the stopband attenuation for the filter.
- f Press the **Apply** button to compute the new filter.

When the new filter is computed, the magnitude response of the filter is displayed with a solid line in the main axes display area:



The resulting filter is an order 78 bandpass equiripple filter.

Apply the Filter to a Signal

In this step, you apply the filter to the signal in SPTool. The new, filtered signal is automatically created in SPTool.

- 1 Activate SPTool from the **Window** menu in the Filter Designer.
- 2 Click to select the signal `noi se[vector]` from the **Signals** list and click to select the filter (named `fi l t 1[design]`) from the **Filters** list, as shown below:



- 3 Click **Apply** to apply the filter `fi l t 1` to the signal `noi se`.

The **Apply Filter** dialog box is displayed.

- 4 Name the new signal by clicking in the **Output Signal** field and typing `bl noi se`.
- 5 Click **OK**.

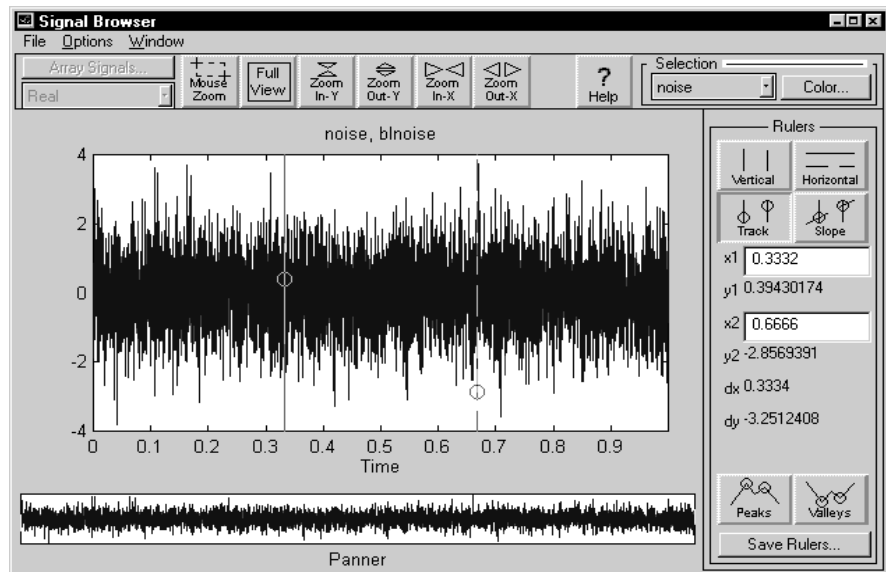
The filter is applied to the selected signal. The new, filtered signal `bl noi se[vector]` is displayed in the **Signals** list.

View and Play the Signals

You can view the time domain information of the signals using the Signal Browser. You can also play the signals, if your computer has audio output capabilities. In this step, you'll display both signals in the Signal Browser and select and play first one signal and then the other.

- 1 **Shift**-click on the noi se and bl noi se signals in the **Signals** list of SPTool to select both signals.
- 2 Click **View** in the **Signals** panel.

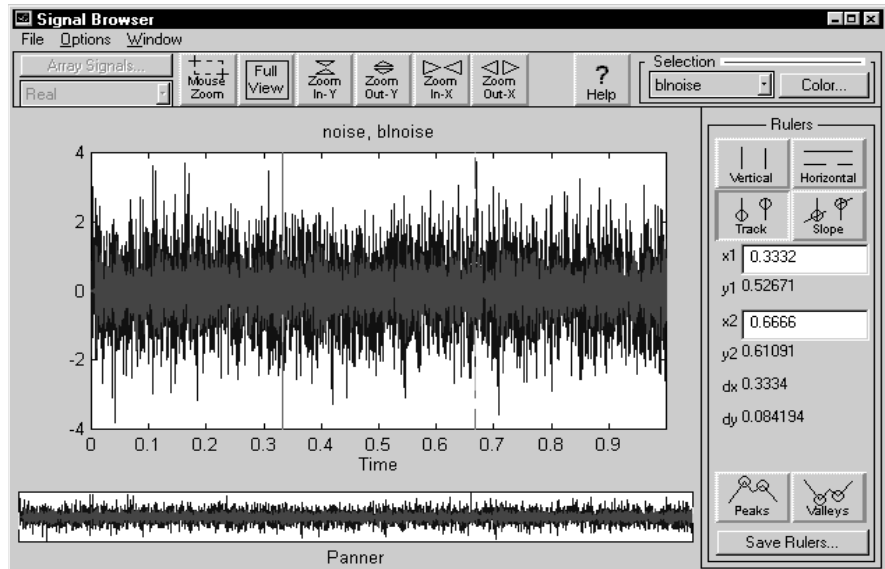
The Signal Browser is activated and both signals are displayed in the main axes display area. Initially, the noi se signal covers up the bl noi se signal, but you can see that both signals are displayed because the names of both signals are shown above the main axes display area:



- 3 Click-and-drag in the **Selection** pop-up menu to select the bl noi se signal.

The main axes display area is redisplayed. Now you can see the bl noi se signal superimposed on top of the noi se signal. The signals are displayed in different colors in both the main axes display area and the panner. Notice

that the color of the line in the **Selection** display changes to correspond to the color of the signal that you've selected:



The signal that's displayed in the **Selection** pop-up menu and in the **Selection** display is the active signal. When you select **Play**, or use the rulers, the active signal is the one that is played or measured.

- 4 To hear the active signal, select **Play** from the **Options** menu.
- 5 To hear the other signal, select it as in step 3 above and then select **Play** from the **Options** menu again. You can also select the signal by clicking on it in the main axes display area.

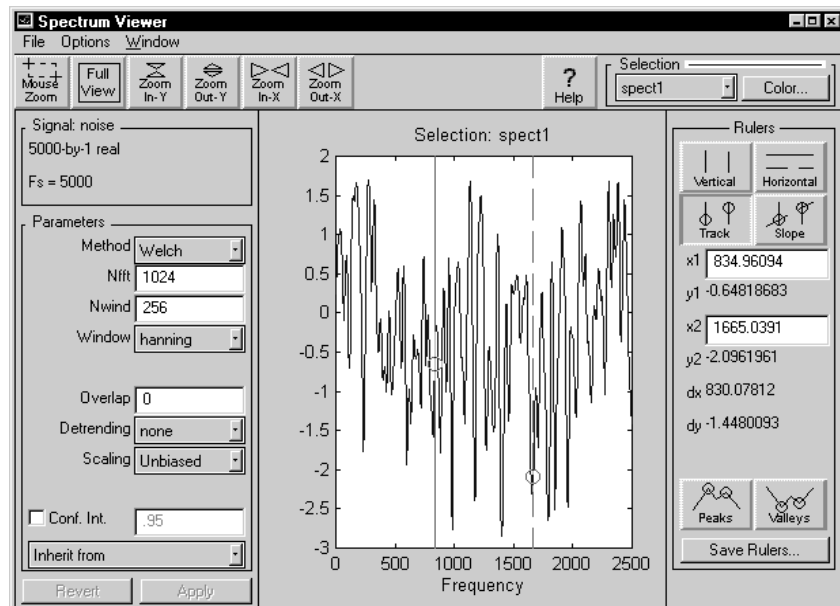
Compare Spectra of Both Signals

You can get an idea of the frequency content of the two signals by displaying their power spectra using the Spectrum Viewer, as described below.

- 1 Reactivate SPTool by selecting it from the **Window** menu of the Signal Browser.
- 2 Click on the `noise[vector]` signal in the **Signals** list of SPTool to select it.
- 3 Click **Create** in the **Spectra** panel.

The Spectrum Viewer is activated, and a spectrum object (spect1) corresponding to the `noise` signal is created in the **Spectra** list. The spectrum is not computed or displayed yet.

- 4 Click **Apply** in the Spectrum Viewer to compute and display spect1. The spectrum of the `noise` signal is displayed in the main axes display area:



Notice that the spectrum's signal identification information – including its name, its type, and its sampling frequency – is displayed above the

Parameters panel, and the spectrum's name is displayed both above the main axes display area and in the **Selection** pop-up menu.

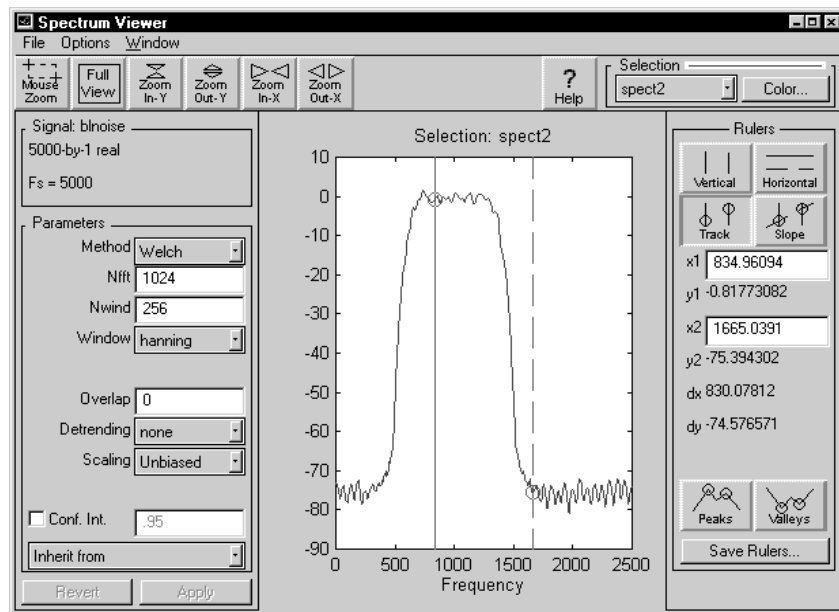
The spectrum estimate is within 2 or 3 dB of 0, so the noise has a fairly “flat” spectrum.

- 5 Reactivate SPTool by selecting it from the **Window** menu in the Spectrum Viewer.
- 6 Click on the bl noi se signal in the **Signals** list of SPTool to select it.
- 7 Click **Create** in the **Spectra** panel.

The Spectrum Viewer is again activated, and a spectrum object (spect2) corresponding to the bl noi se signal is created in the **Spectra** list. The spectrum is not computed or displayed yet.

- 8 Click **Apply** in the Spectrum Viewer to display spect2.

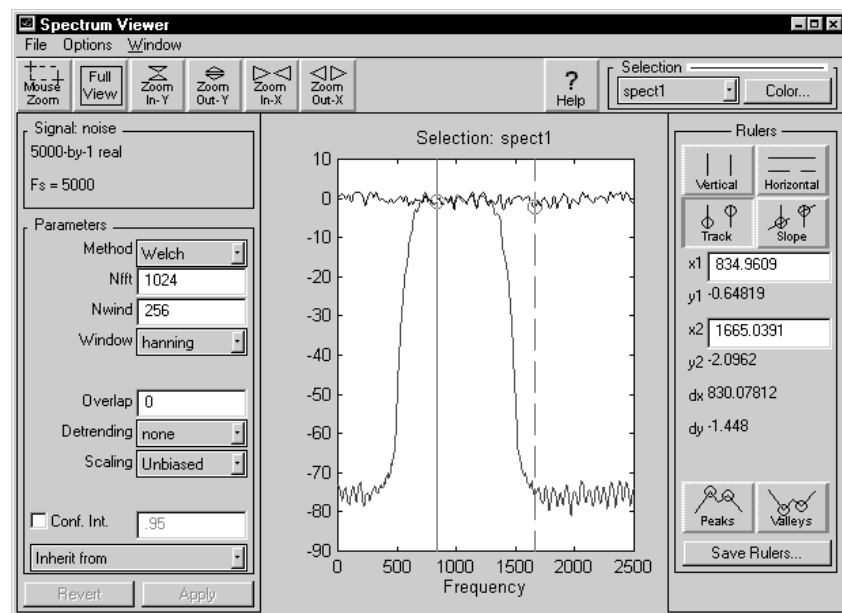
The spectrum of the bl noi se signal is displayed in the main axes display area:



The new spectrum's signal identification information – including its name, its type, and its sampling frequency – is displayed above the **Parameters** panel, and the spectrum's name is displayed both above the main axes display area and in the **Selection** pop-up menu.

This spectrum is flat between 750 and 1250 Hz and has 75 dB less power in the stopband regions of `filter`.

- 9 Reactivate SPTool again, as in step 5 above.
- 10 **Shift**-click on `spect1` and `spect2` in the **Spectra** list to select them both.
- 11 Click **View** in the **Spectra** panel to reactivate the Spectrum Viewer and display both spectra together.



12 To select one of the spectra for measuring or editing, use the **Selection** pop-up menu, or click on the spectrum in the main axes display area.

The color of the line in the **Selection** display changes to correspond to the color of the spectrum that you've selected.

The spectrum that's displayed in the **Selection** pop-up menu and in the **Selection** display is the active spectrum. When you use the rulers or change parameters, the active spectrum is the one that is measured or modified.

Reference

This chapter contains detailed descriptions of all Signal Processing Toolbox functions. It begins with a list of functions grouped by subject area and continues with the reference entries in alphabetical order. For more information, see the online *MATLAB Function Reference*.

Waveform Generation and Plotting

chi rp	Swept-frequency cosine generator.
di ri c	Dirichlet or periodic sinc function.
gauspul s	Gaussian-modulated sinusoidal pulse generator.
pul stran	Pulse train generator.
rectpul s	Sampled aperiodic rectangle generator.
sawtooth	Sawtooth or triangle wave generator.
si nc	Sinc function.
square	Square wave generator.
stri ps	Strip plot.
tri pul s	Sampled aperiodic triangle generator.

Filter Analysis and Implementation

abs	Absolute value (magnitude).
angl e	Phase angle.
conv	Convolution and polynomial multiplication.
conv2	Two-dimensional convolution.
fftfilt	FFT-based FIR filtering using the overlap-add method.
filter	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.

Filter Analysis and Implementation	
<code>filter2</code>	Two-dimensional digital filtering.
<code>filtfilt</code>	Zero-phase digital filtering.
<code>filtic</code>	Find initial conditions for a transposed direct form II filter implementation.
<code>freqs</code>	Frequency response of analog filters.
<code>freqspace</code>	Frequency spacing for frequency response.
<code>freqz</code>	Frequency response of digital filters.
<code>grpdelay</code>	Average filter delay (group delay).
<code>impz</code>	Impulse response of digital filters.
<code>latcfilt</code>	Lattice and lattice-ladder filter implementation.
<code>unwrap</code>	Unwrap phase angles.
<code>zplane</code>	Zero-pole plot.

Linear System Transformations	
<code>convmtx</code>	Convolution matrix.
<code>latc2tf</code>	Lattice filter to transfer function conversion.
<code>poly2rc</code>	Reflection coefficients from polynomial coefficients.
<code>rc2poly</code>	Polynomial coefficients from reflection coefficients.
<code>residuez</code>	z-transform partial-fraction expansion.
<code>sos2ss</code>	Second-order section to state-space conversion.
<code>sos2tf</code>	Second-order section to transfer function conversion.
<code>sos2zp</code>	Second-order section to zero-pole-gain conversion.

Linear System Transformations	
ss2sos	State-space to second-order section conversion.
ss2tf	State-space to transfer function conversion.
ss2zp	State-space to zero-pole-gain conversion.
tf2latc	Transfer function to lattice filter conversion.
tf2ss	Transfer function to state-space conversion.
tf2zp	Transfer function to zero-pole-gain conversion.
zp2sos	Zero-pole-gain to second-order section conversion.
zp2ss	Zero-pole-gain to state-space conversion.
zp2tf	Zero-pole-gain to transfer function conversion.

IIR Filter Design—Classical and Direct	
bessel f	Bessel analog filter design.
butter	Butterworth analog and digital filter design.
cheby1	Chebyshev type I filter design (passband ripple).
cheby2	Chebyshev type II filter design (stopband ripple).
ellip	Elliptic (Cauer) filter design.
maxflat	Generalized digital Butterworth filter design.
yulewalk	Recursive digital filter design.

IIR Filter Order Selection	
<code>buttord</code>	Butterworth filter order selection.
<code>cheb1ord</code>	Chebyshev type I filter order selection.
<code>cheb2ord</code>	Chebyshev type II filter order selection.
<code>ellipord</code>	Elliptic filter order selection.

FIR Filter Design	
<code>cremez</code>	Complex and nonlinear-phase equiripple FIR filter design
<code>fir1</code>	Window-based finite impulse response filter design—standard response.
<code>fir2</code>	Window-based finite impulse response filter design—arbitrary response.
<code>fircls</code>	Constrained least square FIR filter design for multiband filters.
<code>fircls1</code>	Constrained least square filter design for lowpass and highpass linear phase FIR filters.
<code>firls</code>	Least square linear-phase FIR filter design.
<code>firrcos</code>	Raised cosine FIR filter design.
<code>intfilt</code>	Interpolation FIR filter design.
<code>kaiserord</code>	Estimate parameters for an FIR filter design with Kaiser window.
<code>remez</code>	Parks-McClellan optimal FIR filter design.
<code>remezord</code>	Parks-McClellan optimal FIR filter order estimation.

Transforms	
<code>czft</code>	Chirp z-transform.
<code>dct</code>	Discrete cosine transform (DCT).
<code>dftmtx</code>	Discrete Fourier transform matrix.
<code>fft</code>	One-dimensional fast Fourier transform.
<code>fft2</code>	Two-dimensional fast Fourier transform.
<code>fftshift</code>	Rearrange the outputs of the FFT functions.
<code>hilbert</code>	Hilbert transform.
<code>idct</code>	Inverse discrete cosine transform.
<code>ifft</code>	One-dimensional inverse fast Fourier transform.
<code>ifft2</code>	Two-dimensional inverse fast Fourier transform.

Statistical Signal Processing	
<code>cohere</code>	Estimate magnitude squared coherence function between two signals.
<code>corrcoef</code>	Correlation coefficient matrix.
<code>cov</code>	Covariance matrix.
<code>csd</code>	Estimate the cross spectral density (CSD) of two signals.
<code>pburg</code>	Power spectrum estimate using the Burg method.
<code>pmtm</code>	Power spectrum estimate using the multitaper method (MTM).
<code>pmusic</code>	Power spectrum estimate using MUSIC eigenvector method.

Statistical Signal Processing	
psd	Estimate the power spectral density (PSD) of a signal using Welch's method.
pyul ear	Power spectrum estimate using Yule-Walker AR method.
tfe	Transfer function estimate from input and output.
xcorr	Cross-correlation function estimate.
xcorr2	Two-dimensional cross-correlation.
xcov	Cross-covariance function estimate (equal to mean-removed cross-correlation).

Windows	
bartlett	Bartlett window.
blackman	Blackman window.
boxcar	Rectangular window.
chebwin	Chebyshev window.
hamming	Hamming window.
hanning	Hanning window.
kaiser	Kaiser window.
triang	Triangular window.

Parametric Modeling	
<code>invfreqs</code>	Continuous-time (analog) filter identification from frequency data.
<code>invfreqz</code>	Discrete-time filter identification from frequency data.
<code>levinson</code>	Levinson-Durbin recursion.
<code>lpc</code>	Linear prediction coefficients.
<code>prony</code>	Prony's method for time domain IIR filter design.
<code>stmcb</code>	Linear model using Steiglitz-McBride iteration.

Specialized Operations	
<code>cceps</code>	Complex cepstral analysis.
<code>cplxpair</code>	Group complex numbers into complex conjugate pairs.
<code>decimate</code>	Decrease the sampling rate for a sequence (decimation).
<code>deconv</code>	Deconvolution and polynomial division.
<code>demod</code>	Demodulation for communications simulation.
<code>detrend</code>	Remove linear trends.
<code>dpss</code>	Discrete prolate spheroidal sequences (Slepian sequences).
<code>dpssclear</code>	Remove discrete prolate spheroidal sequences from database.
<code>dpssdir</code>	Discrete prolate spheroidal sequences database directory.
<code>dpssload</code>	Load discrete prolate spheroidal sequences from database.
<code>dpsssave</code>	Save discrete prolate spheroidal sequences in database.
<code>icceps</code>	Inverse complex cepstrum.

Specialized Operations	
<code>interp</code>	Increase sampling rate by an integer factor (interpolation).
<code>medfilt1</code>	One-dimensional median filtering.
<code>modulate</code>	Modulation for communications simulation.
<code>polystab</code>	Stabilize polynomial.
<code>rceps</code>	Real cepstrum and minimum phase reconstruction.
<code>resample</code>	Change sampling rate by any factor.
<code>specgram</code>	Time-dependent frequency analysis (spectrogram).
<code>upfirdn</code>	Upsample, apply an FIR filter, and downsample.
<code>vco</code>	Voltage controlled oscillator.

Analog Prototype Design	
<code>besselap</code>	Bessel analog lowpass filter prototype.
<code>buttap</code>	Butterworth analog lowpass filter prototype.
<code>cheb1ap</code>	Chebyshev type I analog lowpass filter prototype.
<code>cheb2ap</code>	Chebyshev type II analog lowpass filter prototype.
<code>ellipap</code>	Elliptic analog lowpass filter prototype.

Frequency Translation	
l p2bp	Lowpass to bandpass analog filter transformation.
l p2bs	Lowpass to bandstop analog filter transformation.
l p2hp	Lowpass to highpass analog filter transformation.
l p2l p	Lowpass to lowpass analog filter transformation.

Filter Discretization	
bi l i near	Map variables using bilinear transformation.
i mpi nvar	Impulse invariance method of analog-to-digital filter conversion.

Interactive Tools	
sptool	Interactive digital signal processing tool (SPTool).

Purpose	Absolute value (magnitude).
Syntax	<code>y = abs(x)</code>
Description	<p><code>y = abs(x)</code> returns the absolute value of the elements of <code>x</code>. If <code>x</code> is complex, <code>abs</code> returns the complex modulus (magnitude):</p> $\text{abs}(x) = \sqrt{\text{real}(x)^2 + \text{imag}(x)^2}$ <p>If <code>x</code> is a MATLAB string, <code>abs</code> returns the numeric values of the ASCII characters in the string. The display format of the string changes; the internal representation does not.</p> <p>This function is part of the standard MATLAB environment.</p>
Example	<p>Calculate the magnitude of the FFT of a sequence:</p> <pre> t = (0:99)/100; % time vector x = sin(2*pi*15*t) + sin(2*pi*40*t); % signal y = fft(x); % compute DFT of x m = abs(y); % magnitude </pre> <p>Plot the magnitude:</p> <pre> f = (0:length(y)-1)/length(y)*100; % frequency vector plot(f, m) </pre>
See Also	<code>angle</code> Phase angle.

angle

Purpose Phase angle.

Syntax `p = angle(h)`

Description `p = angle(h)` returns the phase angles, in radians, of the elements of complex vector or array `h`. The phase angles lie between $-\pi$ and π .

For complex sequence $h = x + iy = me^{ip}$, the magnitude and phase are given by

```
m = abs(h)
p = angle(h)
```

To convert back to the original `h` from its magnitude and phase:

```
i = sqrt(-1)
h = m.*exp(i*p)
```

This function is part of the standard MATLAB environment.

Example Calculate the phase of the FFT of a sequence:

```
t = (0:99)/100; % time vector
x = sin(2*pi*15*t) + sin(2*pi*40*t); % signal
y = fft(x); % compute DFT of x
p = unwrap(angle(y)); % phase
```

Plot the phase:

```
f = (0:length(y)-1)/length(y)*100; % frequency vector
plot(f, p)
```

Algorithm `angle` can be expressed as:

```
angle(x) = imag(log(x)) = atan2(imag(x), real(x))
```

See Also `abs` Absolute value (magnitude).

Purpose Bartlett window.

Syntax `w = bartlett(n)`

Description `w = bartlett(n)` returns an n-point Bartlett window in the column vector `w`. The coefficients of a Bartlett window are

For `n` odd

$$w[k] = \begin{cases} \frac{2(k-1)}{n-1}, & 1 \leq k \leq \frac{n+1}{2} \\ 2 - \frac{2(k-1)}{n-1}, & \frac{n+1}{2} \leq k \leq n \end{cases}$$

For `n` even

$$w[k] = \begin{cases} \frac{2(k-1)}{n-1}, & 1 \leq k \leq \frac{n}{2} \\ \frac{2(n-k)}{n-1}, & \frac{n}{2} + 1 \leq k \leq n \end{cases}$$

The Bartlett window is very similar to a triangular window as returned by the `triang` function. The Bartlett window always ends with zeros at samples 1 and `n`, however, while the triangular window is nonzero at those points. For `n` odd, the center `n-2` points of `bartlett(n)` are equivalent to `triang(n-2)`.

See Also

<code>blackman</code>	Blackman window.
<code>boxcar</code>	Rectangular window.
<code>chebwin</code>	Chebyshev window.
<code>hamming</code>	Hamming window.
<code>hanning</code>	Hanning window.
<code>kaiser</code>	Kaiser window.
<code>triang</code>	Triangular window.

References

[1] Oppenheim, A.V., and R.W. Schaffer, *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

Purpose Bessel analog lowpass filter prototype.

Syntax [z, p, k] = besselap(n)

Description [z, p, k] = besselap(n) returns the zeros, poles, and gain of an order n Bessel analog lowpass filter prototype. It returns the poles in the length n column vector p and the gain in scalar k. z is an empty matrix, because there are no zeros. n must be less than or equal to 25. The transfer function is

$$H(s) = \frac{k}{(s - p(1))(s - p(2)) \cdots (s - p(n))}$$

besselap normalizes the poles and gain so that at low frequency and high frequency the Bessel prototype is asymptotically equivalent to the Butterworth prototype of the same order [1]. The magnitude of the filter is less than $\sqrt{1/2}$ at the unity cutoff frequency $\Omega_c = 1$.

Analog Bessel filters are characterized by a group delay that is maximally flat at zero frequency and almost constant throughout the passband. The group delay at zero frequency is

$$\left(\frac{(2n)!}{2^n n!} \right)^{1/n}$$

Algorithm besselap finds the filter roots from a look-up table constructed using the Symbolic Math Toolbox.

See Also	bessel f	Bessel analog filter design.
	butt ap	Butterworth analog lowpass filter prototype.
	cheb1ap	Chebyshev type I analog lowpass filter prototype.
	cheb2ap	Chebyshev type II analog lowpass filter prototype.
	ell i ap	Elliptic analog lowpass filter prototype.

Also see the *Symbolic Math Toolbox User's Guide*.

References [1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pgs. 228-230.

besself

Purpose Bessel analog filter design.

Syntax

```
[b, a] = besself(n, Wn)
[b, a] = besself(n, Wn, 'ftype')
[z, p, k] = besself(...)
[A, B, C, D] = besself(...)
```

Description besself designs lowpass, bandpass, highpass, and bandstop analog Bessel filters. Analog Bessel filters are characterized by almost constant group delay across the entire passband, thus preserving the wave shape of filtered signals in the passband. Digital Bessel filters do not retain this quality, and besself therefore does not support the design of digital Bessel filters.

[b, a] = besself(n, Wn) designs an order n lowpass analog filter with cutoff frequency Wn. It returns the filter coefficients in the length n+1 row vectors b and a, with coefficients in descending powers of s:

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \dots + b(n+1)}{s^n + a(2)s^{n-1} + \dots + a(n+1)}$$

Cutoff frequency is the frequency at which the magnitude response of the filter begins to decrease significantly. For besself, the cutoff frequency Wn must be greater than 0. The magnitude response of a Bessel filter designed by besself is always less than sqrt(1/2) at the cutoff frequency, and it decreases as the order n increases.

If Wn is a two-element vector, Wn = [w1 w2] with w1 < w2, besself(n, Wn) returns an order 2*n bandpass analog filter with passband w1 < ω < w2.

[b, a] = besself(n, Wn, 'ftype') designs a highpass or bandstop filter, where ftype is

- hi gh for a highpass analog filter with cutoff frequency Wn
- stop for an order 2*n bandstop analog filter if Wn is a two-element vector, Wn = [w1 w2]

The stopband is w1 < ω < w2.

With different numbers of output arguments, `besself` directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = besself(n, Wn)` or

`[z, p, k] = besself(n, Wn, 'ftype')`

`besself` returns the zeros and poles in length `n` or `2*n` column vectors `z` and `p` and the gain in the scalar `k`.

To obtain state-space form, use four output arguments:

`[A, B, C, D] = besself(n, Wn)` or

`[A, B, C, D] = besself(n, Wn, 'ftype')` where `A`, `B`, `C`, and `D` are

$$\dot{x} = Ax + Bu$$

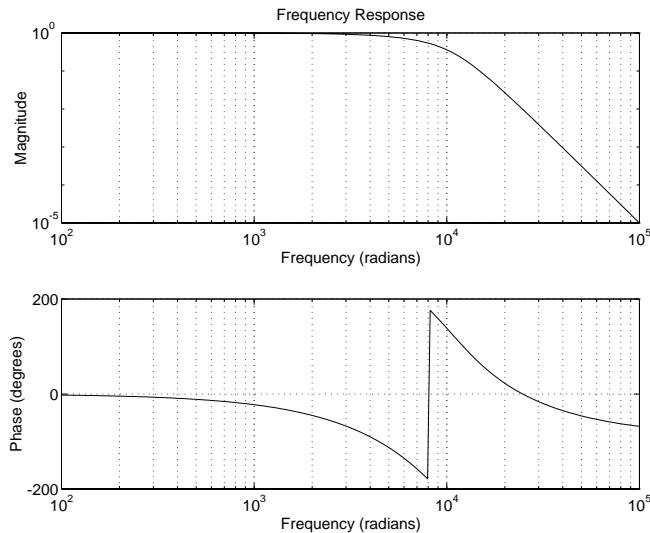
$$y = Cx + Du$$

and u is the input, x is the state vector, and y is the output.

Example

Design a fifth-order analog lowpass Bessel filter that suppresses frequencies greater than 10,000 rad/sec and plot the frequency response of the filter using freqs:

```
[b, a] = besself(5, 10000);  
freqs(b, a) % plot frequency response
```



Limitations

Lowpass Bessel filters have a monotonically decreasing magnitude response, as do lowpass Butterworth filters. Compared to the Butterworth, Chebyshev, and elliptic filters, the Bessel filter has the slowest rolloff and requires the highest order to meet an attenuation specification.

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function coefficient form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm

`besself` performs a four-step algorithm:

- 1 It finds lowpass analog prototype poles, zeros, and gain using the `besselap` function.
- 2 It converts the poles, zeros, and gain into state-space form.
- 3 It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
- 4 It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

<code>besselap</code>	Bessel analog lowpass filter prototype.
<code>butter</code>	Butterworth analog and digital filter design.
<code>cheby1</code>	Chebyshev type I filter design (passband ripple).
<code>cheby2</code>	Chebyshev type II filter design (stopband ripple).
<code>ellip</code>	Elliptic (Cauer) filter design.

bilinear

Purpose Map variables using bilinear transformation.

Syntax

```
[zd, pd, kd] = bilinear(z, p, k, Fs)
[zd, pd, kd] = bilinear(z, p, k, Fs, Fp)
[numd, dend] = bilinear(num, den, Fs)
[numd, dend] = bilinear(num, den, Fs, Fp)
[Ad, Bd, Cd, Dd] = bilinear(A, B, C, D, Fs)
[Ad, Bd, Cd, Dd] = bilinear(A, B, C, D, Fs, Fp)
```

Description The *bilinear transformation* is a mathematical mapping of variables. In digital filtering, it is a standard method of mapping the s or analog plane into the z or digital plane. It transforms analog filters, designed using classical filter design techniques, into their discrete equivalents.

The bilinear transformation maps the s -plane into the z -plane by

$$H(z) = H(s) \Big|_{s=2f_s \frac{z-1}{z+1}}$$

This transformation maps the $j\Omega$ axis (from $\Omega = -\infty$ to $+\infty$) repeatedly around the unit circle ($\exp(j\omega)$, from $\omega = -\pi$ to π) by

$$\omega = 2 \tan^{-1} \left(\frac{\Omega}{2f_s} \right)$$

`bilinear` can accept an optional parameter `Fp` that specifies prewarping. `Fp`, in Hertz, indicates a “match” frequency, that is, a frequency for which the frequency responses before and after mapping match exactly. In prewarped mode, the bilinear transformation maps the s -plane into the z -plane with

$$H(z) = H(s) \Big|_{s = \frac{2\pi f_p}{\tan\left(\pi \frac{f_p}{f_s}\right)} \frac{(z-1)}{(z+1)}}$$

With the prewarping option, `bilinear` maps the $j\Omega$ axis (from $\Omega = -\infty$ to $+\infty$) repeatedly around the unit circle ($\exp(j\omega)$, from $\omega = -\pi$ to π) by

$$\omega = 2 \tan^{-1} \left(\frac{\Omega \tan\left(\pi \frac{f_p}{f_s}\right)}{2\pi f_p} \right)$$

In prewarped mode, `bilinear` matches the frequency $2\pi f_p$ (in radians per second) in the s -plane to the normalized frequency $2\pi f_p/f_s$ (in radians per second) in the z -plane.

The `bilinear` function works with three different linear system representations: zero-pole-gain, transfer function, and state-space form.

Zero-Pole-Gain

`[zd, pd, kd] = bilinear(z, p, k, Fs)` and

`[zd, pd, kd] = bilinear(z, p, k, Fs, Fp)` convert the s -domain transfer function specified by `z`, `p`, and `k` to a discrete equivalent. Inputs `z` and `p` are column vectors containing the zeros and poles, and `k` is a scalar gain. `Fs` is the sampling frequency in Hertz. `bilinear` returns the discrete equivalent in column vectors `zd` and `pd` and scalar `kd`. `Fp` is the optional match frequency, in Hertz, for prewarping.

Transfer Function

`[numd, dend] = bilinear(num, den, Fs)` and

`[numd, dend] = bilinear(num, den, Fs, Fp)` convert an s -domain transfer function given by `num` and `den` to a discrete equivalent. Row vectors `num` and `den` specify the coefficients of the numerator and denominator, respectively, in descending powers of s

$$\frac{num(s)}{den(s)} = \frac{num(1)s^{nn} + \dots + num(nn)s + num(nn+1)}{den(1)s^{nd} + \dots + den(nd)s + den(nd+1)}$$

`Fs` is the sampling frequency in Hertz. `bilinear` returns the discrete equivalent in row vectors `numd` and `dend` in descending powers of z (ascending powers of z^{-1}). `Fp` is the optional match frequency, in Hertz, for prewarping.

State-Space

`[Ad, Bd, Cd, Dd] = bilinear(A, B, C, D, Fs)` and

`[Ad, Bd, Cd, Dd] = bilinear(A, B, C, D, Fs, Fp)` convert the continuous-time state-space system in matrices A, B, C, D,

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

to the discrete-time system

$$x[n+1] = A_d x[n] + B_d u[n]$$

$$y[n] = C_d x[n] + D_d u[n]$$

Fs is the sampling frequency in Hertz. `bilinear` returns the discrete equivalent in matrices Ad, Bd, Cd, Dd. Fp is the optional match frequency, in Hertz, for prewarping.

Algorithm

`bilinear` uses one of two algorithms, depending on the format of the input linear system you supply. One algorithm works on the zero-pole-gain format and the other on the state-space format. For transfer function representations, `bilinear` converts to state-space form, performs the transformation, and converts the resulting state-space system back to transfer function form.

Zero-Pole-Gain Algorithm

For a system in zero-pole-gain form, `bilinear` performs four steps:

- 1 If Fp is present, $k = 2\pi \cdot \text{Fp} / \tan(\pi \cdot \text{Fp} / \text{Fs})$; otherwise $k = 2 \cdot \text{Fs}$.
- 2 It strips any zeros at plus or minus infinity using
$$z = z(\text{find}(\text{finite}(z)));$$
- 3 It transforms the zeros, poles, and gain using
$$\begin{aligned}pd &= (1+p/k) ./ (1-p/k); \\zd &= (1+z/k) ./ (1-z/k); \\kd &= \text{real}(k \cdot \text{prod}(fs-z) ./ \text{prod}(fs-p));\end{aligned}$$
- 4 It adds extra zeros at -1 so the resulting system has equivalent numerator and denominator order.

State-Space Algorithm

For a system in state-space form, `bilinear` performs two steps:

- 1 If `Fp` is present, $k = 2\pi \cdot \text{Fp} / \tan(\pi \cdot \text{Fp} / \text{Fs})$; else $k = 2 \cdot \text{Fs}$.
- 2 It computes A_d , B_d , C_d , and D_d in terms of A , B , C , and D using

$$A_d = \left(I + \left(\frac{1}{k} \right) A \right) \left(I - \left(\frac{1}{k} \right) A \right)^{-1}$$

$$B_d = \frac{2k}{r} \left(I - \left(\frac{1}{k} \right) A \right)^{-1} B$$

$$C_d = r C \left(I - \left(\frac{1}{k} \right) A \right)^{-1}$$

$$D_d = \left(\frac{1}{k} \right) C \left(I - \left(\frac{1}{k} \right) A \right)^{-1} B + D$$

`bilinear` implements these relations using conventional MATLAB statements. The scalar `r` is arbitrary; `bilinear` uses `sqrt(2/k)` to ensure good quantization noise properties in the resulting system.

Diagnostics

`bilinear` requires that the numerator order be no greater than the denominator order. If this is not the case, `bilinear` displays:

Numerator cannot be higher order than denominator.

For `bilinear` to distinguish between the zero-pole-gain and transfer function linear system formats, the first two input parameters must be vectors with the same orientation in these cases. If this is not the case, `bilinear` displays:

First two arguments must have the same orientation.

See Also

<code>impinvar</code>	Impulse invariance method of analog-to-digital filter conversion.
<code>lp2bp</code>	Lowpass to bandpass analog filter transformation.
<code>lp2bs</code>	Lowpass to bandstop analog filter transformation.
<code>lp2hp</code>	Lowpass to highpass analog filter transformation.
<code>lp2lp</code>	Lowpass to lowpass analog filter transformation.

References

- [1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Pgs. 209-213.
- [2] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989. Pgs. 415-430.

Purpose Blackman window.

Syntax
`w = blackman(n)`
`w = blackman(n, sflag)`

Description `w = blackman(n)` returns the n -point symmetrically sampled Blackman window in the column vector `w`. n should be a nonnegative integer. The equation for a Blackman window is

$$w[k] = 0.42 - 0.5 \cos\left(2\pi \frac{k-1}{n-1}\right) + 0.08 \cos\left(4\pi \frac{k-1}{n-1}\right), \quad k = 1, \dots, n$$

Blackman windows have slightly wider central lobes and less sideband leakage than equivalent length Hamming and Hanning windows.

`w = blackman(n, sflag)` returns an n -point Blackman window using the window sampling specified by `sflag`, which can be either `'periodic'` or `'symmetric'` (the default). When `'periodic'` is specified, `blackman` computes a length $n+1$ window and returns the first n points.

Algorithm
`w = (0.42 - 0.5*cos(2*pi*(0:N-1)/(N-1)) + ...`
`0.08*cos(4*pi*(0:N-1)/(N-1)))';`

Diagnostics An error message is displayed when incorrect arguments are used:

Order cannot be less than zero.

Sampling must be either `'symmetric'` or `'periodic'`.

A warning message is displayed for noninteger n :

Warning: Rounding order to nearest integer.

See Also

bartlett	Bartlett window.
boxcar	Rectangular window.
chebwin	Chebyshev window.
hamming	Hamming window.
hanning	Hanning window.
kaiser	Kaiser window.
triang	Triangular window.

References

[1] Oppenheim, A.V., and R.W. Schaffer, *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

Purpose	Rectangular window.	
Syntax	<code>w = boxcar(n)</code>	
Description	<code>w = boxcar(n)</code> returns a rectangular window of length <code>n</code> in the column vector <code>w</code> . This function is provided for completeness; a rectangular window is equivalent to no window at all.	
Algorithm	<code>w = ones(n, 1);</code>	
See Also	<code>bartlett</code>	Bartlett window.
	<code>blackman</code>	Blackman window.
	<code>chebwin</code>	Chebyshev window.
	<code>hamming</code>	Hamming window.
	<code>hanning</code>	Hanning window.
	<code>kaiser</code>	Kaiser window.
References	<code>triang</code>	Triangular window.
	[1] Oppenheim, A.V., and R.W. Schafer, <i>Discrete-Time Signal Processing</i> . Englewood Cliffs, NJ: Prentice-Hall, 1989.	

buttap

Purpose Butterworth analog lowpass filter prototype.

Syntax [z, p, k] = buttap(n)

Description [z, p, k] = buttap(n) returns the zeros, poles, and gain of an order n Butterworth analog lowpass filter prototype. It returns the poles in the length n column vector p and the gain in scalar k. z is an empty matrix, because there are no zeros. The transfer function is

$$H(s) = \frac{z(s)}{p(s)} = \frac{k}{(s - p(1))(s - p(2)) \cdots (s - p(n))}$$

Butterworth filters are characterized by a magnitude response that is maximally flat in the passband and monotonic overall. In the lowpass case, the first $2n-1$ derivatives of the squared magnitude response are zero at $\omega = 0$. The squared magnitude response function is

$$|H(\omega)|^2 = \frac{1}{1 + (\omega/\omega_0)^{2n}}$$

corresponding to a transfer function with poles equally spaced around a circle in the left half plane. The magnitude response at the cutoff frequency ω_0 is always $1/\sqrt{2}$, regardless of the filter order. buttap sets ω_0 to 1 for a normalized result.

Algorithm

```
z = [];  
p = exp(sqrt(-1)*(pi*(1:2:2*n-1)/(2*n)+pi/2)).';  
k = real(prod(-p));
```

See Also

besselap	Bessel analog lowpass filter prototype.
butter	Butterworth analog and digital filter design.
cheb1ap	Chebyshev type I analog lowpass filter prototype.
cheb2ap	Chebyshev type II analog lowpass filter prototype.
ellipap	Elliptic analog lowpass filter prototype.

References [1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Chapter 7.

Purpose	Butterworth analog and digital filter design.
Syntax	<pre> [b, a] = butter(n, Wn) [b, a] = butter(n, Wn, 'ftype') [b, a] = butter(n, Wn, 's') [b, a] = butter(n, Wn, 'ftype', 's') [z, p, k] = butter(...) [A, B, C, D] = butter(...) </pre>
Description	<p>butter designs lowpass, bandpass, highpass, and bandstop digital and analog Butterworth filters. Butterworth filters are characterized by a magnitude response that is maximally flat in the passband and monotonic overall.</p> <p>Butterworth filters sacrifice rolloff steepness for monotonicity in the pass- and stopbands. Unless the smoothness of the Butterworth filter is needed, an elliptic or Chebyshev filter can generally provide steeper rolloff characteristics with a lower filter order.</p> <p>Digital Domain</p> <p><code>[b, a] = butter(n, Wn)</code> designs an order n lowpass digital Butterworth filter with cutoff frequency W_n. It returns the filter coefficients in length $n + 1$ row vectors b and a, with coefficients in descending powers of z:</p> $H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \dots + a(n+1)z^{-n}}$ <p><i>Cutoff frequency</i> is that frequency where the magnitude response of the filter is $\sqrt{1/2}$. For <code>butter</code>, the cutoff frequency W_n must be a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).</p> <p>If W_n is a two-element vector, $W_n = [w1 \ w2]$, <code>butter</code> returns an order $2*n$ digital bandpass filter with passband $w1 < \omega < w2$.</p>

`[b, a] = butter(n, Wn, 'ftype')` designs a highpass or bandstop filter, where *ftype* is

- `high` for a highpass digital filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop digital filter if W_n is a two-element vector,
 $W_n = [w1 \ w2]$

The stopband is $w1 < \omega < w2$.

With different numbers of output arguments, `butter` directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = butter(n, Wn)` or

`[z, p, k] = butter(n, Wn, 'ftype')`

`butter` returns the zeros and poles in length n column vectors z and p , and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = butter(n, Wn)` or

`[A, B, C, D] = butter(n, Wn, 'ftype')` where A , B , C , and D are

$$x[n+1] = Ax[n] + Bu[n]$$

$$y[n] = Cx[n] + Du[n]$$

and u is the input, x is the state vector, and y is the output.

Analog Domain

`[b, a] = butter(n, Wn, 's')` designs an order n lowpass analog Butterworth filter with cutoff frequency W_n . It returns the filter coefficients in the length $n + 1$ row vectors b and a , in descending powers of s :

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \dots + b(n+1)}{s^n + a(2)s^{n-1} + \dots + a(n+1)}$$

`butter`'s cutoff frequency W_n must be greater than 0.

If W_n is a two-element vector with $w_1 < w_2$, `butter(n, Wn, 's')` returns an order $2*n$ bandpass analog filter with passband $w_1 < \omega < w_2$.

`[b, a] = butter(n, Wn, 'ftype', 's')` designs a highpass or bandstop filter, where *ftype* is

- `hi gh` for a highpass analog filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop analog filter if W_n is a two-element vector, $W_n = [w_1 \ w_2]$

The stopband is $w_1 < \omega < w_2$.

With different numbers of output arguments, `butter` directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = butter(n, Wn, 's')` or

`[z, p, k] = butter(n, Wn, 'ftype', 's')` returns the zeros and poles in length n or $2*n$ column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = butter(n, Wn, 's')` or

`[A, B, C, D] = butter(n, Wn, 'ftype', 's')` where A , B , C , and D are

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

and u is the input, x is the state vector, and y is the output.

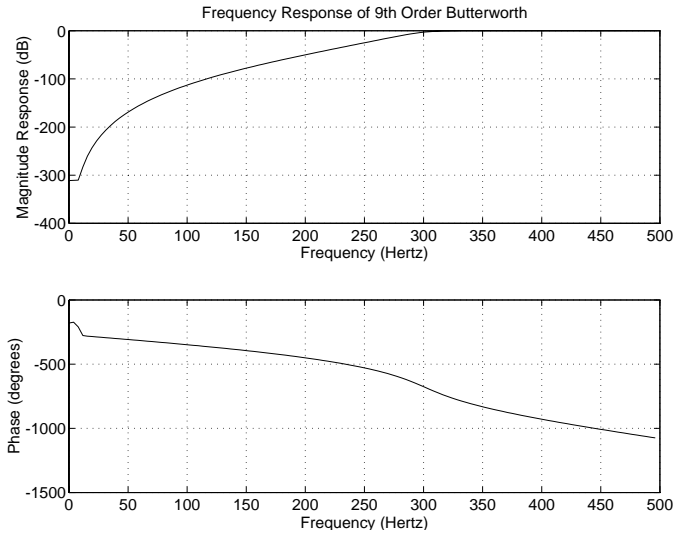
Examples

For data sampled at 1000 Hz, design a 9th-order highpass Butterworth filter with cutoff frequency of 300 Hz:

```
[b, a] = butter(9, 300/500, 'hi gh')
```

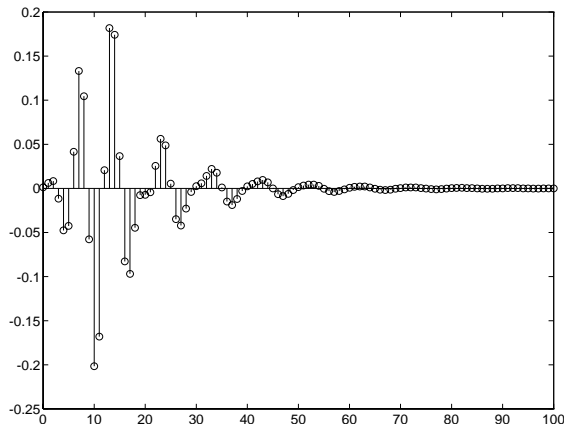
The filter's frequency response is

```
freqz(b, a, 128, 1000)
```



Design a 10th-order bandpass Butterworth filter with a passband from 100 to 200 Hz and plot its impulse response, or *unit sample response*:

```
n = 5; Wn = [100 200]/500;
[b, a] = butter(n, Wn);
[y, t] = impz(b, a, 101);
stem(t, y)
```



Limitations

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function coefficient form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm

`butter` uses a five-step algorithm:

- 1 It finds the lowpass analog prototype poles, zeros, and gain using the `buttap` function.
- 2 It converts the poles, zeros, and gain into state-space form.
- 3 It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
- 4 For digital filter design, `butter` uses `bilinear` to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at ω_n or ω_1 and ω_2 .
- 5 It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

<code>bessel f</code>	Bessel analog filter design.
<code>buttap</code>	Butterworth analog lowpass filter prototype.
<code>buttford</code>	Butterworth filter order selection.
<code>cheby1</code>	Chebyshev type I filter design (passband ripple).
<code>cheby2</code>	Chebyshev type II filter design (stopband ripple).
<code>ellip</code>	Elliptic (Cauer) filter design.
<code>maxflat</code>	Generalized digital Butterworth filter design.

buttord

Purpose Butterworth filter order selection.

Syntax `[n, Wn] = buttord(Wp, Ws, Rp, Rs)`
`[n, Wn] = buttord(Wp, Ws, Rp, Rs, ' s ')`

Description `buttord` selects the minimum order digital or analog Butterworth filter required to meet a set of filter design specifications:

Wp	Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
Ws	Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
Rp	Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is $0 < w < Wp$.
Rs	Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is $Ws < w < 1$.

Digital Domain

`[n, Wn] = buttord(Wp, Ws, Rp, Rs)` returns the order n of the lowest order digital Butterworth filter that loses no more than R_p dB in the passband and has at least R_s dB of attenuation in the stopband. The passband runs from 0 to W_p and the stopband runs from W_s to 1, the Nyquist frequency. `buttord` also returns W_n , the Butterworth cutoff frequency that allows butter to achieve the given specifications (the “3 dB” frequency).

Use `buttord` for highpass, bandpass, and bandstop filters. For highpass filters, W_p is greater than W_s . For bandpass and bandstop filters, W_p and W_s are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, `buttord` returns W_n as a two-element row vector for input to `butter`.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.

Analog Domain

`[n, Wn] = butterd(Wp, Ws, Rp, Rs, 's')` finds the minimum order n and cutoff frequencies Wn for an analog filter. In this case the frequencies in Wp and Ws are in radians per second and may be greater than 1.

Use `butterd` for highpass, bandpass, and bandstop filters, as described under “Digital Domain.”

Examples

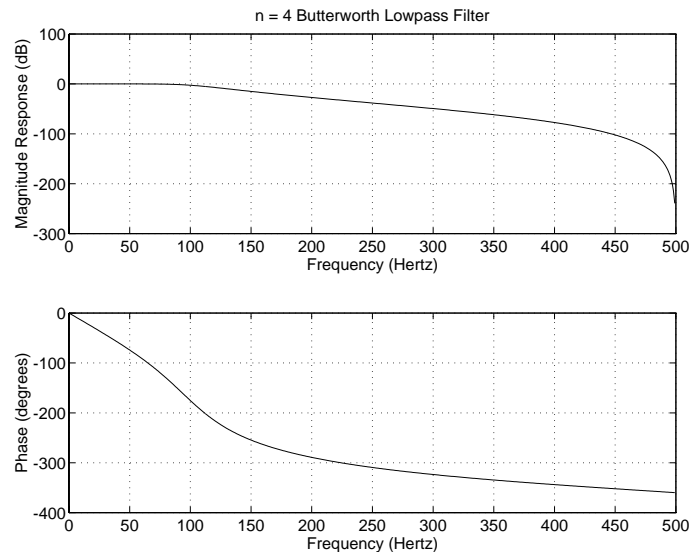
For data sampled at 1000 Hz, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz, and attenuation at least 15 dB from 150 Hz to the Nyquist frequency. Plot the filter’s frequency response:

```
Wp = 100/500; Ws = 150/500;
[n, Wn] = butterd(Wp, Ws, 3, 15)
```

```
n =
    4
```

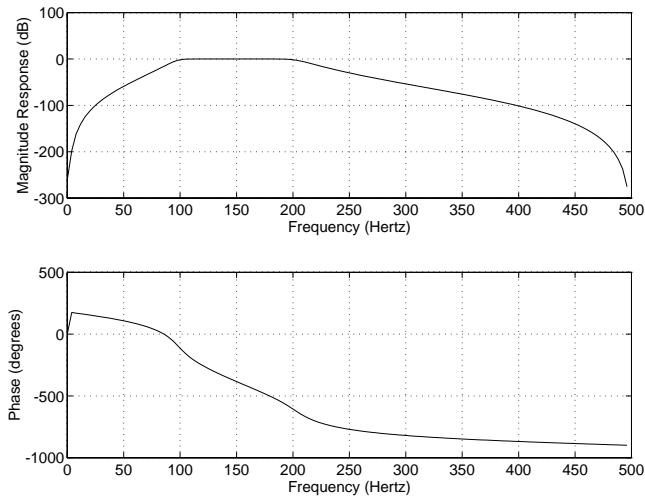
```
Wn =
    0.2042
```

```
[b, a] = butter(n, Wn);
freqz(b, a, 512, 1000)
```



Next design a bandpass filter with passband of 100 Hz to 200 Hz, less than 3 dB of attenuation at the passband corners, and attenuation down 30 dB by 50 Hz out on both sides of the passband:

```
Wp = [100 200]/500; Ws = [50 250]/500;  
Rp = 3; Rs = 30;  
[n, Wn] = buttord(Wp, Ws, Rp, Rs); [b, a] = butter(n, Wn);  
freqz(b, a, 128, 1000)
```



Algorithm

`buttord`'s order prediction formula is described in [1]. It operates in the analog domain for both analog and digital cases. For the digital case, it converts the frequency parameters to the s -domain before estimating the order and natural frequency, then converts back to the z -domain.

`buttord` initially develops a lowpass filter prototype by transforming the passband frequencies of the desired filter to 1 rad/sec (for low- and highpass filters) and to -1 and 1 rad/sec (for bandpass and bandstop filters). It then computes the minimum order required for a lowpass filter to meet the stopband specification.

See Also

buttf	Butterworth analog and digital filter design.
cheb1ord	Chebyshev type I filter order selection.
cheb2ord	Chebyshev type II filter order selection.
ellipord	Elliptic filter order selection.
kaiserord	Estimate parameters for an FIR filter design with Kaiser window.

References

[1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pg. 227.

Purpose Complex cepstral analysis.

Syntax

```
xhat = cceps(x)
[xhat, nd] = cceps(x)
[xhat, nd, xhat1] = cceps(x)
[... ] = cceps(x, n)
```

Description Cepstral analysis is a nonlinear signal processing technique that is applied most commonly in speech processing and homomorphic filtering [1].

`xhat = cceps(x)` returns the complex cepstrum of the (assumed real) sequence `x`. The input is altered, by the application of a linear phase term, to have no phase discontinuity at $\pm\pi$ radians. That is, it is circularly shifted (after zero padding) by some samples, if necessary, to have zero phase at π radians.

`[xhat, nd] = cceps(x)` returns the number of samples `nd` of (circular) delay added to `x` prior to finding the complex cepstrum.

`[xhat, nd, xhat1] = cceps(x)` returns a second complex cepstrum, computed using an alternate rooting algorithm, in `xhat1`. The alternate method ([1] p.795) is useful for short sequences that can be rooted and do not have zeros on the unit circle. For these signals, `xhat1` can provide a verification of `xhat`.

`[...] = cceps(x, n)` zero pads `x` to length `n` and returns the length `n` complex cepstrum of `x`.

Algorithm `cceps`, in its basic form, is an M-file implementation of algorithm 7.1 in [2]. A lengthy Fortran program reduces to three lines of MATLAB code:

```
h = fft(x);
logh = log(abs(h)) + sqrt(-1)*rcunwrap(angle(h));
y = real(iffth(logh));
```

`rcunwrap` is a special version of `unwrap` that subtracts a straight line from the phase.

See Also

i cceps	Inverse complex cepstrum.
hi l bert	Hilbert transform.
r ceps	Real cepstrum and minimum phase reconstruction.
unwrap	Unwrap phase angles.

References

- [1] Oppenheim, A.V., and R.W. Schafer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [2] IEEE. *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979.

cheb1ap

Purpose Chebyshev type I analog lowpass filter prototype.

Syntax [z, p, k] = cheb1ap(n, Rp)

Description [z, p, k] = cheb1ap(n, Rp) returns the zeros, poles, and gain of an order n Chebyshev type I analog lowpass filter prototype with Rp dB of ripple in the passband. It returns the poles in the length n column vector p and the gain in scalar k. z is an empty matrix, because there are no zeros. The transfer function is

$$H(s) = \frac{z(s)}{p(s)} = \frac{k}{(s - p(1))(s - p(2)) \cdots (s - p(n))}$$

Chebyshev type I filters are equiripple in the passband and monotonic in the stopband. The poles are evenly spaced about an ellipse in the left half plane. The Chebyshev type I cutoff frequency ω_0 is set to 1.0 for a normalized result. This is the frequency at which the passband ends and the filter has magnitude response of $10^{-Rp/20}$.

See Also	bessel ap	Bessel analog lowpass filter prototype.
	butt ap	Butterworth analog and digital filter design.
	cheb2ap	Chebyshev type II analog lowpass filter prototype.
	cheby1	Chebyshev type I filter design (passband ripple).
	ellip ap	Elliptic analog lowpass filter prototype.

References [1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Chapter 7.

Purpose	Chebyshev type I filter order selection.
Syntax	<pre>[n, Wn] = cheb1ord(Wp, Ws, Rp, Rs) [n, Wn] = cheb1ord(Wp, Ws, Rp, Rs, 's')</pre>
Description	<p>cheb1ord selects the minimum order digital or analog Chebyshev type I filter required to meet a set of filter design specifications:</p> <p>Wp Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).</p> <p>Ws Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).</p> <p>Rp Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is $0 < w < Wp$.</p> <p>Rs Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is $Ws < w < 1$.</p>

Digital Domain

`[n, Wn] = cheb1ord(Wp, Ws, Rp, Rs)` returns the order *n* of the lowest order Chebyshev filter that loses no more than *Rp* dB in the passband and has at least *Rs* dB of attenuation in the stopband. The passband runs from 0 to *Wp* and the stopband runs from *Ws* to 1, the Nyquist frequency. cheb1ord also returns *Wn*, the Chebyshev type I cutoff frequency that allows cheby1 to achieve the given specifications.

Use cheb1ord for lowpass, highpass, bandpass, and bandstop filters. For highpass filters, $Wp > Ws$. For bandpass and bandstop filters, *Wp* and *Ws* are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, cheb1ord returns *Wn* as a two-element row vector for input to cheby1.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.

Analog Domain

`[n, Wn] = cheb1ord(Wp, Ws, Rp, Rs, 's')` finds the minimum order n and cutoff frequencies W_n for an analog filter. In this case the frequencies in W_p and W_s are in radians per second and may be greater than 1.

Use `cheb1ord` for lowpass, highpass, bandpass, and bandstop filters, as described under “Digital Domain.”

Examples

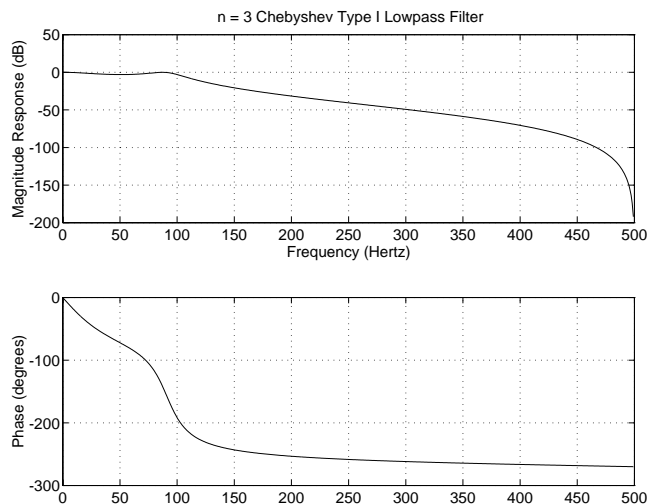
For data sampled at 1000 Hz, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz and attenuation at least 15 dB from 150 Hz to the Nyquist frequency:

```
Wp = 100/500; Ws = 150/500;  
Rp = 3; Rs = 15;  
[n, Wn] = cheb1ord(Wp, Ws, Rp, Rs)
```

```
n =  
    3
```

```
Wn =  
    0.2000
```

```
[b, a] = cheby1(n, Rp, Wn);  
freqz(b, a, 512, 1000)
```



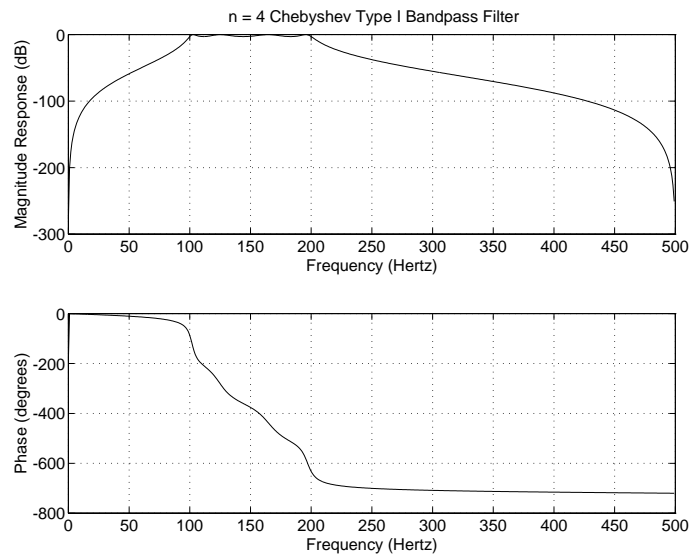
Next design a bandpass filter with a passband of 100 Hz to 200 Hz, less than 3 dB of attenuation throughout the passband, and 30 dB stopbands 50 Hz out on both sides of the passband:

```
Wp = [ 100 200 ] / 500; Ws = [ 50 250 ] / 500;
Rp = 3; Rs = 30;
[ n, Wn ] = cheb1ord(Wp, Ws, Rp, Rs)
```

```
n =
    4
```

```
Wn =
    0.2000    0.4000
```

```
[ b, a ] = cheby1(n, Rp, Wn);
freqz(b, a, 512, 1000)
```



Algorithm

cheb1ord uses the Chebyshev lowpass filter order prediction formula described in [1]. The function performs its calculations in the analog domain for both analog and digital cases. For the digital case, it converts the frequency parameters to the s -domain before the order and natural frequency estimation process, then converts them back to the z -domain.

cheb1ord initially develops a lowpass filter prototype by transforming the passband frequencies of the desired filter to 1 rad/sec (for low- or highpass filters) or to -1 and 1 rad/sec (for bandpass or bandstop filters). It then computes the minimum order required for a lowpass filter to meet the stopband specification.

See Also	buttord	Butterworth filter order selection.
	cheby1	Chebyshev type I filter design (passband ripple).
	cheb2ord	Chebyshev type II filter order selection.
	ellipord	Elliptic filter order selection.
	kaiserord	Estimate parameters for an FIR filter design with Kaiser window.

References [1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pg. 241.

Purpose	Chebyshev type II analog lowpass filter prototype.	
Syntax	<code>[z, p, k] = cheb2ap(n, Rs)</code>	
Description	<p><code>[z, p, k] = cheb2ap(n, Rs)</code> finds the zeros, poles, and gain of an order <code>n</code> Chebyshev type II analog lowpass filter prototype with stopband ripple <code>Rs</code> dB down from the passband peak value. <code>cheb2ap</code> returns the zeros and poles in length <code>n</code> column vectors <code>z</code> and <code>p</code> and the gain in scalar <code>k</code>. If <code>n</code> is odd, <code>z</code> is length <code>n-1</code>. The transfer function is</p> $H(s) = \frac{z(s)}{p(s)} = k \frac{(s - z(1))(s - z(2)) \cdots (s - z(n))}{(s - p(1))(s - p(2)) \cdots (s - p(n))}$ <p>Chebyshev type II filters are monotonic in the passband and equiripple in the stopband. The pole locations are the inverse of the pole locations of <code>cheb1ap</code>, whose poles are evenly spaced about an ellipse in the left half plane. The Chebyshev type II cutoff frequency ω_0 is set to 1 for a normalized result. This is the frequency at which the stopband begins and the filter has magnitude response of $10^{-Rs/20}$.</p>	
Algorithm	<p>Chebyshev type II filters are sometimes called <i>inverse Chebyshev</i> filters because of their relationship to Chebyshev type I filters. The <code>cheb2ap</code> function is a modification of the Chebyshev type I prototype algorithm:</p> <ol style="list-style-type: none"> 1 <code>cheb2ap</code> replaces the frequency variable ω with $1/\omega$, turning the lowpass filter into a highpass filter while preserving the performance at $\omega = 1$. 2 <code>cheb2ap</code> subtracts the filter transfer function from unity. 	
See Also	<p><code>bessel ap</code> Bessel analog lowpass filter prototype.</p> <p><code>butt ap</code> Butterworth analog lowpass filter prototype.</p> <p><code>cheb1ap</code> Chebyshev type I analog lowpass filter prototype.</p> <p><code>cheby2</code> Chebyshev type II filter design (stopband ripple).</p> <p><code>ell ip ap</code> Elliptic analog lowpass filter prototype.</p>	
References	<p>[1] Parks, T.W., and C.S. Burrus. <i>Digital Filter Design</i>. New York: John Wiley & Sons, 1987. Chapter 7.</p>	

cheb2ord

Purpose Chebyshev type II filter order selection.

Syntax `[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs)`
`[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs, ' s')`

Description `cheb2ord` selects the minimum order digital or analog Chebyshev type II filter required to meet a set of filter design specifications:

Wp	Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
Ws	Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).
Rp	Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is $0 < w < Wp$.
Rs	Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is $Ws < w < 1$.

Digital Domain

`[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs)` returns the order `n` of the lowest order Chebyshev filter that loses no more than `Rp` dB in the passband and has at least `Rs` dB of attenuation in the stopband. The passband runs from 0 to `Wp` and the stopband runs from `Ws` to 1, the Nyquist frequency. `cheb2ord` also returns `Wn`, the Chebyshev type II cutoff frequency that allows `cheby2` to achieve the given specifications.

Use `cheb2ord` for lowpass, highpass, bandpass, and bandstop filters. For highpass filters, `Wp` is greater than `Ws`. For bandpass and bandstop filters, `Wp` and `Ws` are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, `cheb2ord` returns `Wn` as a two-element row vector for input to `cheby2`.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.

Analog Domain

`[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs, 's')` finds the minimum order n and cutoff frequencies Wn for an analog filter. In this case the frequencies in Wp and Ws are in radians per second and may be greater than 1.

Use `cheb2ord` for lowpass, highpass, bandpass, and bandstop filters, as described under “Digital Domain.”

Examples

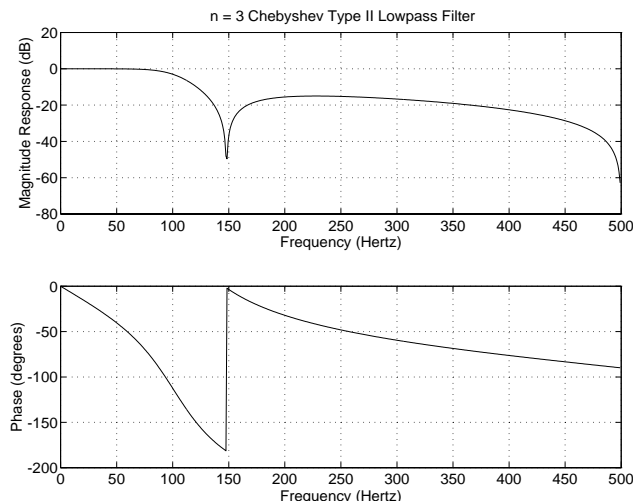
For data sampled at 1000 Hz, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz, and attenuation at least 15 dB from 150 Hz to the Nyquist frequency:

```
Wp = 100/500; Ws = 150/500;
Rp = 3; Rs = 15;
[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs)
```

```
n =
     3
```

```
Wn =
     0.2609
```

```
[b, a] = cheby2(n, Rs, Wn);
freqz(b, a, 512, 1000)
```



Next design a bandpass filter with a passband of 100 Hz to 200 Hz, less than 3 dB of attenuation throughout the passband, and 30 dB stopbands 50 Hz out on both sides of the passband:

```
Wp = [ 100 200]/500; Ws = [ 50 250]/500;
```

```
Rp = 3; Rs = 30;
```

```
[n, Wn] = cheb2ord(Wp, Ws, Rp, Rs)
```

```
n =
```

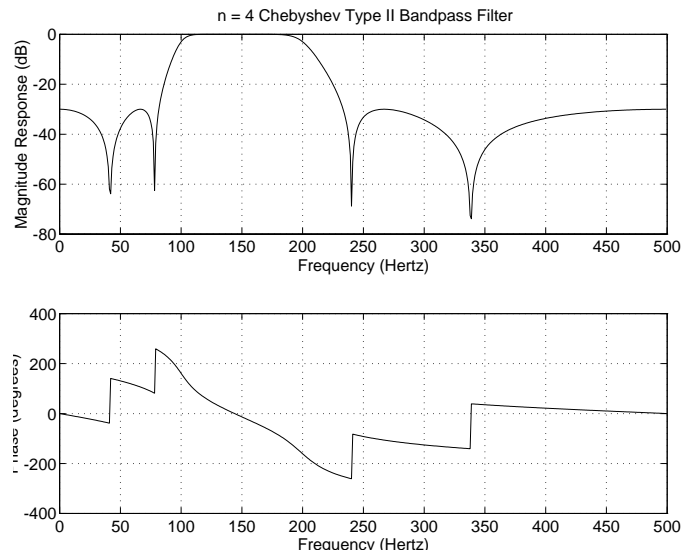
```
4
```

```
Wn =
```

```
0.1633    0.4665
```

```
[b, a] = cheby2(n, Rs, Wn);
```

```
freqz(b, a, 512, 1000)
```



Algorithm

cheb2ord uses the Chebyshev lowpass filter order prediction formula described in [1]. The function performs its calculations in the analog domain for both analog and digital cases. For the digital case, it converts the frequency parameters to the s -domain before the order and natural frequency estimation process, then converts them back to the z -domain.

cheb2ord initially develops a lowpass filter prototype by transforming the stopband frequencies of the desired filter to 1 rad/sec (for low- and highpass filters) and to -1 and 1 rad/sec (for bandpass and bandstop filters). It then computes the minimum order required for a lowpass filter to meet the passband specification.

See Also

buttord	Butterworth filter order selection.
cheb1ord	Chebyshev type I filter order selection.
cheby2	Chebyshev type II filter design (stopband ripple).
ellipord	Elliptic filter order selection.
kaiserord	Estimate parameters for an FIR filter design with Kaiser window.

References

[1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pg. 241.

chebwin

Purpose	Chebyshev window.	
Syntax	<code>w = chebwin(n, r)</code>	
Description	<code>w = chebwin(n, r)</code> returns the column vector <code>w</code> , containing the length <code>n</code> Chebyshev window whose Fourier transform magnitude sidelobe ripple is <code>r</code> dB below the mainlobe magnitude.	
See Also	<code>bartlett</code>	Bartlett window.
	<code>blackman</code>	Blackman window.
	<code>boxcar</code>	Rectangular window.
	<code>hamming</code>	Hamming window.
	<code>hanning</code>	Hanning window.
	<code>kaiser</code>	Kaiser window.
	<code>triang</code>	Triangular window.
References	[1] IEEE. <i>Programs for Digital Signal Processing</i> . IEEE Press. New York: John Wiley & Sons, 1979. Program 5.2.	

Purpose	Chebyshev type I filter design (passband ripple).
Syntax	<pre>[b, a] = cheby1(n, Rp, Wn) [b, a] = cheby1(n, Rp, Wn, ' ftype') [b, a] = cheby1(n, Rp, Wn, ' s') [b, a] = cheby1(n, Rp, Wn, ' ftype' , ' s') [z, p, k] = cheby1(. . .) [A, B, C, D] = cheby1(. . .)</pre>
Description	<p>cheby1 designs lowpass, bandpass, highpass, and bandstop digital and analog Chebyshev type I filters. Chebyshev type I filters are equiripple in the passband and monotonic in the stopband. Type I filters roll off faster than type II filters, but at the expense of greater deviation from unity in the passband.</p>

Digital Domain

[b, a] = cheby1(n, Rp, Wn) designs an order n lowpass digital Chebyshev filter with cutoff frequency Wn and Rp dB of ripple in the passband. It returns the filter coefficients in the length n+1 row vectors b and a, with coefficients in descending powers of z.

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \dots + a(n+1)z^{-n}}$$

Cutoff frequency is the frequency at which the magnitude response of the filter is equal to -Rp dB. For cheby1, the cutoff frequency Wn is a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). Smaller values of passband ripple Rp lead to wider transition widths (shallower rolloff characteristics).

If Wn is a two-element vector, Wn = [w1 w2], cheby1 returns an order 2*n bandpass filter with passband w1 < ω < w2.

`[b, a] = cheby1(n, Rp, Wn, 'ftype')` designs a highpass or bandstop filter, where *ftype* is

- `hi gh` for a highpass digital filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop digital filter if W_n is a two-element vector, $W_n = [w1 \ w2]$

The stopband is $w1 < \omega < w2$.

With different numbers of output arguments, `cheby1` directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = cheby1(n, Rp, Wn)` or

`[z, p, k] = cheby1(n, Rp, Wn, 'ftype')` returns the zeros and poles in length n column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = cheby1(n, Rp, Wn)` or

`[A, B, C, D] = cheby1(n, Rp, Wn, 'ftype')` where A , B , C , and D are

$$x[n+1] = Ax[n] + Bu[n]$$

$$y[n] = Cx[n] + Du[n]$$

and u is the input, x is the state vector, and y is the output.

Analog Domain

`[b, a] = cheby1(n, Rp, Wn, 's')` designs an order n lowpass analog Chebyshev type I filter with cutoff frequency W_n . It returns the filter coefficients in length $n + 1$ row vectors b and a , in descending powers of s :

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \dots + b(n+1)}{s^n + a(2)s^{n-1} + \dots + a(n+1)}$$

Cutoff frequency is the frequency at which the magnitude response of the filter is $-R_p$ dB. For `cheby1`, the cutoff frequency W_n must be greater than 0.

If W_n is a two-element vector, $W_n = [w_1 \ w_2]$, with $w_1 < w_2$, then `cheby1(n, Rp, Wn, 's')` returns an order $2*n$ bandpass analog filter with passband $w_1 < \omega < w_2$.

`[b, a] = cheby1(n, Rp, Wn, 'ftype', 's')` designs a highpass or bandstop filter, where *ftype* is

- `high` for a highpass analog filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop analog filter if W_n is a two-element vector, $W_n = [w_1 \ w_2]$

The stopband is $w_1 < \omega < w_2$.

You can supply different numbers of output arguments for `cheby1` to directly obtain other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = cheby1(n, Rp, Wn, 's')` or

`[z, p, k] = cheby1(n, Rp, Wn, 'ftype', 's')` returns the zeros and poles in length n or $2*n$ column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = cheby1(n, Rp, Wn, 's')` or

`[A, B, C, D] = cheby1(n, Rp, Wn, 'ftype', 's')` where A , B , C , and D are defined as

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

and u is the input, x is the state vector, and y is the output.

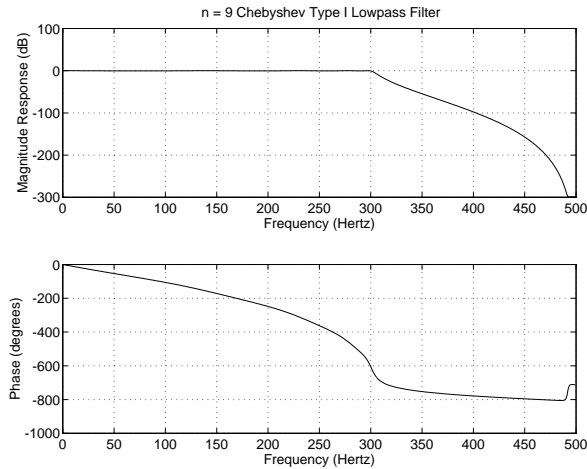
Examples

For data sampled at 1000 Hz, design a 9th-order lowpass Chebyshev type I filter with 0.5 dB of ripple in the passband and a cutoff frequency of 300 Hz:

```
[b, a] = cheby1(9, 0.5, 300/500);
```

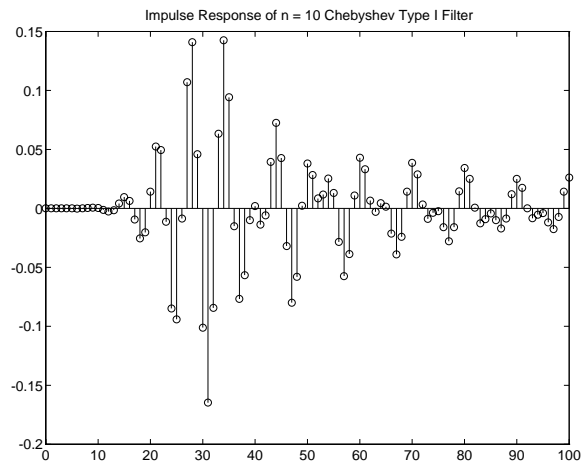
The frequency response of the filter is

```
freqz(b, a, 512, 1000)
```



Design a 10th-order bandpass Chebyshev type I filter with a passband from 100 to 200 Hz and plot its impulse response:

```
n = 10; Rp = 0.5;
Wn = [100 200]/500;
[b, a] = cheby1(n, Rp, Wn);
[y, t] = impz(b, a, 101); stem(t, y)
```



Limitations For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm cheby1 uses a five-step algorithm:

- 1 It finds the lowpass analog prototype poles, zeros, and gain using the `cheb1ap` function.
- 2 It converts the poles, zeros, and gain into state-space form.
- 3 It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
- 4 For digital filter design, `cheby1` uses `bi1inear` to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at ω_n or ω_1 and ω_2 .
- 5 It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

<code>bessel f</code>	Bessel analog filter design.
<code>butter</code>	Butterworth analog and digital filter design.
<code>cheb1ap</code>	Chebyshev type I analog lowpass filter prototype.
<code>cheb1ord</code>	Chebyshev type I filter order selection.
<code>cheby2</code>	Chebyshev type II filter design (stopband ripple).
<code>ellip</code>	Elliptic (Cauer) filter design.

cheby2

Purpose Chebyshev type II filter design (stopband ripple).

Syntax

```
[b, a] = cheby2(n, Rs, Wn)
[b, a] = cheby2(n, Rs, Wn, 'ftype')
[b, a] = cheby2(n, Rs, Wn, 's')
[b, a] = cheby2(n, Rs, Wn, 'ftype', 's')
[z, p, k] = cheby2(...)
[A, B, C, D] = cheby2(...)
```

Description cheby2 designs lowpass, highpass, bandpass, and bandstop digital and analog Chebyshev type II filters. Chebyshev type II filters are monotonic in the passband and equiripple in the stopband. Type II filters do not roll off as fast as type I filters, but are free of passband ripple.

Digital Domain

`[b, a] = cheby2(n, Rs, Wn)` designs an order n lowpass digital Chebyshev type II filter with cutoff frequency W_n and stopband ripple R_s dB down from the peak passband value. It returns the filter coefficients in the length $n + 1$ row vectors b and a , with coefficients in descending powers of z :

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \dots + a(n+1)z^{-n}}$$

Cutoff frequency is the beginning of the stopband, where the magnitude response of the filter is equal to $-R_s$ dB. For cheby2, the cutoff frequency W_n is a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). Larger values of stopband attenuation R_s lead to wider transition widths (shallower rolloff characteristics).

If W_n is a two-element vector, $W_n = [w1 \ w2]$, cheby2 returns an order $2*n$ bandpass filter with passband $w1 < \omega < w2$.

`[b, a] = cheby2(n, Rs, Wn, 'ftype')` designs a highpass or bandstop filter, where *ftype* is

- `hi gh` for a highpass digital filter with cutoff frequency W_n
- `stop` for an order $2 \times n$ bandstop digital filter if W_n is a two-element vector, $W_n = [w1 \ w2]$.

The stopband is $w1 < \omega < w2$.

With different numbers of output arguments, `cheby2` directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = cheby2(n, Rs, Wn)` or

`[z, p, k] = cheby2(n, Rs, Wn, 'ftype')` returns the zeros and poles in length n column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = cheby2(n, Rs, Wn)` or

`[A, B, C, D] = cheby2(n, Rs, Wn, 'ftype')` where A , B , C , and D are

$$x[n+1] = Ax[n] + Bu[n]$$

$$y[n] = Cx[n] + Du[n]$$

and u is the input, x is the state vector, and y is the output.

Analog Domain

`[b, a] = cheby2(n, Rs, Wn, 's')` designs an order n lowpass analog Chebyshev type II filter with cutoff frequency W_n . It returns the filter coefficients in the length $n + 1$ row vectors b and a , with coefficients in descending powers of s :

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \dots + b(n+1)}{s^n + a(2)s^{n-1} + \dots + a(n+1)}$$

Cutoff frequency is the frequency at which the magnitude response of the filter is equal to $-R_s$ dB. For `cheby2`, the cutoff frequency W_n must be greater than 0.

If W_n is a two-element vector, $W_n = [w1 \ w2]$, with $w1 < w2$, then `cheby2(n, Rs, Wn, 's')` returns an order $2*n$ bandpass analog filter with passband $w1 < \omega < w2$.

`[b, a] = cheby2(n, Rs, Wn, 'ftype', 's')` designs a highpass or bandstop filter, where *ftype* is

- `high` for a highpass analog filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop analog filter if W_n is a two-element vector, $W_n = [w1 \ w2]$

The stopband is $w1 < \omega < w2$.

With different numbers of output arguments, `cheby2` directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = cheby2(n, Rs, Wn, 's')` or

`[z, p, k] = cheby2(n, Rs, Wn, 'ftype', 's')` returns the zeros and poles in length n or $2*n$ column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = cheby2(n, Rs, Wn, 's')` or

`[A, B, C, D] = cheby2(n, Rs, Wn, 'ftype', 's')` where A , B , C , and D are

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

and u is the input, x is the state vector, and y is the output.

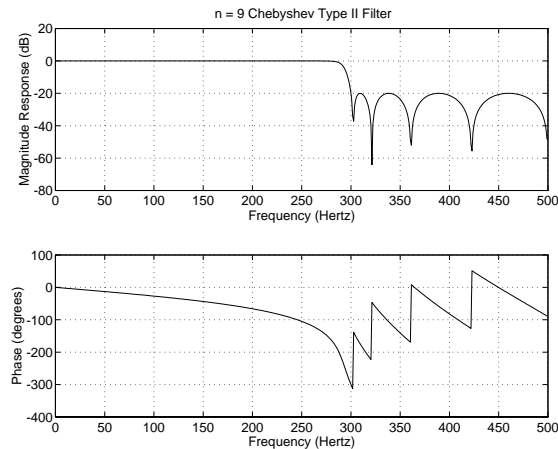
Examples

For data sampled at 1000 Hz, design a ninth-order lowpass Chebyshev type II filter with stopband attenuation 20 dB down from the passband and a cutoff frequency of 300 Hz:

```
[b, a] = cheby2(9, 20, 300/500);
```

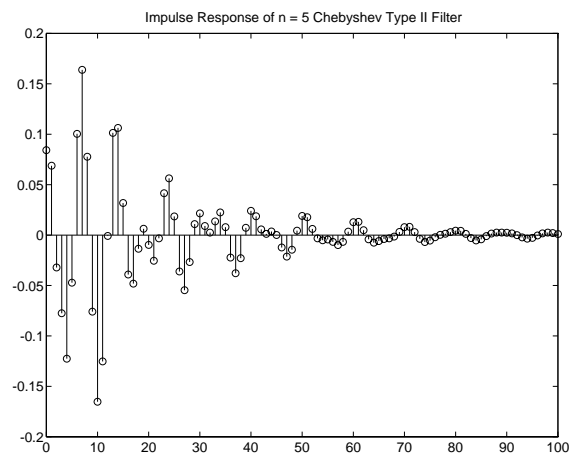
The frequency response of the filter is

`freqz(b, a, 512, 1000)`



Design a fifth-order bandpass Chebyshev type II filter with passband from 100 to 200 Hz and plot the impulse response of the filter:

```
n = 5; r = 20;
Wn = [100 200]/500;
[b, a] = cheby2(n, r, Wn);
[y, t] = impz(b, a, 101); stem(t, y)
```



Limitations

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function coefficient form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm

`cheby2` uses a five-step algorithm:

- 1 It finds the lowpass analog prototype poles, zeros, and gain using the `cheb2ap` function.
- 2 It converts poles, zeros, and gain into state-space form.
- 3 It transforms the lowpass filter into a bandpass, highpass, or bandstop filter with desired cutoff frequencies, using a state-space transformation.
- 4 For digital filter design, `cheby2` uses `bi1inear` to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at ω_n or ω_1 and ω_2 .
- 5 It converts the state-space filter back to transfer function or zero-pole-gain form, as required.

See Also

<code>bessel f</code>	Bessel analog filter design.
<code>butter</code>	Butterworth analog and digital filter design.
<code>cheb2ap</code>	Chebyshev type II analog lowpass filter prototype.
<code>cheb2ord</code>	Chebyshev type II filter order selection.
<code>cheby1</code>	Chebyshev type I filter design (passband ripple).
<code>ellip</code>	Elliptic (Cauer) filter design.

Purpose Swept-frequency cosine generator.

Syntax

```
y = chirp(t, f0, t1, f1)
y = chirp(t, f0, t1, f1, 'method')
y = chirp(t, f0, t1, f1, 'method', phi)
```

Description `y = chirp(t, f0, t1, f1)` generates samples of a linear swept-frequency cosine signal at the time instances defined in array `t`, where `f0` is the instantaneous frequency at time 0, and `f1` is the instantaneous frequency at time `t1`. `f0` and `f1` are both in Hertz. If unspecified, `f0` is 0, `t1` is 1, and `f1` is 100.

`y = chirp(t, f0, t1, f1, 'method')` specifies alternative sweep method options, where *method* can be

- `linear`, which specifies an instantaneous frequency sweep $f_i(t)$ given by

$$f_i(t) = f_0 + \beta t$$

where

$$\beta = (f_1 - f_0) / t_1$$

β ensures that the desired frequency breakpoint f_1 at time t_1 is maintained.

- `quadratic`, which specifies an instantaneous frequency sweep $f_i(t)$ given by

$$f_i(t) = f_0 + \beta t^2$$

where

$$\beta = (f_1 - f_0) / t_1$$

- `logarithmic` specifies an instantaneous frequency sweep $f_i(t)$ given by

$$f_i(t) = f_0 + 10^{\beta t}$$

where

$$\beta = [\log_{10}(f_1 - f_0)] / t_1$$

For a log-sweep, `f1` must be greater than `f0`.

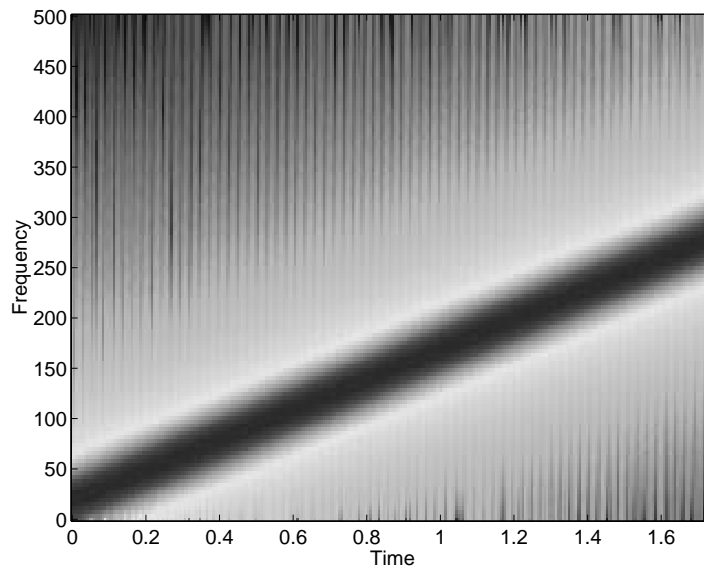
chirp

`y = chirp(t, f0, t1, f1, 'method', phi)` allows an initial phase `phi` to be specified in degrees. If unspecified, `phi` is 0. Default values are substituted for empty or omitted trailing input arguments.

Examples

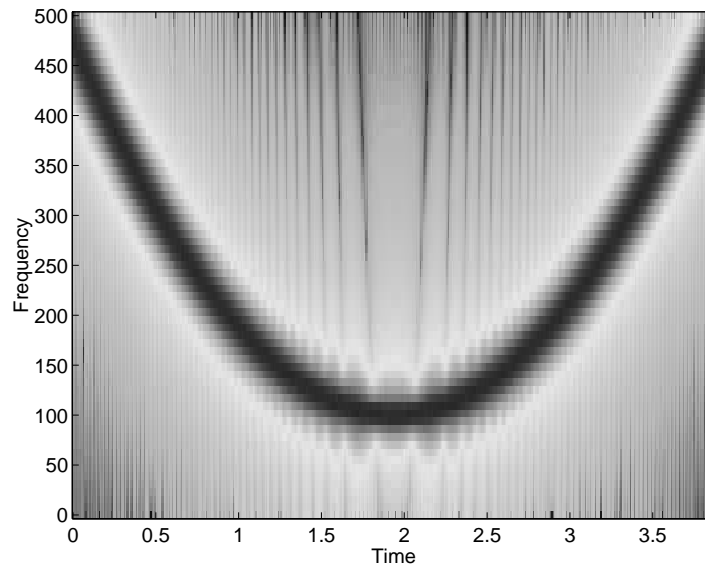
Compute the spectrogram of a chirp with linear instantaneous frequency deviation:

```
t = 0:0.001:2;           % 2 secs @ 1kHz sample rate
y = chirp(t, 0, 1, 150);  % Start @ DC, cross 150Hz at t=1 sec
spectrogram(y, 256, 1e3, 256, 250) % Display the spectrogram
```



Compute the spectrogram of a chirp with quadratic instantaneous frequency deviation:

```
t = -2:0.001:2; % ±2 secs @ 1kHz sample rate
y = chirp(t, 100, 1, 200, 'quadratic'); % Start @ 100Hz, cross 200Hz
% at t=1 sec
specgram(y, 128, 1e3, 128, 120) % Display the spectrogram
```



See Also

cos	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
di ric	Dirichlet or periodic sinc function.
gauspul s	Gaussian-modulated sinusoidal pulse generator.
pul stran	Pulse train generator.
rectpul s	Sampled aperiodic rectangle generator.
sawtooth	Sawtooth or triangle wave generator.
si n	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
si nc	Sinc function.
square	Square wave generator.
tri pul s	Sampled aperiodic triangle generator.

Purpose Estimate magnitude squared coherence function between two signals.

Syntax

```
Cxy = cohere(x, y)
Cxy = cohere(x, y, nfft)
[Cxy, f] = cohere(x, y, nfft, Fs)
Cxy = cohere(x, y, nfft, Fs, window)
Cxy = cohere(x, y, nfft, Fs, window, overlap)
Cxy = cohere(x, y, ..., 'dflag')
cohere(x, y)
```

Description `Cxy = cohere(x, y)` finds the magnitude squared coherence between length `n` signal vectors `x` and `y`. The coherence is a function of the power spectra of `x` and `y` and the cross spectrum of `x` and `y`:

$$C_{xy}(f) = \frac{|P_{xy}(f)|^2}{P_{xx}(f)P_{yy}(f)}$$

`x` and `y` must be the same length. `Cxy = cohere(x, y)` uses the following default values:

- `nfft = min(256, length(x))`
- `Fs = 2`
- `window = hanning(nfft)`
- `overlap = 0`

`nfft` specifies the FFT length that `cohere` uses. This value determines the frequencies at which the coherence is estimated. `Fs` is a scalar that specifies the sampling frequency. `window` specifies a windowing function and the number of samples `cohere` uses in its sectioning of the `x` and `y` vectors. `overlap` is the number of samples by which the sections overlap. Any arguments that you omit from the end of the parameter list use the default values shown above.

If `x` is real, `cohere` estimates the coherence function at positive frequencies only; in this case, the output `Cxy` is a column vector of length `nfft/2 + 1` for `nfft` even and `(nfft + 1)/2` for `n` odd. If `x` or `y` is complex, `cohere` estimates the coherence function at both positive and negative frequencies, and `Cxy` has length `nfft`.

`Cxy = cohere(x, y, nfft)` uses the FFT length `nfft` in estimating the power spectrum for `x`. Specify `nfft` as a power of 2 for fastest execution.

`[Cxy, f] = cohere(x, y, nfft, Fs)` returns a vector `f` of frequencies at which the function evaluates the coherence. `Fs` is the sampling frequency. `f` is the same size as `Cxy`, so `plot(f, Cxy)` plots the coherence function versus properly scaled frequency. `Fs` has no effect on the output `Cxy`; it is a frequency scaling multiplier.

`Cxy = cohere(x, y, nfft, Fs, window)` specifies a windowing function and the number of samples per section of the vectors `x` and `y`. If you supply a scalar for `window`, `cohere` uses a Hanning window of that length. The length of the window must be less than or equal to `nfft`; `cohere` zero pads the sections if the window length exceeds `nfft`.

`Cxy = cohere(x, y, nfft, Fs, window, overlap)` overlaps the sections of `x` by `overlap` samples.

You can use the empty matrix `[]` to specify the default value for any input argument except `x` or `y`. For example,

```
Cxy = cohere(x, y, [], [], kaiser(128, 5));
```

uses 256 as the value for `nfft` and 2 as the value for `Fs`.

`Cxy = cohere(x, y, ..., 'dflag')` specifies a detrend option, where `dflag` is

- `linear`, to remove the best straight-line fit from the prewindowed sections of `x` and `y`
- `mean`, to remove the mean from the prewindowed sections of `x` and `y`
- `none`, for no detrending (default)

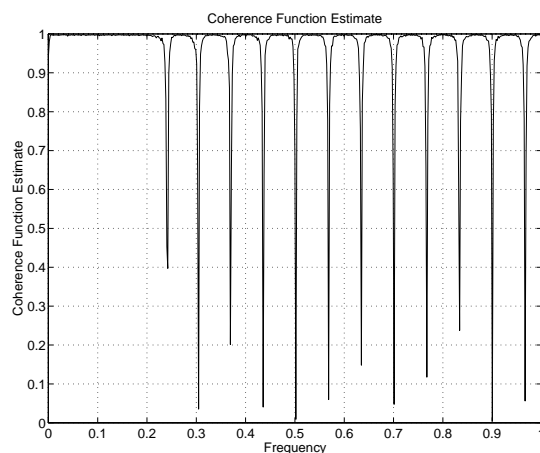
The `dflag` parameter must appear last in the list of input arguments. `cohere` recognizes a `dflag` string no matter how many intermediate arguments are omitted.

`cohere` with no output arguments plots the coherence estimate versus frequency in the current figure window.

Example

Compute and plot the coherence estimate between two colored noise sequences x and y :

```
h = fir1(30, 0.2, boxcar(31));
h1 = ones(1, 10)/sqrt(10);
r = randn(16384, 1);
x = filter(h1, 1, r);
y = filter(h, 1, x);
cohere(x, y, 1024, [], [], 512)
```

**Diagnostics**

An appropriate diagnostic messages is displayed when incorrect arguments are used:

- Requires window's length to be no greater than the FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires vector (either row or column) input.
- Requires inputs X and Y to have the same length.

Algorithm cohere estimates the magnitude squared coherence function [1] using Welch's method of power spectrum estimation (see references [2] and [3]), as follows:

- 1 It divides the signals x and y into overlapping sections, detrends each section, and multiplies each section by window.
- 2 It calculates the length nfft fast Fourier transform of each section.
- 3 It averages the squares of the spectra of the x sections to form Pxx, averages the squares of the spectra of the y sections to form Pyy, and averages the products of the spectra of the x and y sections to form Pxy. It calculates Cxy by
$$C_{xy} = \text{abs}(P_{xy}) . ^2 / (P_{xx} . * P_{yy})$$

See Also	csd	Estimate the cross spectral density (CSD) of two signals.
	psd	Estimate the power spectral density (PSD) of a signal using Welch's method.
	tfe	Transfer function estimate from input and output.

References

[1] Kay, S.M. *Modern Spectral Estimation*. Englewood Cliffs, NJ: Prentice-Hall, 1988. Pg. 454.

[2] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975.

[3] Welch, P.D. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms." *IEEE Trans. Audio Electroacoust.* Vol. AU-15 (June 1967). Pgs. 70-73.

Purpose Convolution and polynomial multiplication.

Syntax `c = conv(a, b)`

Description `conv(a, b)` convolves vectors `a` and `b`. The convolution sum is

$$c(n+1) = \sum_{k=0}^{N-1} a(k+1)b(n-k)$$

where N is the maximum sequence length. The series is indexed from $n+1$ and $k+1$ instead of the usual n and k because MATLAB vectors run from 1 to n instead of from 0 to $n-1$.

Example The convolution of `a = [1 2 3]` and `b = [4 5 6]` is

```
c = conv(a, b)

c =
     4     13     28     27     18
```

Algorithm This function is part of the MATLAB environment. It is an M-file that uses the `filter` primitive. `conv` computes the convolution operation as FIR filtering with an appropriate number of zeros appended to the input.

See Also	<code>conv2</code>	Two-dimensional convolution.
	<code>convmtx</code>	Convolution matrix.
	<code>convn</code>	N -dimensional convolution (see the online <i>MATLAB Function Reference</i>).
	<code>deconv</code>	Deconvolution and polynomial division.
	<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
	<code>residuez</code>	z -transform partial fraction expansion.
	<code>xcorr</code>	Cross-correlation function estimate.

conv2

Purpose Two-dimensional convolution.

Syntax `C = conv2(A, B)`
`C = conv2(A, B, 'shape')`

Description `C = conv2(A, B)` computes the two-dimensional convolution of matrices A and B. If one of these matrices describes a two-dimensional FIR filter, the other matrix is filtered in two dimensions.

Each dimension of the output matrix C is equal in size to the sum of the corresponding dimensions of the input matrices minus 1. For $[ma, na] = \text{size}(A)$ and $[mb, nb] = \text{size}(B)$, then $\text{size}(C) = [ma+mb-1, na+nb-1]$.

`C = conv2(A, B, 'shape')` returns a subsection of the two-dimensional convolution with size specified by *shape*, where:

- `full` returns the full two-dimensional convolution (default)
- `same` returns the central part of the convolution that is the same size as A
- `valid` returns only those parts of the convolution that are computed without the zero-padded edges. Using this option, $\text{size}(C) = [ma-mb+1, na-nb+1]$ when $\text{size}(A) > \text{size}(B)$

`conv2` executes most quickly when $\text{size}(A) > \text{size}(B)$.

Examples In image processing, the Sobel edge-finding operation is a two-dimensional convolution of an input array with the special matrix

```
s = [ 1 2 1; 0 0 0; -1 -2 -1];
```

Given any image, the following line extracts the horizontal edges:

```
h = conv2(I, s);
```

The lines below extract first the vertical edges, then both horizontal and vertical edges combined:

```
v = conv2(I, s');  
v2 = (sqrt(h.^2 + v.^2))
```

See Also

conv	Convolution and polynomial multiplication.
convn	<i>N</i> -dimensional convolution (see the online <i>MATLAB Function Reference</i>).
deconv	Deconvolution and polynomial division.
filter2	Two-dimensional digital filtering.
xcorr	Cross-correlation function estimate.
xcorr2	Two-dimensional cross-correlation.

convmtx

Purpose Convolution matrix.

Syntax $A = \text{convmtx}(c, n)$
 $A = \text{convmtx}(r, n)$

Description A *convolution matrix* is a matrix, formed from a vector, whose inner product with another vector is the convolution of the two vectors.

$A = \text{convmtx}(c, n)$ where c is a length m column vector returns a matrix A of size $(m + n - 1)$ -by- n . The product of A and another column vector x of length n is the convolution of c with x .

$A = \text{convmtx}(r, n)$ where r is a length m row vector returns a matrix A of size n -by- $(m + n - 1)$. The product of A and another row vector x of length n is the convolution of r with x .

Example Generate a simple convolution matrix:

```
h = [1 2 3 2 1];  
convmtx(h, 7)  
ans =  
  
1 2 3 2 1 0 0 0 0 0 0  
0 1 2 3 2 1 0 0 0 0 0  
0 0 1 2 3 2 1 0 0 0 0  
0 0 0 1 2 3 2 1 0 0 0  
0 0 0 0 1 2 3 2 1 0 0  
0 0 0 0 0 1 2 3 2 1 0  
0 0 0 0 0 0 1 2 3 2 1
```

Note that `convmtx` handles edge conditions by zero padding.

In practice, it is more efficient to compute convolution using

```
y = conv(c, x)
```

than by using a convolution matrix:

```
n = length(x);  
y = convmtx(c, n)*x
```

Algorithm `convmtx` uses the function `toeplitz` to generate the convolution matrix.

See Also

conv	Convolution and polynomial multiplication.
convn	<i>N</i> -dimensional convolution (see the online <i>MATLAB Function Reference</i>).
conv2	Two-dimensional convolution.
dftmtx	Discrete Fourier transform matrix.

corrcoef

Purpose Correlation coefficient matrix.

Syntax `C = corrcoef(X)`
`C = corrcoef(X, Y)`

Description `corrcoef` returns a matrix of correlation coefficients calculated from an input matrix whose rows are observations and whose columns are variables. If `C = cov(X)`, then `corrcoef(X)` is the matrix whose element (i, j) is

$$\text{corrcoef}(i, j) = \frac{C(i, j)}{\sqrt{C(i, i)C(j, j)}}$$

`C = corrcoef(X)` is the zeroth lag of the covariance function, that is, the zeroth lag of `xcov(x, 'coeff')` packed into a square array.

`C = corrcoef(X, Y)` is the same as `corrcoef([X Y])`, that is, it concatenates `X` and `Y` in the row direction before its computation.

`corrcoef` removes the mean from each column before calculating the results. See the `xcorr` function for cross-correlation options.

This function is part of the standard MATLAB environment.

See Also	<code>cov</code>	Covariance matrix.
	<code>mean</code>	Average value (see the online <i>MATLAB Function Reference</i>).
	<code>medi an</code>	Median value (see the online <i>MATLAB Function Reference</i>).
	<code>std</code>	Standard deviation (see the online <i>MATLAB Function Reference</i>).
	<code>xcorr</code>	Cross-correlation function estimate.
	<code>xcov</code>	Cross-covariance function estimate (equal to mean-removed cross-correlation).

Purpose	Covariance matrix.	
Syntax	$c = \text{cov}(x)$ $c = \text{cov}(x, y)$	
Description	<p><code>cov</code> computes the covariance matrix. If x is a vector, c is a scalar containing the variance. For an array where each row is an observation and each column a variable, <code>cov(X)</code> is the covariance matrix. <code>diag(cov(X))</code> is a vector of variances for each column, and <code>sqrt(diag(cov(X)))</code> is a vector of standard deviations.</p> <p><code>cov(x)</code> is the zeroth lag of the covariance function, that is, the zeroth lag of <code>xcov(x)/(n-1)</code> packed into a square array.</p> <p><code>cov(x, y)</code> where x and y are column vectors of equal length is equivalent to <code>cov([x y])</code>, that is, it concatenates x and y in the row direction before its computation.</p> <p><code>cov</code> removes the mean from each column before calculating the results.</p> <p>This function is part of the standard MATLAB environment.</p>	
Algorithm	<pre>[n, p] = size(x); x = x-ones(n, 1)*(sum(x)/n); y = x'*x/(n-1);</pre>	
See Also	<code>corrcoef</code>	Correlation coefficient matrix.
	<code>mean</code>	Average value (see the online <i>MATLAB Function Reference</i>).
	<code>medi an</code>	Median value (see the online <i>MATLAB Function Reference</i>).
	<code>std</code>	Standard deviation (see the online <i>MATLAB Function Reference</i>).
	<code>xcorr</code>	Cross-correlation function estimate.
	<code>xcov</code>	Cross-covariance function estimate (equal to mean-removed cross-correlation).

cplxpair

Purpose Group complex numbers into complex conjugate pairs.

Syntax `y = cplxpair(x)`
`y = cplxpair(x, tol)`

Description `y = cplxpair(x)` returns `x` with complex conjugate pairs grouped together. `cplxpair` orders the conjugate pairs by increasing real part. Within a pair, the element with negative imaginary part comes first. The function returns all purely real values following all the complex pairs.

`y = cplxpair(x, tol)` includes a tolerance, `tol`, for determining which numbers are real and which are paired complex conjugates. By default, `cplxpair` uses a tolerance of $100 \times \text{eps}$ relative to $\text{abs}(x(i))$. `cplxpair` forces the complex conjugate pairs to be exact complex conjugates.

This function is part of the standard MATLAB environment.

Example Order five poles evenly spaced around the unit circle into complex pairs:

```
cplxpair(exp(2*pi*sqrt(-1)*(0:4)/5)')  
  
ans =  
-0.8090 - 0.5878i  
-0.8090 + 0.5878i  
0.3090 - 0.9511i  
0.3090 + 0.9511i  
1.0000
```

Diagnostics If there is an odd number of complex numbers, or if the complex numbers cannot be grouped into complex conjugate pairs within the tolerance, `cplxpair` generates the error message:

Complex numbers can't be paired.

Purpose Complex and nonlinear-phase equiripple FIR filter design

Syntax

```

b = cremez(n, f, 'fresp')
b = cremez(n, f, 'fresp', w)
b = cremez(n, f, {'fresp', p1, p2, ...}, w)
b = cremez(n, f, a, w)
b = cremez(..., 'sym')
b = cremez(..., 'skip_stage2')
b = cremez(..., 'debug')
b = cremez(..., {lgrid})
[b, delta, opt] = cremez(...)

```

Description cremez allows arbitrary frequency-domain constraints to be specified for the design of a possibly complex FIR filter. The Chebyshev (or minimax) filter error is optimized, producing equiripple FIR filter designs.

`b = cremez(n, f, 'fresp')` returns a length $n+1$ FIR filter with the best approximation to the desired frequency response as returned by function `fresp`. `f` is a vector of frequency band edge pairs, specified in the range -1 and 1 , where 1 corresponds to half the sampling frequency (the Nyquist frequency). The frequencies must be in increasing order, and `f` must have even length. The frequency bands span $f(k)$ to $f(k+1)$ for k odd; the intervals $f(k+1)$ to $f(k+2)$ for k even are “transition bands” or “don’t care” regions during optimization.

`b = cremez(n, f, 'fresp', w)` uses the real, non-negative weights in vector `w` to weight the fit in each frequency band. The length of `w` is half the length of `f`, so there is exactly one weight per band.

`b = cremez(n, f, {'fresp', p1, p2, ...}, ...)` supplies optional parameters `p1`, `p2`, ..., to the frequency response function `fresp`. Predefined `'fresp'` frequency response functions are included for a number of common filter designs, as described below. For all of the predefined frequency response functions, the symmetry option `'sym'` defaults to `'even'` if no negative frequencies are contained in `f` and `d = 0`; otherwise `'sym'` defaults to `'none'`. (See the `'sym'` option below for details.) For all of the predefined frequency response functions, `d` specifies a group-delay offset such that the filter response

has a group delay of $n/2+d$ in units of the sample interval. Negative values create less delay; positive values create more delay. By default, $d = 0$.

- `lowpass`, `highpass`, `bandpass`, `bandstop`

These functions share a common syntax, exemplified here by '`lowpass`' :

`b = cremez(n, f, 'lowpass', ...)` and

`b = cremez(n, f, {'lowpass', d}, ...)` design a linear-phase ($n/2+d$ delay) filter.

- `multiband` designs a linear-phase frequency response filter with arbitrary band amplitudes.

`b = cremez(n, f, {'multiband', a}, ...)` and

`b = cremez(n, f, {'multiband', a, d}, ...)` specify vector `a` containing the desired amplitudes at the band edges in `f`. The desired amplitude at frequencies between pairs of points `f(k)` and `f(k+1)` for `k` odd is the line segment connecting the points `(f(k), a(k))` and `(f(k+1), a(k+1))`.

- `differentiator` designs a linear-phase differentiator. For these designs, zero-frequency must be in a transition band, and band weighting is set to be inversely proportional to frequency.

`b = cremez(n, f, {'differentiator', Fs}, ...)` and

`b = cremez(n, f, {'differentiator', Fs, d}, ...)` specify the sample rate `Fs` used to determine the slope of the differentiator response. If omitted, `Fs` defaults to 1.

- `hilbfilt` designs a linear-phase Hilbert transform filter response. For Hilbert designs, zero-frequency must be in a transition band.

`b = cremez(n, f, 'hilbfilt', ...)` and

`b = cremez(N, F, {'hilbfilt', d}, ...)` design a linear-phase ($n/2+d$ delay) Hilbert transform filter.

`b = cremez(n, f, a, w)` is a synonym for

`b = cremez(n, f, {'multiband', a}, w)`.

`b = cremez(..., 'sym')` imposes a symmetry constraint on the impulse response of the design, where `'sym'` may be one of the following:

- `'none'` indicates no symmetry constraint
This is the default if any negative band edge frequencies are passed, or if `'fresp'` does not supply a default.
- `'even'` indicates a real and even impulse response
This is the default for highpass, lowpass, bandpass, bandstop, and multiband designs.
- `'odd'` indicates a real and odd impulse response
This is the default for Hilbert and differentiator designs.
- `'real'` indicates conjugate symmetry for the frequency response

If any `'sym'` option other than `'none'` is specified, the band edges should only be specified over positive frequencies; the negative frequency region is filled in from symmetry. If a `'sym'` option is not specified, the `'fresp'` function is queried for a default setting.

`b = cremez(..., 'skip_stage2')` disables the second-stage optimization algorithm, which executes only when `cremez` determines that an optimal solution has not been reached by the standard Remez error-exchange. Disabling this algorithm may increase the speed of computation, but may incur a reduction in accuracy. By default, the second-stage optimization is enabled.

`b = cremez(..., 'debug')` enables the display of intermediate results during the filter design, where `'debug'` may be one of `'trace'`, `'plots'`, `'both'`, or `'off'`. By default, it is set to `'off'`.

`b = cremez(..., {lgrid})` uses the integer `lgrid` to control the density of the frequency grid, which has roughly $2^{\text{nextpow2}(lgrid \cdot n)}$ frequency points. The default value for `lgrid` is 25. Note that the `{lgrid}` argument must be a 1-by-1 cell array.

Any combination of the `'sym'`, `'skip_stage2'`, `'debug'`, and `{lgrid}` options may be specified.

`[b, delta] = cremez(...)` returns the maximum ripple height `delta`.

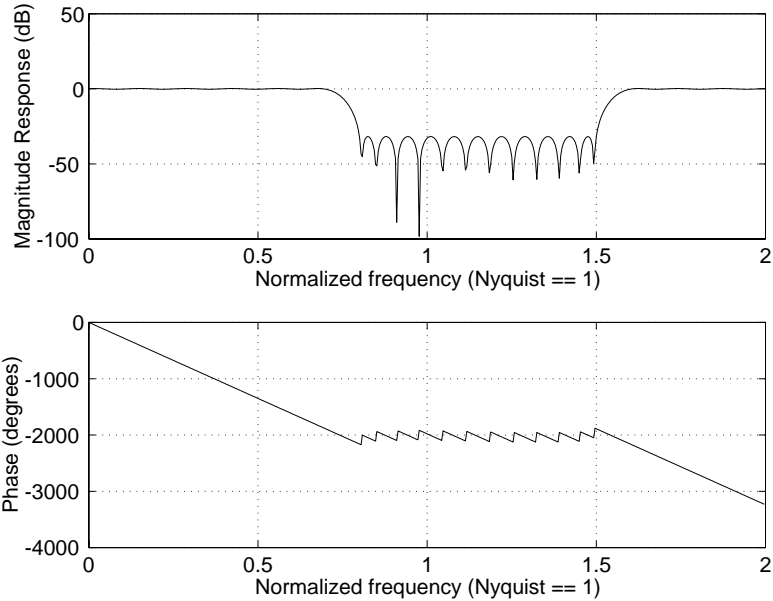
[b, del ta, opt] = cremez(. . .) returns a structure opt of optional results computed by cremez and contains the following fields:

opt. fgri d	Frequency grid vector used for the filter design optimization
opt. des	Desired frequency response for each point in opt. fgri d
opt. wt	Weighting for each point in opt. fgri d
opt. H	Actual frequency response for each point in opt. fgri d
opt. error	Error at each point in opt. fgri d
opt. i extr	Vector of indices into opt. fgri d for extremal frequencies
opt. fextr	Vector of extremal frequencies

Example

Design a 31-tap, linear-phase, lowpass filter:

```
b = cremez(30, [-1 -0.5 -0.4 0.7 0.8 1], 'lowpass');
freqz(b, 1, 512, 'whole');
```



Remarks

User-definable functions may be used, instead of the predefined frequency response functions for '*fresp*'. The function is called from within *cremez* using the following syntax:

$[dh, dw] = fresp(n, f, gf, w, p1, p2, \dots)$ where

- *n* is the filter order.
- *f* is the vector of frequency band edges that appear monotonically between -1 and 1, where 1 is the Nyquist frequency.
- *gf* is a vector of grid points that have been linearly interpolated over each specified frequency band by *cremez*. *gf* determines the frequency grid at which the response function must be evaluated. This is the same data returned by *cremez* in the *fgri d* field of the *opt* structure.
- *w* is a vector of real, positive weights, one per band, used during optimization. *w* is optional in the call to *cremez*; if not specified, it is set to unity weighting before being passed to '*fresp*'.
- *dh* and *dw* are the desired complex frequency response and band weight vectors, respectively, evaluated at each frequency in grid *gf*.
- *p1*, *p2*, ..., are optional parameters that may be passed to '*fresp*'.

Additionally, a preliminary call is made to '*fresp*' to determine the default symmetry property '*sym*'. This call is made using the syntax:

sym = *fresp*('default s', {*n*, *f*, [], *w*, *p1*, *p2*, ...})

The arguments may be used in determining an appropriate symmetry default as necessary. The function *private/lowpass.m* may be useful as a template for generating new frequency response functions.

Algorithm

An extended version of the Remez exchange method is implemented for the complex case. This exchange method obtains the optimal filter when the equiripple nature of the filter is restricted to have *n*+2 extremals. When it does not converge, the algorithm switches to an ascent-descent algorithm that takes over to finish the convergence to the optimal solution. See the references for further details.

Diagnostics

The following diagnostic messages arise from incorrect usage of cremez:

Not enough input arguments.

F must contain an even number of band edge entries.

Band edges must be monotonically increasing.

Expecting a string argument.

Invalid argument *arg* specified.

Invalid default symmetry option *sym* returned from response function *fresp*. Must be one of 'none', 'real', 'even', or 'odd'.

Frequency band edges must be in the range $[-1, +1]$ for designs with $Sym = 'sym'$.

Frequency band edges must be in the range $[0, +1]$ for designs with $Sym = 'sym'$.

Incorrect size of results from response function *fresp*. Sizes must be the same size as the frequency grid GF.

Both -1 and 1 have been specified as frequencies in F, and the frequency spacing is too close to move either of them toward its neighbor.

Internal error: Grid frequencies out of range.

Internal error: domain must be "whole" or "half".

Internal error: obtained a negative bandwidth.

Internal error: two extremal frequencies at the same grid point.

Internal error: dBrange must be > 0 .

See Also	<code>fir1</code>	Window-based finite impulse response filter design—standard response.
	<code>fir2</code>	Window-based finite impulse response filter design—arbitrary response.
	<code>firls</code>	Least square linear-phase FIR filter design.
	<code>remez</code>	Parks-McClellan optimal FIR filter design.
	<code>private/bandpass</code>	Bandpass filter design function.
	<code>private/bandstop</code>	Bandstop filter design function.
	<code>private/differentiator</code>	Differentiator filter design function.
	<code>private/highpass</code>	Highpass filter design function.
	<code>private/hilbfilter</code>	Hilbert filter design function.
	<code>private/lowpass</code>	Lowpass filter design function.
	<code>private/multiband</code>	Multiband filter design function.

References

[1] Karam, L.J., and J.H. McClellan. "Complex Chebyshev Approximation for FIR Filter Design." *IEEE Trans. on Circuits and Systems II*. March 1995. Pgs. 207-216.

[2] Karam, L.J. *Design of Complex Digital FIR Filters in the Chebyshev Sense*. Ph.D. Thesis, Georgia Institute of Technology, March 1995.

[3] Demjanjov, V.F., and V.N. Malozemov. *Introduction to Minimax*. New York: John Wiley & Sons, 1974.

Purpose Estimate the cross spectral density (CSD) of two signals.

Syntax

```
Pxy = csd(x, y)
Pxy = csd(x, y, nfft)
[Pxy, f] = csd(x, y, nfft, Fs)
Pxy = csd(x, y, nfft, Fs, window)
Pxy = csd(x, y, nfft, Fs, window, overlap)
Pxy = csd(x, y, ..., 'dfлаг')
[Pxy, Pxy, f] = csd(x, y, nfft, Fs, window, overlap, p)
csd(x, y, ...)
```

Description `Pxy = csd(x, y)` estimates the cross spectral density of the length n sequences x and y using the Welch method of spectral estimation. `Pxy = csd(x, y)` uses the following default values:

- `nfft = min(256, length(x))`
- `Fs = 2`
- `window = hanning(nfft)`
- `overlap = 0`

`nfft` specifies the FFT length that `csd` uses. This value determines the frequencies at which the cross spectrum is estimated. `Fs` is a scalar that specifies the sampling frequency. `window` specifies a windowing function and the number of samples `csd` uses in its sectioning of the x and y vectors. `overlap` is the number of samples by which the sections overlap. Any arguments omitted from the end of the parameter list use the default values shown above.

If x and y are real, `csd` estimates the cross spectral density at positive frequencies only; in this case, the output `Pxy` is a column vector of length $nfft/2 + 1$ for `nfft` even and $(nfft + 1)/2$ for `nfft` odd. If x or y is complex, `csd` estimates the cross spectral density at both positive and negative frequencies and `Pxy` has length `nfft`.

`Pxy = csd(x, y, nfft)` uses the FFT length `nfft` in estimating the cross spectral density of x and y . Specify `nfft` as a power of 2 for fastest execution.

`[Pxy, f] = csd(x, y, nfft, Fs)` returns a vector `f` of frequencies at which the function evaluates the CSD. `f` is the same size as `Pxy`, so `plot(f, Pxy)` plots the

spectrum versus properly scaled frequency. *Fs* has no effect on the output *Pxy*; it is a frequency scaling multiplier.

Pxy = *csd*(*x*, *y*, *nfft*, *Fs*, *window*) specifies a windowing function and the number of samples per section of the *x* vector. If you supply a scalar for *window*, *csd* uses a Hanning window of that length. The length of the window must be less than or equal to *nfft*; *csd* zero pads the sections if the length of the window is less than *nfft*. *csd* returns an error if the length of the window is greater than *nfft*.

Pxy = *csd*(*x*, *y*, *nfft*, *Fs*, *window*, *noverlap*) overlaps the sections of *x* and *y* by *noverlap* samples.

You can use the empty matrix `[]` to specify the default value for any input argument except *x* or *y*. For example,

```
csd(x, y, [], 10000)
```

is equivalent to

```
csd(x)
```

but with a sampling frequency of 10,000 Hz instead of the default of 2 Hz.

Pxy = *csd*(*x*, *y*, . . . , ' *dflag* ') specifies a detrend option, where *dflag* is

- *linear*, to remove the best straight-line fit from the prewindowed sections of *x* and *y*
- *mean*, to remove the mean from the prewindowed sections of *x* and *y*
- *none*, for no detrending (default)

The *dflag* parameter must appear last in the list of input arguments. *csd* recognizes a *dflag* string no matter how many intermediate arguments are omitted.

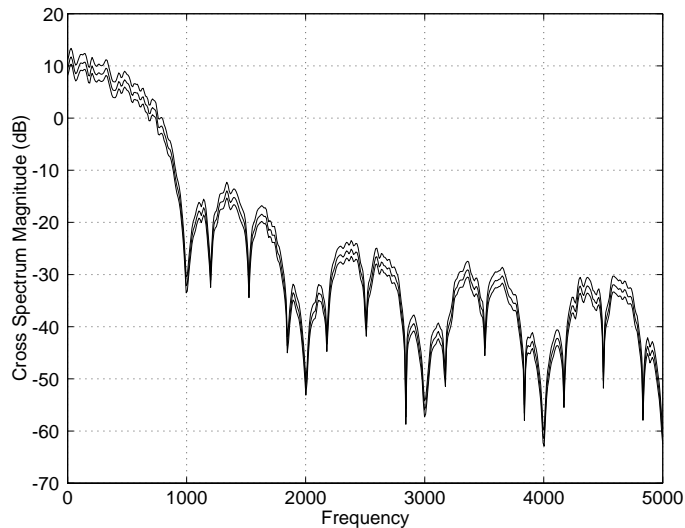
[*Pxy*, *Pxyc*, *f*] = *csd*(*x*, *y*, *nfft*, *Fs*, *window*, *noverlap*, *p*) where *p* is a positive scalar between 0 and 1 returns a vector *Pxyc* that contains an estimate of the *p**100 percent confidence interval for *Pxy*. *Pxyc* is a two-column matrix the same length as *Pxy*. The interval [*Pxyc*(:, 1), *Pxyc*(:, 2)] covers the true CSD with probability *p*. *plot*(*f*, [*Pxy* *Pxyc*]) plots the cross spectrum inside the *p**100 percent confidence interval. If unspecified, *p* defaults to 0.95.

`csd(x, y, ...)` plots the CSD versus frequency in the current figure window. If the `p` parameter is specified, the plot includes the confidence interval.

Example

Generate two colored noise signals and plot their CSD with a confidence interval of 95%. Specify a length 1024 FFT, a 500 point triangular window with no overlap, and a sampling frequency of 10 Hz:

```
h = fir1(30, 0.2, boxcar(31));  
h1 = ones(1, 10) / sqrt(10);  
r = randn(16384, 1);  
x = filter(h1, 1, r);  
y = filter(h, 1, x);  
csd(x, y, 1024, 10000, triang(500), 0, [])
```



Algorithm

csd implements the Welch method of spectral density estimation (see references [1] and [2]):

- 1 It applies the window specified by the `window` vector to each successive detrended section.
- 2 It transforms each section with an `nfft`-point FFT.
- 3 It forms the periodogram of each section by scaling the product of the transform of the `y` section and the conjugate of the transformed `x` section.
- 4 It averages the periodograms of the successive overlapping sections to form `Pxy`, the cross spectral density of `x` and `y`.

The number of sections that `csd` averages is `k`, where `k` is

$$\text{fix}((\text{length}(x) - \text{noverlap}) / (\text{length}(\text{window}) - \text{noverlap}))$$

Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments to `csd` are used:

Requires `window`'s length to be no greater than the FFT length.
 Requires `NOVERLAP` to be strictly less than the window length.
 Requires positive integer values for `NFFT` and `NOVERLAP`.
 Requires vector (either row or column) input.
 Requires inputs `X` and `Y` to have the same length.
 Requires confidence parameter to be a scalar between 0 and 1.

See Also

<code>cohere</code>	Estimate magnitude squared coherence function between two signals.
<code>pburg</code>	Power spectrum estimate using the Burg method.
<code>pmtm</code>	Power spectrum estimate using the multitaper method (MTM).
<code>pmusic</code>	Power spectrum estimate using MUSIC eigenvector method.
<code>psd</code>	Estimate the power spectral density (PSD) of a signal using Welch's method.
<code>pyulear</code>	Power spectrum estimate using Yule-Walker AR method.
<code>tfe</code>	Transfer function estimate from input and output.

References

- [1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pgs. 414-419.
- [2] Welch, P.D. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms." *IEEE Trans. Audio Electroacoust.* Vol. AU-15 (June 1967). Pgs. 70-73.
- [3] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989. Pg. 737.

Purpose Chirp z -transform.

Syntax

```
y = czt(x, m, w, a)
y = czt(x)
```

Description `y = czt(x, m, w, a)` returns the chirp z -transform of signal x . The chirp z -transform is the z -transform of x along a spiral contour defined by w and a . m is a scalar that specifies the length of the transform, w is the ratio between points along the z -plane spiral contour of interest, and scalar a is the complex starting point on that contour. The contour, a spiral or “chirp” in the z -plane, is given by

$$z = a \cdot (w.^{(0:m-1)})$$

`y = czt(x)` uses the following default values:

- $m = \text{length}(x)$
- $w = \exp(j \cdot 2 \cdot \pi / m)$
- $a = 1$

With these defaults, `czt` returns the z -transform of x at m equally spaced points around the unit circle. This is equivalent to the discrete Fourier transform of x , or `fft(x)`. The empty matrix `[]` specifies the default value for a parameter.

If x is a matrix, `czt(x, m, w, a)` transforms the columns of x .

Examples Create a random vector x of length 1013 and compute its DFT using `czt`. This is faster than the `fft` function on the same sequence.

```
x = randn(1013, 1);
y = czt(x);
```

Use `czt` to zoom in on a narrow-band section (100 to 150 Hz) of a filter's frequency response. First design the filter:

```
h = fir1(30, 125/500, boxcar(31)); % filter
```

Establish frequency and CZT parameters:

```

Fs = 1000; f1 = 100; f2 = 150; % in Hertz
m = 1024;
w = exp(-j*2*pi*(f2-f1)/(m*Fs));
a = exp(j*2*pi*f1/Fs);

```

Compute both the DFT and CZT of the filter:

```

y = fft(h, 1000);
z = czt(h, m, w, a);

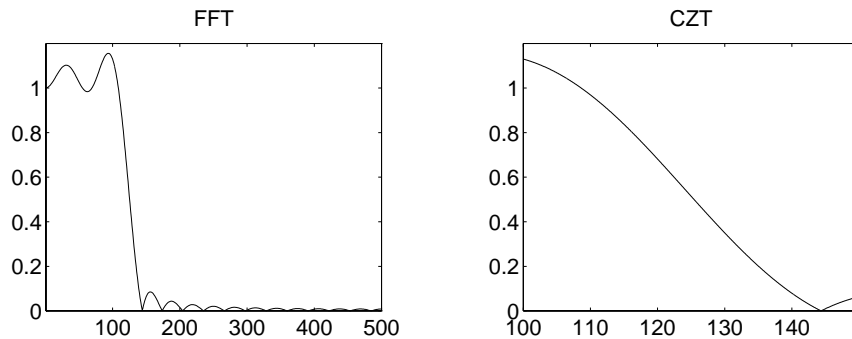
```

Create frequency vectors and compare the results:

```

fy = (0:length(y)-1)' * 1000/length(y);
fz = ((0:length(z)-1)' * (f2-f1)/length(z)) + f1;
plot(fy(1:500), abs(y(1:500))); axis([1 500 0 1.2])
plot(fz, abs(z)); axis([f1 f2 0 1.2])

```



Algorithm

`czt` uses the next power-of-2 length FFT to perform a fast convolution when computing the z -transform on a specified chirp contour [1]. `czt` can be significantly faster than `fft` for large, prime-length sequences.

Diagnostics

If `m`, `w`, or `a` is not a scalar, `czt` gives the following error message:

Inputs M, W, and A must be scalars.

See Also

<code>fft</code>	One-dimensional fast Fourier transform.
<code>freqz</code>	Frequency response of digital filters.

References

[1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pgs. 393-399.

dct

Purpose Discrete cosine transform (DCT).

Syntax $y = \text{dct}(x)$
 $y = \text{dct}(x, n)$

Description $y = \text{dct}(x)$ returns the unitary discrete cosine transform of x

$$y(k) = \sum_{n=1}^N w(n)x(n) \cos \frac{\pi(2n-1)(k-1)}{2N}, \quad k = 1, \dots, N$$

where

$$w(n) = \begin{cases} \frac{1}{\sqrt{N}}, & n = 1 \\ \sqrt{\frac{2}{N}}, & 2 \leq n \leq N \end{cases}$$

N is the length of x , and x and y are the same size. If x is a matrix, `dct` transforms its columns. The series is indexed from $n = 1$ and $k = 1$ instead of the usual $n = 0$ and $k = 0$ because MATLAB vectors run from 1 to N instead of from 0 to $N-1$.

$y = \text{dct}(x, n)$ pads or truncates x to length n before transforming.

The DCT is closely related to the discrete Fourier transform. You can often reconstruct a sequence very accurately from only a few DCT coefficients, a useful property for applications requiring data reduction.

Example

Find how many DCT coefficients represent 99% of the energy in a sequence:

```
x = (1:100) + 50*cos((1:100)*2*pi/40);
X = dct(x);
[XX,ind] = sort(abs(X)); ind = flipr(ind);
i = 1;
while (norm([X(ind(1:i)) zeros(1,100-i)]) / norm(X) < .99)
    i = i + 1;
end

i =
    3
```

See Also

fft	One-dimensional fast Fourier transform.
idct	Inverse discrete cosine transform.
dct2	Two-dimensional DCT (see <i>Image Processing Toolbox User's Guide</i>).
idct2	Two-dimensional inverse DCT (see <i>Image Processing Toolbox User's Guide</i>).

References

- [1] Jain, A.K. *Fundamentals of Digital Image Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [2] Pennebaker, W.B., and J.L. Mitchell. *JPEG Still Image Data Compression Standard*. New York, NY: Van Nostrand Reinhold, 1993. Chapter 4.

decimate

Purpose Decrease the sampling rate for a sequence (decimation).

Syntax

```
y = decimate(x, r)
y = decimate(x, r, n)
y = decimate(x, r, 'fir')
y = decimate(x, r, n, 'fir')
```

Description Decimation reduces the original sampling rate for a sequence to a lower rate. It is the opposite of interpolation. The decimation process filters the input data with a lowpass filter and then resamples the resulting smoothed signal at a lower rate.

`y = decimate(x, r)` reduces the sample rate of `x` by a factor `r`. The decimated vector `y` is `r` times shorter in length than the input vector `x`. By default, `decimate` employs an eighth-order lowpass Chebyshev type I filter. It filters the input sequence in both the forward and reverse directions to remove all phase distortion, effectively doubling the filter order.

`y = decimate(x, r, n)` uses an order `n` Chebyshev filter. Orders above 13 are not recommended because of numerical instability. MATLAB displays a warning in this case.

`y = decimate(x, r, 'fir')` uses a 30-point FIR filter, instead of the Chebyshev IIR filter. Here `decimate` filters the input sequence in only one direction. This technique conserves memory and is useful for working with long sequences.

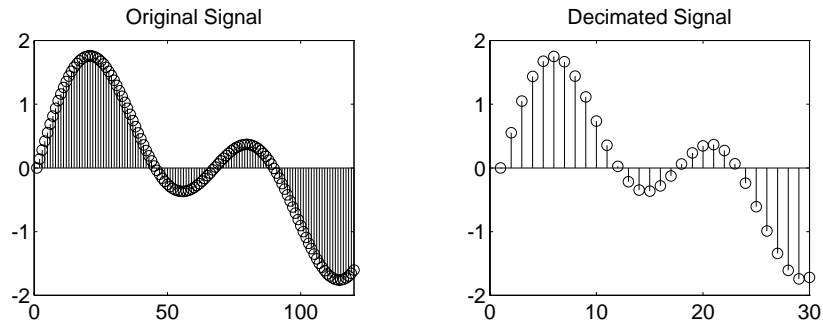
`y = decimate(x, r, n, 'fir')` uses a length `n` FIR filter.

Example Decimate a signal by a factor of four:

```
t = 0: .00025: 1; % time vector
x = sin(2*pi*30*t) + sin(2*pi*60*t);
y = decimate(x, 4);
```

View the original and decimated signals:

```
stem(x(1:120)), axis([0 120 -2 2]) % original signal
stem(y(1:30)) % decimated signal
```



Algorithm

`decimate` uses decimation algorithms 8.2 and 8.3 from [1]:

- 1 It designs a lowpass filter. By default, `decimate` uses a Chebyshev type I filter with normalized cutoff frequency $0.8/r$ and 0.05 dB of passband ripple. For the `fir` option, `decimate` designs a lowpass FIR filter with cutoff frequency $1/r$ using `fir1`.
- 2 For the FIR filter, `decimate` applies the filter to the input vector in one direction. In the IIR case, `decimate` applies the filter in forward and reverse directions with `filtfilt`.
- 3 `decimate` resamples the filtered data by selecting every r -th point.

Diagnostics

If r is not an integer, `decimate` gives the following error message:

Resampling rate R must be an integer.

If n specifies an IIR filter with order greater than 13, `decimate` gives the following warning:

Warning: IIR filters above order 13 may be unreliable.

decimate

See Also	i n t e r p	Increase sampling rate by an integer factor (interpolation).
	r e s a m p l e	Change sampling rate by any factor.
	s p l i n e	Cubic spline interpolation (see the online <i>MATLAB Function Reference</i>).
	u p f i r d n	Upsample, apply an FIR filter, and downsample.

References [1] IEEE. *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979. Chapter 8.

Purpose	Deconvolution and polynomial division.	
Syntax	$[q, r] = \text{deconv}(b, a)$	
Description	$[q, r] = \text{deconv}(b, a)$ deconvolves vector a out of vector b , using long division. The result (quotient) is returned in vector q and the remainder in vector r such that $b = \text{conv}(q, a) + r$. If a and b are vectors of polynomial coefficients, convolving them is equivalent to polynomial multiplication, and deconvolution is equivalent to polynomial division. The result of dividing b by a is quotient q and remainder r . deconv is part of the standard MATLAB environment.	
Example	The convolution of $a = [1 \ 2 \ 3]$ and $b = [4 \ 5 \ 6]$ is $c = \text{conv}(a, b)$ $c =$ $\begin{array}{cccccc} & 4 & 13 & 28 & 27 & 18 \end{array}$ Use deconv to divide b back out: $[q, r] = \text{deconv}(c, a)$ $q =$ $\begin{array}{cccccc} & 4 & 5 & 6 & & \end{array}$ $r =$ $\begin{array}{cccccc} & 0 & 0 & 0 & 0 & 0 \end{array}$	
Algorithm	This function is an M-file in the MATLAB environment that uses the <code>filter</code> primitive. Deconvolution is the impulse response of an IIR filter.	
See Also	<code>conv</code>	Convolution and polynomial multiplication.
	<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
	<code>residuez</code>	z -transform partial fraction expansion.

demod

Purpose

Demodulation for communications simulation.

Syntax

```
x = demod(y, Fc, Fs, 'method')
x = demod(y, Fc, Fs, 'method', opt)
x = demod(y, Fc, Fs, 'pwm', 'centered')
[x1, x2] = demod(y, Fc, Fs, 'qam')
```

Description

demod performs demodulation, that is, it obtains the original signal from a modulated version of the signal. demod undoes the operation performed by modulate.

`x = demod(y, Fc, Fs, 'method', opt)` demodulates the real carrier signal `y` with a carrier frequency `Fc` and sampling frequency `Fs`, using one of the options listed below for `method`. (Note that some methods accept an option, `opt`.)

`amdsb-sc` **Amplitude demodulation, double sideband, suppressed carrier.** Multiplies `y` by a sinusoid of frequency `Fc` and applies a fifth-order Butterworth lowpass filter using `filtfilt`:

`am`

```
x = y.*cos(2*pi*Fc*t);
[b, a] = butter(5, Fc*2/Fs);
x = filtfilt(b, a, x);
```

`amdsb-tc` **Amplitude demodulation, double sideband, transmitted carrier.** Multiplies `y` by a sinusoid of frequency `Fc`, and applies a fifth-order Butterworth lowpass filter using `filtfilt`:

```
x = y.*cos(2*pi*Fc*t);
[b, a] = butter(5, Fc*2/Fs);
x = filtfilt(b, a, x);
```

If you specify `opt`, demod subtracts scalar `opt` from `x`. The default value for `opt` is 0.

`amssb` **Amplitude demodulation, single sideband.** Multiplies `y` by a sinusoid of frequency `Fc` and applies a fifth-order Butterworth lowpass filter using `filtfilt`:

```
x = y.*cos(2*pi*Fc*t);
[b, a] = butter(5, Fc*2/Fs);
x = filtfilt(b, a, x);
```

- fm** **Frequency demodulation.** Demodulates the FM waveform by modulating the Hilbert transform of y by a complex exponential of frequency $-F_c$ Hz and obtains the instantaneous frequency of the result.
- pm** **Phase demodulation.** Demodulates the PM waveform by modulating the Hilbert transform of y by a complex exponential of frequency $-F_c$ Hz and obtains the instantaneous phase of the result.
- ptm** **Pulse-time demodulation.** Finds the pulse times of a pulse-time modulated signal y . For correct demodulation, the pulses cannot overlap. x is length $\text{length}(t) * F_c / F_s$.
- pwm** **Pulse-width demodulation.** Finds the pulse widths of a pulse-width modulated signal y . `demod` returns in x a vector whose elements specify the width of each pulse in fractions of a period. The pulses in y should start at the beginning of each carrier period, that is, they should be left justified.
- qam** **Quadrature amplitude demodulation.**
 $[x1, x2] = \text{demod}(y, F_c, F_s, 'qam')$ multiplies y by a cosine and a sine of frequency F_c and applies a fifth-order Butterworth lowpass filter using `filtfilt`:
 $x1 = y \cdot \cos(2\pi F_c t);$
 $x2 = y \cdot \sin(2\pi F_c t);$
 $[b, a] = \text{butter}(5, F_c * 2 / F_s);$
 $x1 = \text{filtfilt}(b, a, x1);$
 $x2 = \text{filtfilt}(b, a, x2);$

The default method is 'am'. Except for the 'ptm' and 'pwm' cases, x is the same size as y .

If y is a matrix, `demod` demodulates its columns.

$x = \text{demod}(y, F_c, F_s, 'pwm', 'centered')$ finds the pulse widths assuming they are centered at the beginning of each period. x is length $\text{length}(y) * F_c / F_s$.

demod

See Also

modul at e
vco

Modulation for communications simulation.
Voltage controlled oscillator.

Purpose Remove linear trends.

Syntax `y = detrend(x)`
`y = detrend(x, 0)`

Description `detrend` removes the mean value or linear trend from a vector or matrix, usually for FFT processing.

`y = detrend(x)` removes the best straight-line fit from vector `x` and returns it in `y`. If `x` is an array, `detrend` removes the trend from each column.

`y = detrend(x, 0)` removes the mean value from vector `x` or, if `x` is an array, from each column of the array.

Algorithm `detrend` computes the least-squares fit of a straight line to the data and subtracts the resulting function from the data. The main part of the algorithm is

```
m = length(x);
a = [(1:m)' /m ones(m, 1)];
y = x - a*(a\'x);
```

To obtain the equation of the straight-line fit, use `polyfit`.

See Also `polyfit` Polynomial curve fitting (see the online *MATLAB Function Reference*).

dftmtx

Purpose	Discrete Fourier transform matrix.	
Syntax	$A = \text{dftmtx}(n)$	
Description	<p>A <i>discrete Fourier transform matrix</i> is a complex matrix of values around the unit circle, whose matrix product with a vector computes the discrete Fourier transform of the vector.</p> <p>$A = \text{dftmtx}(n)$ returns the n-by-n complex matrix A that, when multiplied into a length n column vector x:</p> $y = A * x$ <p>computes the discrete Fourier transform of x.</p> <p>The inverse discrete Fourier transform matrix is</p> $A_i = \text{conj}(\text{dftmtx}(n)) / n$	
Example	<p>In practice, the discrete Fourier transform is computed more efficiently and uses less memory with an FFT algorithm</p> <pre>x = 1:256; y1 = fft(x);</pre> <p>than by using the Fourier transform matrix</p> <pre>n = length(x); y2 = x*dftmtx(n); norm(y1-y2)</pre> <p>ans =</p> <p>2.0016e-09</p>	
Algorithm	dftmtx uses an outer product to generate the transform matrix.	
See Also	convmtx	Convolution matrix.
	fft	One-dimensional fast Fourier transform.

Purpose Dirichlet or periodic sinc function.

Syntax `y = diric(x, n)`

Description `y = diric(x, n)` returns a vector or array `y` the same size as `x`. The elements of `y` are the Dirichlet function of the elements of `x`. `n` must be a positive integer.

The Dirichlet function, or periodic sinc function, is

$$\text{diric}(x) = \begin{cases} -1^{k(n-1)} & x = 2\pi k, \quad k = 0, \pm 1, \pm 2, \dots \\ \frac{\sin(nx/2)}{n \sin(x/2)} & \text{else} \end{cases}$$

for any nonzero integer `n`. This function has period 2π for `n` odd and period 4π for `n` even. Its peak value is 1, and its minimum value is -1 for `n` even. The magnitude of this function is $(1/n)$ times the magnitude of the discrete-time Fourier transform of the `n`-point rectangular window.

Diagnostics If `n` is not a positive integer, `diric` gives the following error message:

Requires `n` to be a positive integer.

See Also

<code>cos</code>	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
<code>gauspuls</code>	Gaussian-modulated sinusoidal pulse generator.
<code>pulstran</code>	Pulse train generator.
<code>rectpuls</code>	Sampled aperiodic rectangle generator.
<code>sawtooth</code>	Sawtooth or triangle wave generator.
<code>sin</code>	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
<code>sinc</code>	Sinc or $\sin(\pi t)/\pi t$ function.
<code>square</code>	Square wave generator.
<code>tripuls</code>	Sampled aperiodic triangle generator.

Purpose Discrete prolate spheroidal sequences (Slepian sequences).

Syntax

```
[ e, v ] = dpss(n, nw)
[ e, v ] = dpss(n, nw, k)
[ e, v ] = dpss(n, nw, [ k1 k2 ])
[ e, v ] = dpss(n, nw, ' spline' )
[ e, v ] = dpss(n, nw, ' spline' , Ni )
[ e, v ] = dpss(n, nw, ' linear' )
[ e, v ] = dpss(n, nw, ' linear' , Ni )
[ e, v ] = dpss(. . . , ' trace' )
[ e, v ] = dpss(. . . , ' int' , ' trace' )
```

Description [e, v] = dpss(n, nw) generates the first $2 \cdot nw$ *discrete prolate spheroidal sequences* (DPSS) of length n in the columns of e , and their corresponding concentrations in vector v . They are also generated in the DPSS MAT-file database `dpss.mat`. nw must be less than $n/2$.

[e, v] = dpss(n, nw, k) returns the k most band-limited sequences of the $2 \cdot nw$ discrete prolate spheroidal sequences calculated. k must be between 0 and $2 \cdot nw$.

[e, v] = dpss(n, nw, [k1 k2]) returns the $k1$ -th through the $k2$ -th sequences from the $2 \cdot nw$ discrete prolate spheroidal sequences calculated, where $1 \leq k1 \leq k2 \leq (2 \cdot nw)$.

For all of the above forms,

- The Slepian sequences are calculated directly.
- The sequences are generated in the frequency band $|\omega| \leq (2\pi W)$, where $W = nw/n$ is the half-bandwidth and ω is in radians.
- $e(:, 1)$ is the length n signal most concentrated in the frequency band $|\omega| \leq (2\pi W)$ radians, $e(:, 2)$ is the signal orthogonal to $e(:, 1)$ that is most concentrated in this band, $e(:, 3)$ is the signal orthogonal to both $e(:, 1)$ and $e(:, 2)$ that is most concentrated in this band, etc.
- For multitaper spectral analysis, typical choices for nw are 2, $5/2$, 3, $7/2$, or 4.

[e, v] = dpss(n, nw, ' spline') uses spline interpolation to compute e and v from the sequences in `dpss.mat` with length closest to n .

[e, v] = dpss(n, nw, ' spline', Ni) interpolates from existing length Ni sequences.

[e, v] = dpss(n, nw, ' linear') and

[e, v] = dpss(n, nw, ' linear', Ni) use linear interpolation, which is much faster but less accurate than spline interpolation. ' linear' requires Ni > n.

[e, v] = dpss(..., ' trace') and

[e, v] = dpss(..., ' int', ' trace') use a trailing ' trace' argument to find out which method DPSS uses, where ' int' is either ' spline' or ' linear'.

See Also

dpssclear	Remove discrete prolate spheroidal sequences from database.
dpssdir	Discrete prolate spheroidal sequences database directory.
dpssload	Load discrete prolate spheroidal sequences from database.
dpsssave	Save discrete prolate spheroidal sequences in database.
pmtm	Power spectrum estimate using the multitaper method (MTM).

References

[1] Percival, D.B., and A.T. Walden. *Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques*. Cambridge: Cambridge University Press, 1993.

dpsscLEAR

Purpose	Remove discrete prolate spheroidal sequences from database.	
Syntax	dpsscLEAR (n, nw)	
Description	dpsscLEAR (n, nw) removes sequences with length n and time-bandwidth product nw from the DPSS MAT-file database dpss.mat.	
See Also	dpss	Discrete prolate spheroidal sequences (Slepian sequences).
	dpssdir	Discrete prolate spheroidal sequences database directory.
	dpssload	Load discrete prolate spheroidal sequences from database.
	dpsssave	Save discrete prolate spheroidal sequences in database.

Purpose	Discrete prolate spheroidal sequences database directory.	
Syntax	<p>dpssdir</p> <p>dpssdir(n)</p> <p>dpssdir(nw, 'nw')</p> <p>dpssdir(n, nw)</p> <p>index = dpssdir</p>	
Description	<p>dpssdir manages the database directory that contains the generated DPSS samples in the DPSS MAT-file database dpss.mat.</p> <p>dpssdir lists the directory of saved sequences in dpss.mat.</p> <p>dpssdir(n) lists the sequences saved with length n.</p> <p>dpssdir(nw, 'nw') lists the sequences saved with time-bandwidth product nw.</p> <p>dpssdir(n, nw) lists the sequences saved with length n and time-bandwidth product nw.</p> <p>index = dpssdir is a structure array describing the DPSS database. Pass n and nw options as for the no output case to get a filtered index.</p>	
See Also	dpss	Discrete prolate spheroidal sequences (Slepian sequences).
	dpssclear	Remove discrete prolate spheroidal sequences from database.
	dpssload	Load discrete prolate spheroidal sequences from database.
	dpsssave	Save discrete prolate spheroidal sequences in database.

dpssload

Purpose	Load discrete prolate spheroidal sequences from database.	
Syntax	<code>[e, v] = dpssload(n, nw)</code>	
Description	<code>[e, v] = dpssload(n, nw)</code> loads all sequences with length <code>n</code> and time-bandwidth product <code>nw</code> in the columns of <code>e</code> and their corresponding concentrations in vector <code>v</code> from the DPSS MAT-file database <code>dpss.mat</code> .	
See Also	<code>dpss</code>	Discrete prolate spheroidal sequences (Slepian sequences).
	<code>dpssclear</code>	Remove discrete prolate spheroidal sequences from database.
	<code>dpssdir</code>	Discrete prolate spheroidal sequences database directory.
	<code>dpsssave</code>	Save discrete prolate spheroidal sequences in database.

Purpose	Save discrete prolate spheroidal sequences in database.	
Syntax	<pre>dpsssave(nw, e, v) status = dpsssave(nw, e, v)</pre>	
Description	<p><code>dpsssave(nw, e, v)</code> saves the sequences in the columns of <code>e</code> and their corresponding concentrations in vector <code>v</code> in the DPSS MAT-file database <code>dpss.mat</code>.</p> <ul style="list-style-type: none">• It is not necessary to specify sequence length, because the length of the sequence is determined by the number of rows of <code>e</code>.• <code>nw</code> is the <i>time-bandwidth product</i> that was specified when the sequence was created using <code>dpss</code>. <p><code>status = dpsssave(nw, e, v)</code> returns 0 if the save was successful and 1 if there was an error.</p>	
See Also	<code>dpss</code>	Discrete prolate spheroidal sequences (Slepian sequences).
	<code>dpssclear</code>	Remove discrete prolate spheroidal sequences from database.
	<code>dpssdir</code>	Discrete prolate spheroidal sequences database directory.
	<code>dpssload</code>	Load discrete prolate spheroidal sequences from database.

ellip

Purpose Elliptic (Cauer) filter design.

Syntax

```
[b, a] = ellip(n, Rp, Rs, Wn)
[b, a] = ellip(n, Rp, Rs, Wn, 'ftype')
[b, a] = ellip(n, Rp, Rs, Wn, 's')
[b, a] = ellip(n, Rp, Rs, Wn, 'ftype', 's')
[z, p, k] = ellip(...)
[A, B, C, D] = ellip(...)
```

Description `ellip` designs lowpass, bandpass, highpass, and bandstop digital and analog elliptic filters. Elliptic filters offer steeper rolloff characteristics than Butterworth or Chebyshev filters, but are equiripple in both the pass- and stopbands. In general, elliptic filters meet given performance specifications with the lowest order of any filter type.

Digital Domain

`[b, a] = ellip(n, Rp, Rs, Wn)` designs an order n lowpass digital elliptic filter with cutoff frequency W_n , R_p dB of ripple in the passband, and a stopband R_s dB down from the peak value in the passband. It returns the filter coefficients in the length $n + 1$ row vectors b and a , with coefficients in descending powers of z :

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}}{1 + a(2)z^{-1} + \cdots + a(n+1)z^{-n}}$$

The *cutoff frequency* is the edge of the passband, at which the magnitude response of the filter is $-R_p$ dB. For `ellip`, the cutoff frequency W_n is a number between 0 and 1, where 1 corresponds to half the sample frequency (Nyquist frequency). Smaller values of passband ripple R_p and larger values of stopband attenuation R_s both lead to wider transition widths (shallower rolloff characteristics).

If W_n is a two-element vector, $W_n = [w1 \ w2]$, `ellip` returns an order $2*n$ bandpass filter with passband $w1 < \omega < w2$.

`[b, a] = ellip(n, Rp, Rs, Wn, 'ftype')` designs a highpass or bandstop filter, where *ftype* is

- `high` for a highpass digital filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop digital filter if W_n is a two-element vector, $W_n = [w1 \ w2]$

The stopband is $w1 < \omega < w2$.

With different numbers of output arguments, `ellip` directly obtains other realizations of the filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = ellip(n, Rp, Rs, Wn)` or

`[z, p, k] = ellip(n, Rp, Rs, Wn, 'ftype')` returns the zeros and poles in length n column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = ellip(n, Rp, Rs, Wn)` or

`[A, B, C, D] = ellip(n, Rp, Rs, Wn, 'ftype')` where A , B , C , and D are

$$\begin{aligned} x[n+1] &= Ax[n] + Bu[n] \\ y[n] &= Cx[n] + Du[n] \end{aligned}$$

and u is the input, x is the state vector, and y is the output.

Analog Domain

`[b, a] = ellip(n, Rp, Rs, Wn, 's')` designs an order n lowpass analog elliptic filter with cutoff frequency W_n and returns the filter coefficients in the length $n + 1$ row vectors b and a , in descending powers of s :

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^n + b(2)s^{n-1} + \dots + b(n+1)}{s^n + a(2)s^{n-1} + \dots + a(n+1)}$$

The *cutoff frequency* is the edge of the passband, at which the magnitude response of the filter is $-R_p$ dB. For `ellip`, the cutoff frequency W_n must be greater than 0.

If W_n is a two-element vector with $w_1 < w_2$, then `ellip(n, Rp, Rs, Wn, 's')` returns an order $2*n$ bandpass analog filter with passband $w_1 < \omega < w_2$.

`[b, a] = ellip(n, Rp, Rs, Wn, 'ftype', 's')` designs a highpass or bandstop filter, where *ftype* is

- `high` for a highpass analog filter with cutoff frequency W_n
- `stop` for an order $2*n$ bandstop analog filter. W_n is a two-element vector, `[w1 w2]`, specifying the stopband $w_1 < \omega < w_2$.

With different numbers of output arguments, `ellip` directly obtains other realizations of the analog filter. To obtain zero-pole-gain form, use three output arguments:

`[z, p, k] = ellip(n, Rp, Rs, Wn, 's')` or

`[z, p, k] = ellip(n, Rp, Rs, Wn, 'ftype', 's')` returns the zeros and poles in length n or $2*n$ column vectors z and p and the gain in the scalar k .

To obtain state-space form, use four output arguments:

`[A, B, C, D] = ellip(n, Rp, Rs, Wn, 's')` or

`[A, B, C, D] = ellip(n, Rp, Rs, Wn, 'ftype', 's')` where A , B , C , and D are

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

and u is the input, x is the state vector, and y is the output.

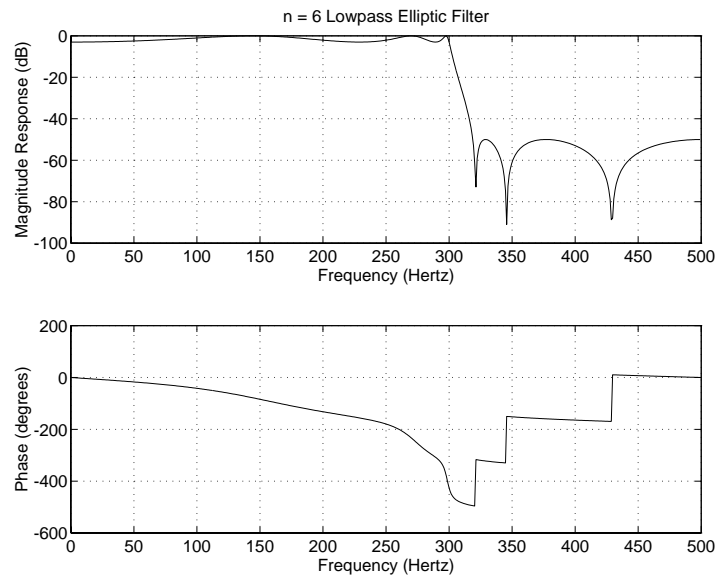
Examples

For data sampled at 1000 Hz, design a sixth-order lowpass elliptic filter with a cutoff frequency of 300 Hz, 3 dB of ripple in the passband, and 50 dB of attenuation in the stopband:

```
[b, a] = ellip(6, 3, 50, 300/500);
```

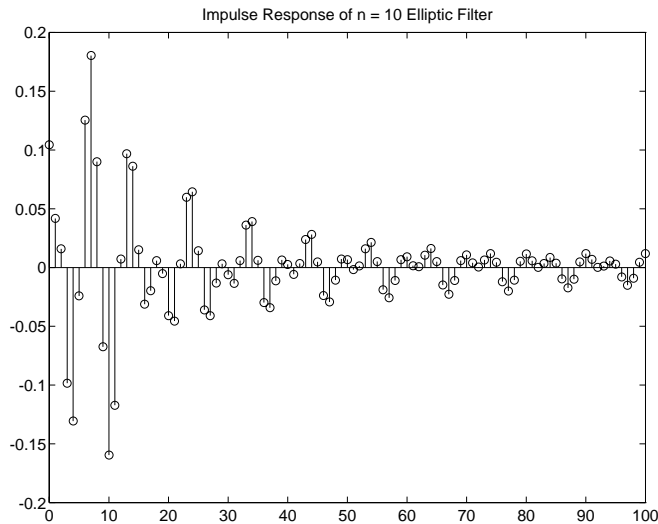

The filter's frequency response is

`freqz(b, a, 512, 1000)`



Design a 20th-order bandpass elliptic filter with a passband from 100 to 200 Hz and plot its impulse response:

```
n = 10; Rp = 0.5; Rs = 20;  
Wn = [100 200]/500;  
[b, a] = ellip(n, Rp, Rs, Wn);  
[y, t] = impz(b, a, 101); stem(t, y)
```



Limitations

For high order filters, the state-space form is the most numerically accurate, followed by the zero-pole-gain form. The transfer function form is the least accurate; numerical problems can arise for filter orders as low as 15.

Algorithm	The design of elliptic filters is the most difficult and computationally intensive of the Butterworth, Chebyshev type I and II, and elliptic designs. <code>ellip</code> uses a five-step algorithm:	
	<ol style="list-style-type: none">1 It finds the lowpass analog prototype poles, zeros, and gain using the <code>ellipap</code> function.2 It converts the poles, zeros, and gain into state-space form.3 It transforms the lowpass filter to a bandpass, highpass, or bandstop filter with the desired cutoff frequencies using a state-space transformation.4 For digital filter design, <code>ellip</code> uses <code>bilinear</code> to convert the analog filter into a digital filter through a bilinear transformation with frequency prewarping. Careful frequency adjustment guarantees that the analog filters and the digital filters will have the same frequency response magnitude at ω_n or ω_1 and ω_2.5 It converts the state-space filter back to transfer function or zero-pole-gain form, as required.	
See Also	<code>bessel f</code>	Bessel analog filter design.
	<code>butter</code>	Butterworth analog and digital filter design.
	<code>cheby1</code>	Chebyshev type I filter design (passband ripple).
	<code>cheby2</code>	Chebyshev type II filter design (stopband ripple).
	<code>ellipap</code>	Elliptic analog lowpass filter prototype.
	<code>ellipord</code>	Elliptic filter order selection.

ellipap

Purpose Elliptic analog lowpass filter prototype.

Syntax [z, p, k] = ellipap(n, Rp, Rs)

Description [z, p, k] = ellipap(n, Rp, Rs) returns the zeros, poles, and gain of an order n elliptic analog lowpass filter prototype, with Rp dB of ripple in the passband, and a stopband Rs dB down from the peak value in the passband. The zeros and poles are returned in length n column vectors z and p and the gain in scalar k. If n is odd, z is length n – 1. The transfer function is

$$H(s) = \frac{z(s)}{p(s)} = k \frac{(s - z(1))(s - z(2)) \cdots (s - z(n))}{(s - p(1))(s - p(2)) \cdots (s - p(n))}$$

Elliptic filters are equiripple in both the passband and stopband. They offer steeper rolloff characteristics than Butterworth and Chebyshev filters, but they are equiripple in both the passband and the stopband. Of the four classical filter types, elliptic filters usually meet a given set of filter performance specifications with the lowest filter order.

ellip sets the cutoff frequency ω_0 of the elliptic filter to 1 for a normalized result. The *cutoff frequency* is the frequency at which the passband ends and the filter has a magnitude response of $10^{-Rp/20}$.

Algorithm ellipap uses the algorithm outlined in [1]. It employs the M-file ellipk to calculate the complete elliptic integral of the first kind and the M-file ellipj to calculate Jacobi elliptic functions.

See Also	besselap	Bessel analog lowpass filter prototype.
	buttap	Butterworth analog lowpass filter prototype.
	cheb1ap	Chebyshev type I analog lowpass filter prototype.
	cheb2ap	Chebyshev type II analog lowpass filter prototype.
	ellip	Elliptic (Cauer) filter design.

References [1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Chapter 7.

Purpose	Elliptic filter order selection.								
Syntax	<pre>[n, Wn] = ellipord(Wp, Ws, Rp, Rs) [n, Wn] = ellipord(Wp, Ws, Rp, Rs, 's')</pre>								
Description	<p><code>ellipord</code> selects the minimum order digital or analog elliptic filter required to meet a set of lowpass filter design specifications:</p> <table> <tr> <td>Wp</td><td>Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).</td></tr> <tr> <td>Ws</td><td>Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).</td></tr> <tr> <td>Rp</td><td>Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is $0 < w < Wp$.</td></tr> <tr> <td>Rs</td><td>Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is $Ws < w < 1$.</td></tr> </table>	Wp	Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).	Ws	Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).	Rp	Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is $0 < w < Wp$.	Rs	Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is $Ws < w < 1$.
Wp	Passband corner frequency. Wp, the cutoff frequency, has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).								
Ws	Stopband corner frequency. Ws is in the same units as Wp; it has a value between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).								
Rp	Passband ripple, in decibels. This value is the maximum permissible passband loss in decibels. The passband is $0 < w < Wp$.								
Rs	Stopband attenuation, in decibels. This value is the number of decibels the stopband is down from the passband. The stopband is $Ws < w < 1$.								

Digital Domain

`[n, Wn] = ellipord(Wp, Ws, Rp, Rs)` returns the order `n` of the lowest order elliptic filter that loses no more than `Rp` dB in the passband and has at least `Rs` dB of attenuation in the stopband. The passband runs from 0 to `Wp` and the stopband extends from `Ws` to 1, the Nyquist frequency. `ellipord` also returns `Wn`, the cutoff frequency that allows `ellip` to achieve the given specifications.

Use `ellipord` for lowpass, highpass, bandpass, and bandstop filters. For highpass filters, `Wp` is greater than `Ws`. For bandpass and bandstop filters, `Wp` and `Ws` are two-element vectors that specify the corner frequencies at both edges of the filter, lower frequency edge first. For the band filters, `ellipord` returns `Wn` as a two-element row vector for input to `ellip`.

If filter specifications call for a bandpass or bandstop filter with unequal ripple in each of the passbands or stopbands, design the filter as separate lowpass and highpass sections and cascade the two filters together.

Analog Domain

`[n, Wn] = ellipord(Wp, Ws, Rp, Rs, 's')` finds the minimum order n and cutoff frequencies W_n for an analog filter. In this case the frequencies in W_p and W_s are in radians per second and may be greater than 1.

Use `ellipord` for lowpass, highpass, bandpass, and bandstop filters as described under “Digital Domain.”

Examples

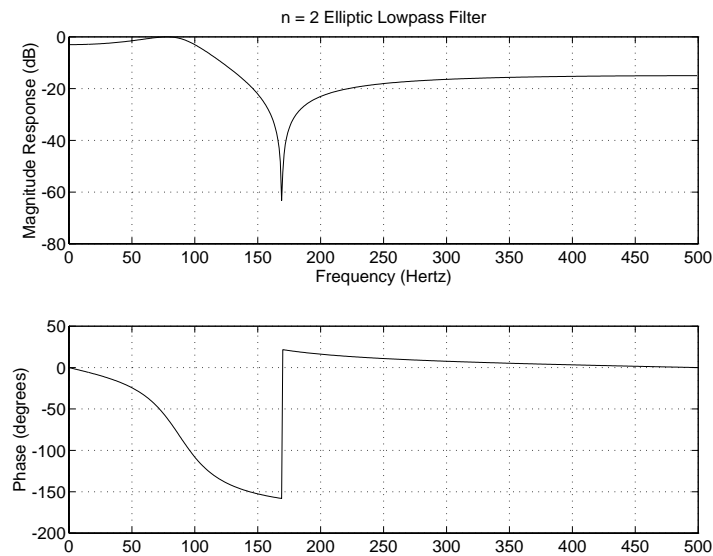
For 1000 Hz data, design a lowpass filter with less than 3 dB of attenuation from 0 to 100 Hz and at least 15 dB of attenuation from 150 Hz to the Nyquist frequency:

```
Wp = 100/500; Ws = 150/500;
Rp = 3; Rs = 15;
[n, Wn] = ellipord(Wp, Ws, Rp, Rs)
```

```
n =
     2
```

```
Wn =
    0.2000
```

```
[b, a] = ellip(n, Rp, Rs, Wn);
freqz(b, a, 512, 1000)
```



Now design a bandpass filter with a passband from 100 Hz to 200 Hz, less than 3 dB of ripple throughout the passband, and 30 dB stopbands 50 Hz out on both sides of the passband:

```
Wp = [100 200]/500; Ws = [50 250]/500;
```

```
Rp = 3; Rs = 30;
```

```
[n, Wn] = ellipord(Wp, Ws, Rp, Rs)
```

```
n =
```

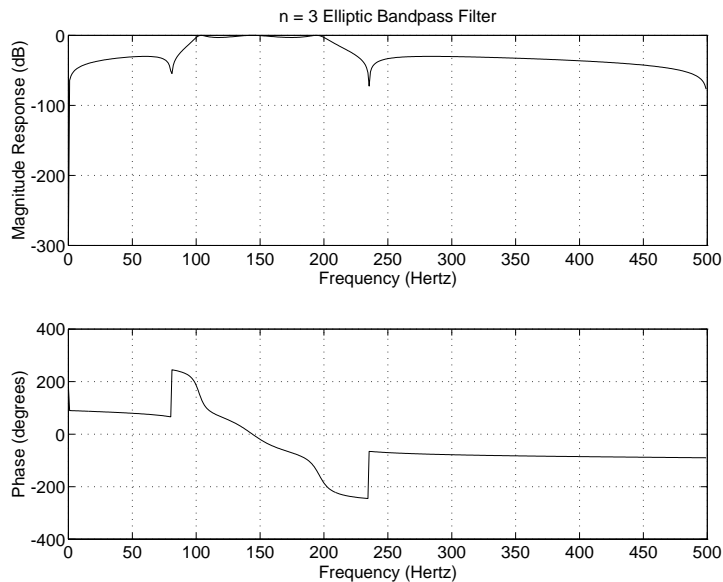
```
3
```

```
Wn =
```

```
0.2000 0.4000
```

```
[b, a] = ellip(n, Rp, Rs, Wn);
```

```
freqz(b, a, 512, 1000)
```



Algorithm

`ellipord` uses the elliptic lowpass filter order prediction formula described in [1]. The function performs its calculations in the analog domain for both the analog and digital cases. For the digital case, it converts the frequency parameters to the s -domain before the order and natural frequency estimation process, then converts them back to the z -domain.

`ellipord` initially develops a lowpass filter prototype by transforming the passband frequencies of the desired filter to 1 rad/sec (for low- and highpass filters) and to -1 and 1 rad/sec (for bandpass and bandstop filters). It then computes the minimum order required for a lowpass filter to meet the stopband specification.

See Also

<code>buttord</code>	Butterworth filter order selection.
<code>cheb1ord</code>	Chebyshev type I filter order selection.
<code>cheb2ord</code>	Chebyshev type II filter order selection.
<code>ellip</code>	Elliptic (Cauer) filter design.

References

[1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1975. Pg. 241.

Purpose One-dimensional fast Fourier transform.

Syntax $y = \text{fft}(x)$
 $y = \text{fft}(x, n)$

Description `fft` computes the discrete Fourier transform of a vector or matrix. This function implements the transform given by

$$X(k+1) = \sum_{n=0}^{N-1} x(n+1)W_N^{kn}$$

where $W_N = e^{-j(2\pi/N)}$ and $N = \text{length}(x)$. Note that the series is indexed as $n+1$ and $k+1$ instead of the usual n and k because MATLAB vectors run from 1 to N instead of from 0 to $N-1$.

$y = \text{fft}(x)$ is the discrete Fourier transform of vector x , computed with a fast Fourier transform (FFT) algorithm. If x is a matrix, y is the FFT of each column of the matrix. The `fft` function employs a radix-2 fast Fourier transform algorithm if the length of the sequence is a power of two, and a slower algorithm if it is not; see the “Algorithm” section for details.

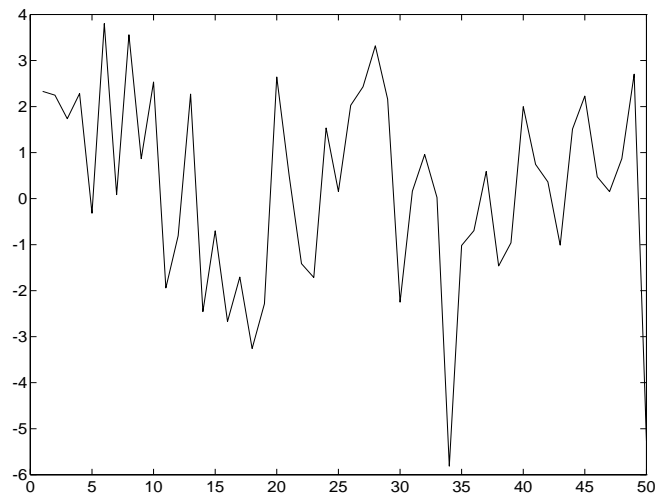
$y = \text{fft}(x, n)$ is the n -point FFT. If the length of x is less than n , `fft` pads x with trailing zeros to length n . If the length of x is greater than n , `fft` truncates the sequence x . If x is an array, `fft` adjusts the length of the columns in the same manner.

`fft` is part of the standard MATLAB environment.

Example A common use of the Fourier transform is to find the frequency components of a time-domain signal buried in noise. Consider data sampled at 1000 Hz. Form

a signal consisting of 50 Hz and 120 Hz sinusoids and corrupt the signal with zero-mean random noise:

```
t = 0:0.001:0.6;
x = sin(2*pi*50*t) + sin(2*pi*120*t);
y = x + 2*randn(1,length(t));
plot(y(1:50))
```



It is difficult to identify the frequency components by studying the original signal. Convert to the frequency domain by taking the discrete Fourier transform of the noisy signal y using a 512-point fast Fourier transform (FFT):

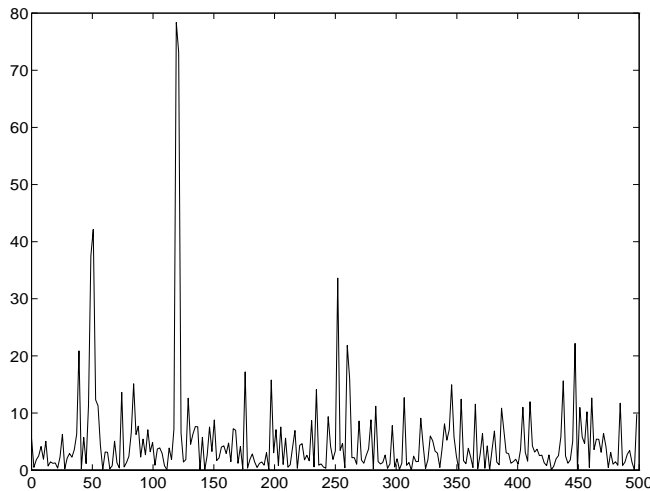
```
Y = fft(y, 512);
```

The power spectral density, a measurement of the energy at various frequencies, is

```
Pyy = Y.*conj(Y) / 512;
```

Graph the first 256 points (the other 256 points are symmetric) on a meaningful frequency axis:

```
f = 1000*(0:255)/512;  
plot(f, Pyy(1:256))
```



See the `psd` function for details on calculating spectral density.

Sometimes it is useful to normalize the output of `fft` so that a unit sinusoid in the time domain corresponds to unit amplitude in the frequency domain. To produce a normalized discrete-time Fourier transform in this manner, use

```
Pn = abs(fft(x))/length(x)
```

Algorithm

`fft` is a built-in MATLAB function. When the sequence length is a power of two, `fft` uses a high-speed radix-2 fast Fourier transform algorithm. The radix-2 FFT routine is optimized to perform a real FFT if the input sequence is purely real; otherwise it computes the complex FFT. This causes a real power-of-two FFT to be about 40% faster than a complex FFT of the same length.

When the sequence length is not an exact power of two, a separate algorithm finds the prime factors of the sequence length and computes the mixed-radix discrete Fourier transforms of the shorter sequences.

The execution time for `fft` depends on the sequence length. If the length of a sequence has many prime factors, the function computes the FFT quickly; if the length has few prime factors, execution is slower. For sequences whose lengths are prime numbers, `fft` uses the raw (and slow) DFT algorithm. For this reason it is usually better to use power-of-two FFTs, if this is supported by your application. For example, on one machine a 4096-point real FFT takes 2.1 seconds and a complex FFT of the same length takes 3.7 seconds. The FFTs of neighboring sequences of length 4095 and 4097, however, take 7 seconds and 58 seconds, respectively.

Suppose a sequence x of N points is obtained at a sample frequency of f_s . Then, for up to the Nyquist frequency, or point $n = N/2 + 1$, the relationship between the actual frequency and the index k into x (out of N possible indices) is

$$f = (k - 1) * f_s / N$$

See Also

<code>dct</code>	Discrete cosine transform (DCT).
<code>dftmtx</code>	Discrete Fourier transform matrix.
<code>fft2</code>	Two-dimensional fast Fourier transform.
<code>fftshift</code>	Rearrange the outputs of <code>fft</code> and <code>fft2</code> .
<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
<code>freqz</code>	Frequency response of digital filters.
<code>ifft</code>	One-dimensional inverse fast Fourier transform.
<code>psd</code>	Estimate the power spectral density (PSD) of a signal using Welch's method.

fft2

Purpose	Two-dimensional fast Fourier transform.	
Syntax	<pre>Y = fft2(X) Y = fft2(X, m, n)</pre>	
Description	<p><code>Y = fft2(X)</code> performs a two-dimensional FFT, producing a result <code>Y</code> the same size as <code>X</code>. If <code>X</code> is a vector, <code>Y</code> has the same orientation as <code>X</code>.</p> <p><code>Y = fft2(X, m, n)</code> truncates or zero pads <code>X</code>, if necessary, to create an <code>m</code>-by-<code>n</code> array before performing the FFT. The result <code>Y</code> is also <code>m</code>-by-<code>n</code>.</p> <p><code>fft2</code> is part of the standard MATLAB environment.</p>	
Algorithm	<p><code>fft2(x)</code> is simply</p> <pre>fft(fft(x) .') .'</pre> <p>This computes the one-dimensional <code>fft</code> of each column of <code>x</code>, then of each row of the result. The time required to compute <code>fft2(x)</code> depends on the number of prime factors in <code>[m, n] = size(x)</code>. <code>fft2</code> is fastest when <code>m</code> and <code>n</code> are powers of 2.</p>	
See Also	<code>fft</code>	One-dimensional fast Fourier transform.
	<code>fftshift</code>	Rearrange the outputs of <code>fft</code> and <code>fft2</code> .
	<code>ifft</code>	One-dimensional inverse fast Fourier transform.
	<code>ifft2</code>	Two-dimensional inverse fast Fourier transform.

Purpose FFT-based FIR filtering using the overlap-add method.

Syntax

```
y = fftfilt(b, x)
y = fftfilt(b, x, n)
```

Description `fftfilt` filters data using the efficient FFT-based method of *overlap-add*, a frequency domain filtering technique that works only for FIR filters.

`y = fftfilt(b, x)` filters the data in vector `x` with the filter described by coefficient vector `b`. It returns the data vector `y`. The operation performed by `fftfilt` is described in the *time domain* by the difference equation

$$y(n) = b(1)x(n) + b(2)x(n-1) + \cdots + b(nb+1)x(n-nb)$$

An equivalent representation is the *z*-transform or *frequency domain* description

$$Y(z) = (b(1) + b(2)z^{-1} + \cdots + b(nb+1)z^{-nb})X(z)$$

By default, `fftfilt` chooses an FFT length and data block length that guarantee efficient execution time.

`y = fftfilt(b, x, n)` uses an FFT length of `nfft = 2^nextpow2(n)` and a data block length of `nfft - length(b) + 1`.

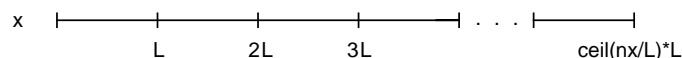
`fftfilt` works for both real and complex inputs.

Example Show that the results from `fftfilt` and `filter` are identical:

```
b = [1 2 3 4];
x = [1 zeros(1, 99)]';
norm(fftfilt(b, x) - filter(b, 1, x))

ans =
    9.5914e-15
```

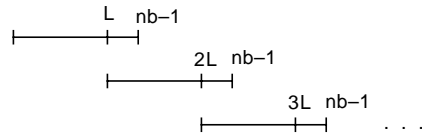
Algorithm `fftfilt` uses `fft` to implement the *overlap-add method* [1], a technique that combines successive frequency domain filtered blocks of an input sequence. `fftfilt` breaks an input sequence `x` into length `L` data blocks:



and convolves each block with the filter `b` by

```
y = ifft(fft(x(i:i+L-1), nfft) .* fft(b, nfft));
```

where `nfft` is the FFT length. `fftfilt` overlaps successive output sections by `nb-1` points, where `nb` is the length of the filter, and sums them:



`fftfilt` chooses the key parameters `L` and `nfft` in different ways, depending on whether you supply an FFT length `n` and on the lengths of the filter and signal. If you do not specify a value for `n` (which determines FFT length), `fftfilt` chooses these key parameters automatically:

- If `length(x) > length(b)`, `fftfilt` chooses values that minimize the number of blocks times the number of flops per FFT.
- If `length(b) >= length(x)`, `fftfilt` uses a single FFT of length

$$2^{\text{nextpow2}(\text{length}(b) + \text{length}(x) - 1)}$$

This essentially computes

```
y = ifft(fft(B, nfft) .* fft(X, nfft))
```

If you supply a value for `n`, `fftfilt` chooses an FFT length, `nfft`, of $2^{\text{nextpow2}(n)}$ and a data block length of `nfft - length(b) + 1`. If `n` is less than `length(b)`, `fftfilt` sets `n` to `length(b)`.

See Also

<code>conv</code>	Convolution and polynomial multiplication.
<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
<code>filtfilt</code>	Zero-phase digital filtering.

References

[1] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989.

Purpose	Rearrange the outputs of the FFT functions.	
Syntax	<code>y = fftshift(x)</code>	
Description	<p><code>y = fftshift(x)</code> rearranges the outputs of <code>fft</code> and <code>fft2</code> by moving the zero frequency component to the center of the spectrum, which is sometimes a more convenient form.</p> <p>For vectors, <code>fftshift(x)</code> returns a vector with the left and right halves swapped.</p> <p>For arrays, <code>fftshift(x)</code> swaps quadrants one and three with quadrants two and four.</p> <p>This function is part of the standard MATLAB environment.</p>	
Example	<p>For any array <code>X</code>,</p> <pre>Y = fft2(X)</pre> <p>has <code>Y(1,1) = sum(sum(X))</code>; the DC component of the signal is in the upper-left corner of the two-dimensional FFT. For</p> <pre>Z = fftshift(Y)</pre> <p>the DC component is near the center of the matrix.</p>	
See Also	<code>fft</code>	One-dimensional fast Fourier transform.
	<code>fft2</code>	Two-dimensional fast Fourier transform.

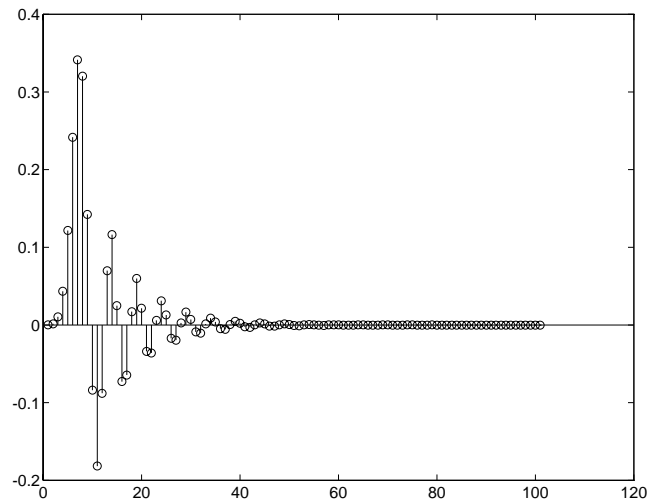
filter

Purpose	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
Syntax	<pre>y = filter(b, a, x) [y, zf] = filter(b, a, x) [...] = filter(b, a, x, zi) [...] = filter(b, a, x, zi, dim)</pre>
Description	<p><code>filter</code> is part of the MATLAB environment. It filters data using a digital filter. The filter realization is the <i>transposed direct form II</i> structure [1], which can handle both FIR and IIR filters.</p> <p>If $a(1) \neq 1$, <code>filter</code> normalizes the filter coefficients by $a(1)$. If $a(1) = 0$, the input is in error.</p> <p><code>y = filter(b, a, x)</code> filters the data in vector <code>x</code> with the filter described by coefficient vectors <code>a</code> and <code>b</code> to create the filtered data vector <code>y</code>. When <code>x</code> is a matrix, <code>filter</code> operates on the columns of <code>x</code>. When <code>x</code> is an N-dimensional array, <code>filter</code> operates on the first non-singleton dimension.</p> <p><code>[y, zf] = filter(b, a, x)</code> returns the final values of the states in the vector <code>zf</code>.</p> <p><code>[...] = filter(b, a, x, zi)</code> specifies initial state conditions in the vector <code>zi</code>. The size of the initial/final condition vector is $\max(\text{length}(b), \text{length}(a)) - 1$. <code>zi</code> or <code>zf</code> can also be an array of such vectors, one for each column of <code>x</code> if <code>x</code> is a matrix. If <code>x</code> is a multidimensional array, <code>filter</code> works across the first nonsingleton dimension of <code>x</code> by default.</p> <p><code>[...] = filter(b, a, x, zi, dim)</code> works across the dimension <code>dim</code> of <code>x</code>. Set <code>zi</code> to empty to get the default initial conditions.</p> <p><code>filter</code> works for both real and complex inputs.</p>

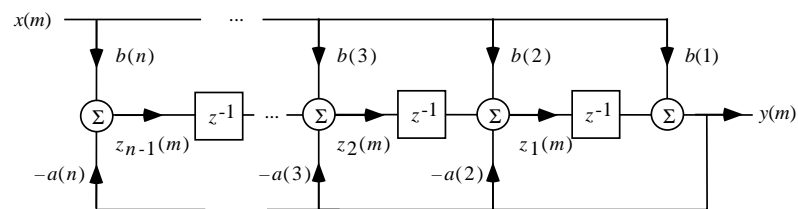
Example

Find and graph the 100-point unit impulse response of a digital filter:

```
x = [ 1 zeros(1, 100) ];
[b, a] = butter(12, 400/1000);
y = filter(b, a, x);
stem(y)
```

**Algorithm**

`filter` is a built-in MATLAB function. `filter` is implemented as a transposed direct form II structure



where $n-1$ is the filter order.

The operation of `filter` at sample m is given by the time domain difference equations for y and the states z_i :

$$\begin{aligned} y(m) &= b(1)x(m) + z_1(m-1) - a(1)y(m) \\ z_1(m) &= b(2)x(m) + z_2(m-1) - a(2)y(m) \\ &\vdots = \vdots \quad \vdots \\ z_{n-2}(m) &= b(n-1)x(m) + z_{n-1}(m-1) - a(n-1)y(m) \\ z_{n-1}(m) &= b(n)x(m) - a(n)y(m) \end{aligned}$$

You can use `filtic` to generate the state vector $z_i(0)$ from past inputs and outputs.

The input-output description of this filtering operation in the z -transform domain is a rational transfer function:

$$Y(z) = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}} X(z)$$

Diagnostics

If $a(1) = 0$, `filter` gives the following error message:

First denominator coefficient must be nonzero.

If the length of the initial condition vector is not the greater of na and nb , `filter` gives the following error message:

Initial condition vector has incorrect dimensions.

See Also

<code>fftfilt</code>	FFT-based FIR filtering using the overlap-add method.
<code>filter2</code>	Two-dimensional digital filtering.
<code>filtfilt</code>	Zero-phase digital filtering.
<code>filtic</code>	Make initial conditions for <code>filter</code> function.

References

[1] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 311-312.

Purpose	Two-dimensional digital filtering.	
Syntax	<code>Y = filter2(B, X)</code> <code>Y = filter2(B, X, 'shape')</code>	
Description	<p><code>Y = filter2(B, X)</code> filters the two-dimensional data in <code>X</code> with the two-dimensional FIR filter in the matrix <code>B</code>. The result, <code>Y</code>, is computed using two-dimensional convolution and is the same size as <code>X</code>.</p> <p><code>Y = filter2(B, X, 'shape')</code> returns <code>Y</code> computed with size specified by <code>shape</code>:</p> <ul style="list-style-type: none"> • <code>same</code> returns the central part of the convolution that is the same size as <code>X</code> (default). • <code>full</code> returns the full two-dimensional convolution, <code>size(Y) > size(X)</code>. • <code>valid</code> returns only those parts of the convolution that are computed without the zero-padded edges, <code>size(Y) < size(X)</code>. 	
Algorithm	<p><code>filter2</code> is part of the MATLAB environment. It uses <code>conv2</code> to compute the full two-dimensional convolution of the FIR filter with the input matrix. By default, <code>filter2</code> extracts and returns the central part of the convolution that is the same size as the input matrix. Use the <code>shape</code> parameter to specify an alternate part of the convolution for return.</p>	
See Also	<code>conv2</code> <code>filter</code>	<p>Two-dimensional convolution.</p> <p>Filter data with a recursive (IIR) or nonrecursive (FIR) filter.</p>

filtfilt

Purpose Zero-phase digital filtering.

Syntax `y = filtfilt(b, a, x)`

Description `y = filtfilt(b, a, x)` performs zero-phase digital filtering by processing the input data in both the forward and reverse directions (see problem 5.39 in [1]). After filtering in the forward direction, it reverses the filtered sequence and runs it back through the filter. The resulting sequence has precisely zero-phase distortion and double the filter order. `filtfilt` minimizes start-up and ending transients by matching initial conditions, and works for both real and complex inputs.

Note that `filtfilt` should not be used with differentiator and Hilbert FIR filters, since the operation of these filters depends heavily on their phase response.

Algorithm `filtfilt` is an M-file that uses the `filter` function. In addition to the forward-reverse filtering, it attempts to minimize startup transients by adjusting initial conditions to match the DC component of the signal and by prepending several filter lengths of a flipped, reflected copy of the input signal.

See Also

<code>fftfilt</code>	FFT-based FIR filtering using the overlap-add method.
<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
<code>filter2</code>	Two-dimensional digital filtering.

References [1] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 311-312.

Purpose Find initial conditions for a transposed direct form II filter implementation.

Syntax

```
z = filtic(b, a, y, x)
z = filtic(b, a, y)
```

Description `z = filtic(b, a, y, x)` finds the initial conditions `z` for the delays in the *transposed direct form II* filter implementation given past outputs `y` and inputs `x`. The vectors `b` and `a` represent the numerator and denominator coefficients, respectively, of the filter's transfer function.

The vectors `x` and `y` contain the most recent input or output first, and oldest input or output last:

$$x = \{x(-1), x(-2), x(-3), \dots, x(-nb), \dots\}$$

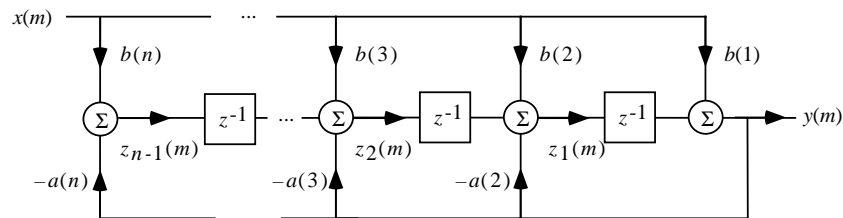
$$y = \{y(-1), y(-2), y(-3), \dots, y(-na), \dots\}$$

where `nb` is `length(b) - 1` (the numerator order) and `na` is `length(a) - 1` (the denominator order). If `length(x)` is less than `nb`, `filtic` pads it with zeros to length `nb`; if `length(y)` is less than `na`, `filtic` pads it with zeros to length `na`. Elements of `x` beyond `x(nb-1)` and elements of `y` beyond `y(na-1)` are unnecessary so `filtic` ignores them.

Output `z` is a column vector of length equal to the larger of `nb` and `na`. `z` describes the state of the delays given past inputs `x` and past outputs `y`.

`z = filtic(b, a, y)` assumes that the input `x` is 0 in the past.

The transposed direct form II structure is



where $n-1$ is the filter order.

`filtic` works for both real and complex inputs.

filtic

Algorithm	filtic performs a reverse difference equation to obtain the delay states z .	
Diagnostics	If any of the input arguments y , x , b , or a is not a vector (that is, if any argument is a scalar or array), filtic gives the following error message: Requires vector inputs.	
See Also	filter	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
	filtfilt	Zero-phase digital filtering.
References	[1] Oppenheim, A.V., and R.W. Schaffer. <i>Discrete-Time Signal Processing</i> . Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 296, 301-302.	

Purpose Window-based finite impulse response filter design—standard response.

Syntax

```

b = fir1(n, Wn)
b = fir1(n, Wn, 'ftype')
b = fir1(n, Wn, window)
b = fir1(n, Wn, 'ftype', window)
b = fir1(..., 'noscale')
```

Description `fir1` implements the classical method of windowed linear-phase FIR digital filter design [1]. It designs filters in standard lowpass, bandpass, highpass, and bandpass configurations. (For windowed filters with arbitrary frequency response, use `fir2`.)

`b = fir1(n, Wn)` returns row vector `b` containing the $n + 1$ coefficients of an order n lowpass FIR filter. This is a Hamming-windowed, linear-phase filter with cutoff frequency `Wn`. The output filter coefficients, `b`, are ordered in descending powers of z :

$$b(z) = b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n}$$

`Wn`, the cutoff frequency, is a number between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).

If `Wn` is a two-element vector, `Wn = [w1 w2]`, `fir1` returns a bandpass filter with passband $w1 < \omega < w2$.

If `Wn` is a multi-element vector, `Wn = [w1 w2 w3 w4 w5 ... wn]`, `fir1` returns an order n multiband filter with bands $0 < w < w1, w1 < w < w2, \dots, wn < w < 1$.

By default, the filter is scaled so that the center of the first passband has magnitude exactly 1 after windowing.

`b = fir1(n, Wn, 'ftype')` specifies a filter type, where `ftype` is

- `high` for a highpass filter with cutoff frequency `Wn`
- `stop` for a bandstop filter, if `Wn = [w1 w2]`

The stopband is $w1 < \omega < w2$.

- `'DC-1'` to make the first band of a multiband filter a passband
- `'DC-0'` to make the first band of a multiband filter a stopband

`fir1` always uses an even filter order for the highpass and bandstop configurations. This is because for odd orders, the frequency response at the Nyquist frequency is 0, which is inappropriate for highpass and bandstop filters. If you specify an odd-valued `n`, `fir1` increments it by 1.

`b = fir1(n, Wn, window)` uses the window specified in column vector `window` for the design. The vector `window` must be `n+1` elements long. If no window is specified, `fir1` employs a Hamming window.

`b = fir1(n, Wn, 'ftype', window)` accepts both `ftype` and `window` parameters.

`b = fir1(..., 'noscale')` turns off the default scaling.

The group delay of the FIR filter designed by `fir1` is $n/2$.

Algorithm

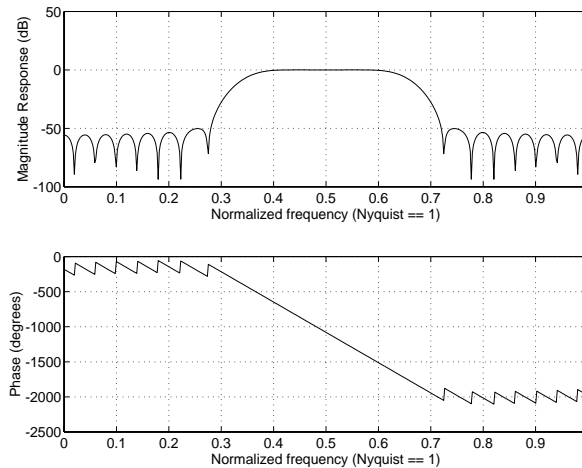
`fir1` uses the window method of FIR filter design [1]. If $w(n)$ denotes a window, where $1 \leq n \leq N$, and the impulse response of the ideal filter is $h(n)$, where $h(n)$ is the inverse Fourier transform of the ideal frequency response, then the windowed digital filter coefficients are given by

$$b(n) = w(n)h(n), \quad 1 \leq n \leq N$$

Examples

Design a 48th-order FIR bandpass filter with passband $0.35 \leq w \leq 0.65$:

```
b = fir1(48, [0.35 0.65]);
freqz(b, 1, 512)
```



Design a 34th order FIR highpass filter with a cutoff frequency of 0.48, using a Chebyshev window with 30 dB of ripple:

```
b = fir1(34, 0.48, 'high', chebwin(35, 30));
xfilt = filter(b, 1, x);
```

Diagnostics

If n is odd and you specify a bandstop or highpass filter, `fir1` gives the following warning message:

For highpass and bandstop filters, N must be even.
Order is being increased by 1.

See Also

<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
<code>fir2</code>	Window-based finite impulse response filter design—arbitrary response.
<code>fircls</code>	Constrained least square FIR filter design for multiband filters.
<code>fircls1</code>	Constrained least square filter design for lowpass and highpass linear phase FIR filters.
<code>firls</code>	Least square linear-phase FIR filter design.
<code>freqz</code>	Frequency response of digital filters.
<code>kaiserord</code>	Estimate parameters for <code>fir1</code> with Kaiser window.
<code>remez</code>	Parks-McClellan optimal FIR filter design.

References

[1] IEEE. *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979. Algorithm 5.2.

Purpose	Window-based finite impulse response filter design—arbitrary response.
Syntax	<pre> b = fir2(n, f, m) b = fir2(n, f, m, window) b = fir2(n, f, m, npt) b = fir2(n, f, m, npt, window) b = fir2(n, f, m, npt, lap) b = fir2(n, f, m, npt, lap, window) </pre>
Description	<p><code>fir2</code> designs windowed digital FIR filters with arbitrarily shaped frequency response. (For standard lowpass, bandpass, highpass, and bandstop configurations, use <code>fir1</code>.)</p> <p><code>b = fir2(n, f, m)</code> returns row vector <code>b</code> containing the $n+1$ coefficients of an order n FIR filter. The frequency-magnitude characteristics of this filter match those given by vectors <code>f</code> and <code>m</code>:</p> <ul style="list-style-type: none"> • <code>f</code> is a vector of frequency points in the range from 0 to 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The first point of <code>f</code> must be 0 and the last point 1. The frequency points must be in increasing order. • <code>m</code> is a vector containing the desired magnitude response at the points specified in <code>f</code>. • <code>f</code> and <code>m</code> must be the same length. • Duplicate frequency points are allowed, corresponding to steps in the frequency response. <p>Use <code>plot(f, m)</code> to view the filter shape.</p> <p>The output filter coefficients, <code>b</code>, are ordered in descending powers of z:</p> $b(z) = b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}$ <p><code>b = fir2(n, f, m, window)</code> uses the window specified in column vector <code>window</code> for the filter design. The vector <code>window</code> must be $n+1$ elements long. If no window is specified, <code>fir2</code> employs a Hamming window.</p>

`b = fir2(n, f, m, npt)` and

`b = fir2(n, f, m, npt, window)` specify the number of points `npt` for the grid onto which `fir2` interpolates the frequency response, with or without a `window` specification.

`b = fir2(n, f, m, npt, lap)` and

`b = fir2(n, f, m, npt, lap, window)` specify the size of the region, `lap`, that `fir2` inserts around duplicate frequency points, with or without a `window` specification.

See the “Algorithm” section for more on `npt` and `lap`.

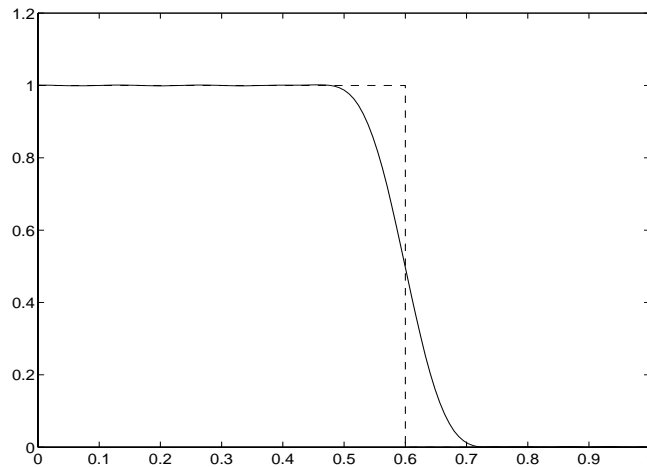
Algorithm

The desired frequency response is interpolated onto a dense, evenly spaced grid of length `npt`. `npt` is 512 by default. If two successive values of `f` are the same, a region of `lap` points is set up around this frequency to provide a smooth but steep transition in the requested frequency response. By default, `lap` is 25. The filter coefficients are obtained by applying an inverse fast Fourier transform to the grid and multiplying by a window; by default, this is a Hamming window.

Example

Design a 30th-order lowpass filter and overplot the desired frequency response with the actual frequency response:

```
f = [0 0.6 0.6 1]; m = [1 1 0 0];
b = fir2(30, f, m);
[h, w] = freqz(b, 1, 128);
plot(f, m, w/pi, abs(h))
```

**See Also**

butter	Butterworth analog and digital filter design.
cheby1	Chebyshev type I filter design (passband ripple).
cheby2	Chebyshev type II filter design (stopband ripple).
ellip	Elliptic (Cauer) filter design.
fir1	Window-based finite impulse response filter design—standard response.
maxflat	Generalized digital Butterworth filter design.
remez	Parks-McClellan optimal FIR filter design.
yulewalk	Recursive digital filter design.

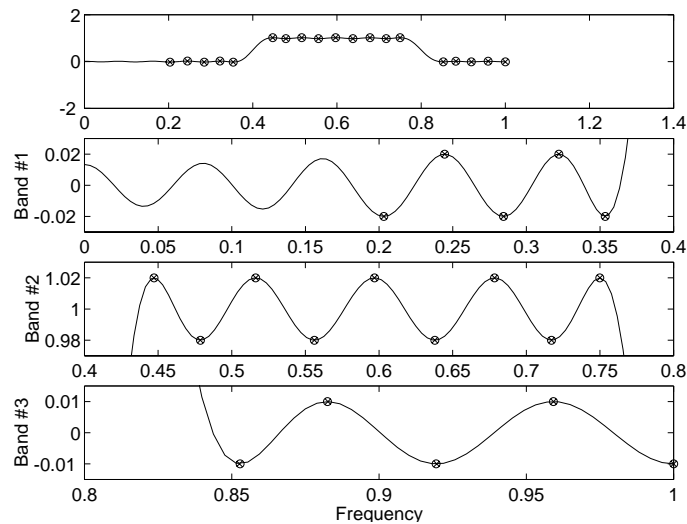
fircls

Purpose	Constrained least square FIR filter design for multiband filters.
Syntax	<pre>b = fircls(n, f, amp, up, lo) fircls(n, f, amp, up, lo, 'design_flag')</pre>
Description	<p><code>b = fircls(n, f, amp, up, lo)</code> generates a length $n+1$ linear phase FIR filter <code>b</code>. The frequency-magnitude characteristics of this filter match those given by vectors <code>f</code> and <code>amp</code>:</p> <ul style="list-style-type: none">• <code>f</code> is a vector of transition frequencies in the range from 0 to 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The first point of <code>f</code> must be 0 and the last point 1. The frequency points must be in increasing order.• <code>amp</code> is a vector describing the piecewise constant desired amplitude of the frequency response. The length of <code>amp</code> is equal to the number of bands in the response and should be equal to $\text{length}(f) - 1$.• <code>up</code> and <code>lo</code> are vectors with the same length as <code>amp</code>. They define the upper and lower bounds for the frequency response in each band. <p><code>fircls(n, f, amp, up, lo, 'design_flag')</code> enables you to monitor the filter design, where <i>design_flag</i> can be</p> <ul style="list-style-type: none">• <code>trace</code>, for a textual display of the design table used in the design• <code>plots</code>, for plots of the filter's magnitude, group delay, and zeros and poles• <code>both</code>, for both the textual display and plots

Example

Design an order 50 bandpass filter:

```
n = 50;
f = [0 0.4 0.8 1];
amp = [0 1 0];
up = [0.02 1.02 0.01];
lo = [-0.02 0.98 -0.01];
b = firls(n, f, amp, up, lo, 'plots') %plots magnitude response
```



NOTE Normally, the lower value in the stopband will be specified as negative. By setting `lo` equal to 0 in the stopbands, a nonnegative frequency response amplitude can be obtained. Such filters can be spectrally factored to obtain minimum phase filters.

Algorithm

The algorithm is a multiple exchange algorithm that uses Lagrange multipliers and Kuhn-Tucker conditions on each iteration.

See Also	<code>fi rcls1</code>	Constrained least square filter design for lowpass and highpass linear phase FIR filters.
	<code>fi rls</code>	Least square linear-phase FIR filter design.
	<code>remez</code>	Parks-McClellan optimal FIR filter design.
References	[1] Selesnick, I.W., M. Lang, and C.S. Burrus. "Constrained Least Square Design of FIR Filters without Specified Transition Bands." <i>Proceedings of the IEEE Int. Conf. Acoust., Speech, Signal Processing</i> . Vol. 2 (May 1995). Pgs. 1260-1263.	
	[2] Selesnick, I.W., M. Lang, and C.S. Burrus. "Constrained Least Square Design of FIR Filters without Specified Transition Bands." <i>IEEE Transactions on Signal Processing</i> , Vol. 44, No. 8 (August 1996).	

Purpose	Constrained least square filter design for lowpass and highpass linear phase FIR filters.
Syntax	<pre> b = fircls1(n, wo, dp, ds) b = fircls1(n, wo, dp, ds, 'hi gh') b = fircls1(n, wo, dp, ds, wt) b = fircls1(n, wo, dp, ds, wt, 'hi gh') b = fircls1(n, wo, dp, ds, wp, ws, k) b = fircls1(n, wo, dp, ds, wp, ws, k, 'hi gh') b = fircls1(n, wo, dp, ds, ..., 'desi gn_flag') </pre>
Description	<p><code>b = fircls1(n, wo, dp, ds)</code> generates a lowpass FIR filter <code>b</code>. <code>n+1</code> is the filter length, <code>wo</code> is the normalized cutoff frequency in the range between 0 and 1 (where 1 corresponds to half the sampling frequency, that is, the Nyquist frequency), <code>dp</code> is the maximum passband deviation from 1 (passband ripple), and <code>ds</code> is the maximum stopband deviation from 0 (stopband ripple).</p> <p><code>b = fircls1(n, wo, dp, ds, 'hi gh')</code> generates a highpass FIR filter <code>b</code>.</p> <p><code>b = fircls1(n, wo, dp, ds, wt)</code> and</p> <p><code>b = fircls1(n, wo, dp, ds, wt, 'hi gh')</code> specify a frequency <code>wt</code> above which (for <code>wt > wo</code>) or below which (for <code>wt < wo</code>) the filter is guaranteed to meet the given band criterion. This will help you design a filter that meets a passband or stopband edge requirement. There are four cases:</p> <ul style="list-style-type: none"> • Lowpass: <ul style="list-style-type: none"> - $0 < wt < wo < 1$: the amplitude of the filter is within <code>dp</code> of 1 over the frequency range $0 < \omega < wt$. - $0 < wo < wt < 1$: the amplitude of the filter is within <code>ds</code> of 0 over the frequency range $wt < \omega < 1$. • Highpass: <ul style="list-style-type: none"> - $0 < wt < wo < 1$: the amplitude of the filter is within <code>ds</code> of 0 over the frequency range $0 < \omega < wt$. - $0 < wo < wt < 1$: the amplitude of the filter is within <code>dp</code> of 1 over the frequency range $wt < \omega < 1$.

`b = fircls1(n, wo, dp, ds, wp, ws, k)` generates a lowpass FIR filter `b` with a weighted function. `n+1` is the filter length, `wo` is the normalized cutoff frequency, `dp` is the maximum passband deviation from 1 (passband ripple), and `ds` is the maximum stopband deviation from 0 (stopband ripple). `wp` is the passband edge of the L2 weight function and `ws` is the stopband edge of the L2 weight function, where $w_p < w_o < w_s$. `k` is the ratio (passband L2 error)/(stopband L2 error):

$$\frac{\int_0^{w_p} |A(\omega) - D(\omega)|^2 d\omega}{\int_{w_s}^{\pi} |A(\omega) - D(\omega)|^2 d\omega} = k$$

`b = fircls1(n, wo, dp, ds, wp, ws, k, 'high')` generates a highpass FIR filter `b` with a weighted function, where $w_s < w_o < w_p$.

`b = fircls1(n, wo, dp, ds, ..., 'design_flag')` enables you to monitor the filter design, where *design_flag* can be

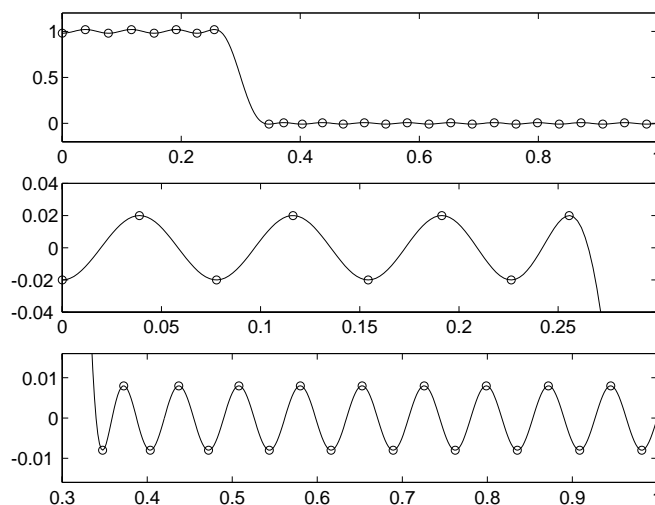
- `trace`, for a textual display of the design table used in the design
- `plots`, for plots of the filter's magnitude, group delay, and zeros and poles
- `both`, for both the textual display and plots

NOTE In the design of very narrow band filters with small `dp` and `ds`, there may not exist a filter of the given length that meets the specifications.

Example

Design an order 55 lowpass filter with a cutoff frequency at 0.3:

```
n = 55; wo = 0.3;
dp = 0.02; ds = 0.008;
b = fircls1(n, wo, dp, ds, 'plots'); % plot magnitude response
```

**Algorithm**

The algorithm is a multiple exchange algorithm that uses Lagrange multipliers and Kuhn-Tucker conditions on each iteration.

See Also

<p><code>fircls</code></p> <p><code>firls</code></p> <p><code>remez</code></p>	<p>Constrained least square FIR filter design for multiband filters.</p> <p>Least square linear-phase FIR filter design.</p> <p>Parks-McClellan optimal FIR filter design.</p>
--------------------------------------------------------------------------------	--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

References

- [1] Selesnick, I.W., M. Lang, and C.S. Burrus. "Constrained Least Square Design of FIR Filters without Specified Transition Bands." *Proceedings of the IEEE Int. Conf. Acoust., Speech, Signal Processing*. Vol. 2 (May 1995). Pgs. 1260-1263.
- [2] Selesnick, I.W., M. Lang, and C.S. Burrus. "Constrained Least Square Design of FIR Filters without Specified Transition Bands." *IEEE Transactions on Signal Processing*, Vol. 44, No. 8 (August 1996).

firls

Purpose Least square linear-phase FIR filter design.

Syntax

```
b = firls(n, f, a)
b = firls(n, f, a, w)
b = firls(n, f, a, 'ftype')
b = firls(n, f, a, w, 'ftype')
```

Description `firls` designs a linear-phase FIR filter that minimizes the weighted, integrated squared error between an ideal piecewise linear function and the magnitude response of the filter over a set of desired frequency bands.

`b = firls(n, f, a)` returns row vector `b` containing the $n+1$ coefficients of the order n FIR filter whose frequency-amplitude characteristics approximately match those given by vectors `f` and `a`. The output filter coefficients, or “taps,” in `b` obey the symmetry relation

$$b(k) = b(n+2-k), \quad k = 1, \dots, n+1$$

These are type I (n odd) and type II (n even) linear-phase filters. Vectors `f` and `a` specify the frequency-amplitude characteristics of the filter:

- `f` is a vector of pairs of frequency points, specified in the range between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The frequencies must be in increasing order. Duplicate frequency points are allowed and, in fact, can be used to design a filter exactly the same as those returned by the `fir1` and `fir2` functions with a rectangular or boxcar window.
- `a` is a vector containing the desired amplitude at the points specified in `f`.

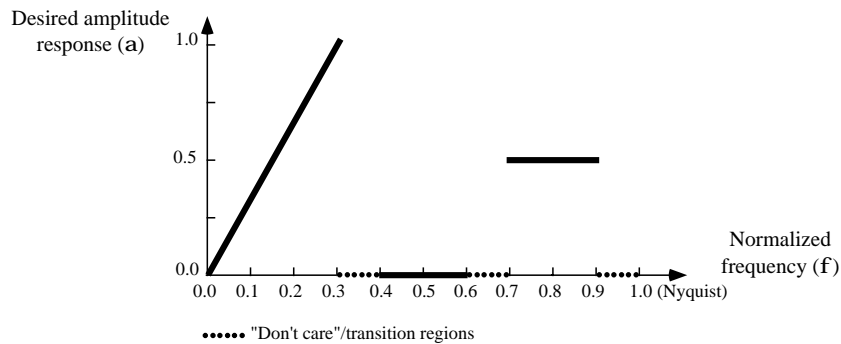
The desired amplitude function at frequencies between pairs of points ($f(k)$, $f(k+1)$) for k odd is the line segment connecting the points ($f(k)$, $a(k)$) and ($f(k+1)$, $a(k+1)$).

The desired amplitude function at frequencies between pairs of points ($f(k)$, $f(k+1)$) for k even is unspecified. These are transition or “don’t care” regions.

- `f` and `a` are the same length. This length must be an even number.

The relationship between the `f` and `a` vectors in defining a desired amplitude response is

```
f = [0 .3 .4 .6 .7 .9]
a = [0 1 0 0 .5 .5]
```



`b = firls(n, f, a, w)` uses the weights in vector `w` to weight the fit in each frequency band. The length of `w` is half the length of `f` and `a`, so there is exactly one weight per band.

`b = firls(n, f, a, 'ftype')` and

`b = firls(n, f, a, w, 'ftype')` specify a filter type, where `ftype` is

- `hilbert` for linear-phase filters with odd symmetry (type III and type IV)
The output coefficients in `b` obey the relation $b(k) = -b(n + 2 - k)$, $k = 1, \dots, n + 1$. This class of filters includes the Hilbert transformer, which has a desired amplitude of 1 across the entire band.
- `differentiator` for type III and type IV filters, using a special weighting technique

For nonzero amplitude bands, the integrated squared error has a weight of $(1/f)^2$ so that the error at low frequencies is much smaller than at high frequencies. For FIR differentiators, which have an amplitude characteristic proportional to frequency, the filters minimize the relative integrated squared error (the integral of the square of the ratio of the error to the desired amplitude).

Examples

Design an order 255 lowpass filter with transition band:

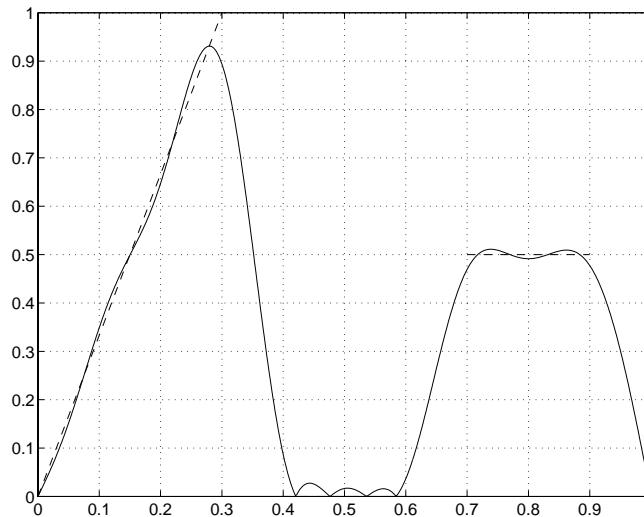
```
b = firls(255, [0 0.25 0.3 1], [1 1 0 0]);
```

Design a 31 coefficient differentiator:

```
b = firls(30, [0 0.9], [0 0.9], 'differentiator');
```

Design a 24th-order anti-symmetric filter with piecewise linear passbands and plot the desired and actual frequency response:

```
F = [0 0.3 0.4 0.6 0.7 0.9];  
A = [0 1 0 0 0.5 0.5];  
b = firls(24, F, A, 'hilbert');  
for i=1:2:6,  
    plot([F(i) F(i+1)], [A(i) A(i+1)], '- -'), hold on  
end  
[H, f] = freqz(b, 1, 512, 2);  
plot(f, abs(H)), grid on, hold off
```



Algorithm

Reference [1] describes the theoretical approach that `firls` takes. The function solves a system of linear equations involving an inner product matrix of size roughly $n/2$ using MATLAB's `\` operator.

This function designs type I, II, III, and IV linear-phase filters. Type I and II are the defaults for `n` even and odd respectively, while the `'hilbert'` and `'differentiator'` flags produce type III (`n` even) and IV (`n` odd) filters. The various filter types have different symmetries and constraints on their frequency responses (see [2] for details).

Linear Phase Filter type	Filter Order <code>n</code>	Symmetry of Coefficients	Response $H(f)$, $f = 0$	Response $H(f)$, $f = 1$ (Nyquist)
Type I	Even	even:	No restriction	No restriction
Type II	Odd	$b(k) = b(n + 2 - k), \quad k = 1, \dots, n + 1$	No restriction	$H(1) = 0$
Type III	Even	odd:	$H(0) = 0$	$H(1) = 0$
Type IV	Odd	$b(k) = -b(n + 2 - k), \quad k = 1, \dots, n + 1$	$H(0) = 0$	No restriction

Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments are used:

- `F` must be even length.
- `F` and `A` must be equal lengths.
- Requires `symmetry` to be `'hilbert'` or `'differentiator'`.
- Requires one weight per band.
- Frequencies in `F` must be nondecreasing.
- Frequencies in `F` must be in range `[0, 1]`.

A more serious warning message is

Warning: `Matrix` is close to singular or badly scaled.

This tends to happen when the filter length times the transition width grows large. In this case, the filter coefficients `b` might not represent the desired filter. You can check the filter by looking at its frequency response.

See Also	f i r 1	Window-based finite impulse response filter design—standard response.
	f i r 2	Window-based finite impulse response filter design—arbitrary response.
	f i r r c o s	Raised cosine FIR filter design.
	r e m e z	Parks-McClellan optimal FIR filter design.

References

[1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Pgs. 54-83.

[2] Oppenheim, A.V., and R.W. Schafer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 256-266.

Purpose Raised cosine FIR filter design.

```
b = firrcos(n, F0, df, Fs)
```

```
b = firrcos(n, F0, df)
```

Description `firrcos(n, F0, df, Fs)` returns an order n lowpass linear-phase FIR filter with a raised cosine transition band. The filter has cutoff frequency $F0$, transition width df , and sampling frequency Fs , all in Hertz. $F0$ must be between 0 and $Fs/2$. df must be small enough so that $F0 \pm df/2$ is between 0 and $Fs/2$. The filter order n must be even.

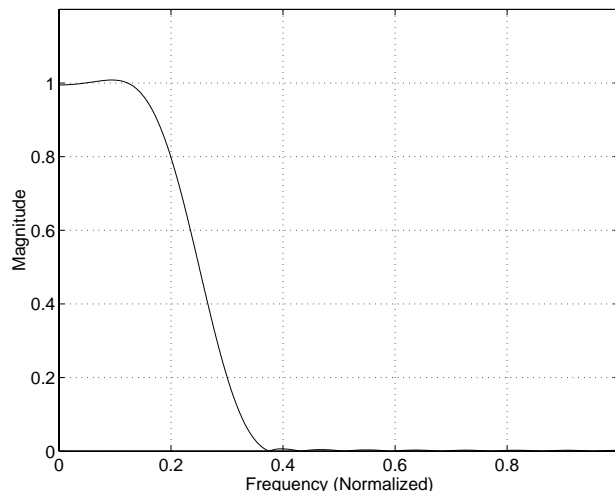
`firrcos(n, F0, df)` uses a default sampling frequency of $Fs = 2$.

Example Design an order 20 raised cosine FIR filter with cutoff frequency 0.25 of the Nyquist frequency and a transition width of 0.25:

```
h = firrcos(20, 0.25, 0.25);
```

```
H = fft(h, 1024);
```

```
plot((0:1023)/1024*2, abs(H), axis([0 1 0 1.2]), grid)
```



Remarks `firrcos` minimizes the integral squared error in the frequency domain.

See Also `firls` Least square linear-phase FIR filter design.
`remez` Parks-McClellan optimal FIR filter design.

freqs

Purpose Frequency response of analog filters.

Syntax

```
h = freqs(b, a, w)
[h, w] = freqs(b, a)
[h, w] = freqs(b, a, n)
freqs(b, a)
```

Description `freqs` returns the complex frequency response $H(jw)$ (Laplace transform) of an analog filter:

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^{nb} + b(2)s^{(nb-1)} + \dots + b(nb+1)}{a(1)s^{na} + a(2)s^{(na-1)} + \dots + a(na+1)}$$

given the numerator and denominator coefficients in vectors `b` and `a`.

`h = freqs(b, a, w)` returns the complex frequency response of the analog filter specified by coefficient vectors `b` and `a`. `freqs` evaluates the frequency response along the imaginary axis in the complex plane at the frequencies specified in real vector `w`.

`[h, w] = freqs(b, a)` automatically picks a set of 200 frequency points `w` on which to compute the frequency response `h`.

`[h, w] = freqs(b, a, n)` picks `n` frequencies on which to compute the frequency response `h`.

`freqs` with no output arguments plots the magnitude and phase response versus frequency in the current figure window.

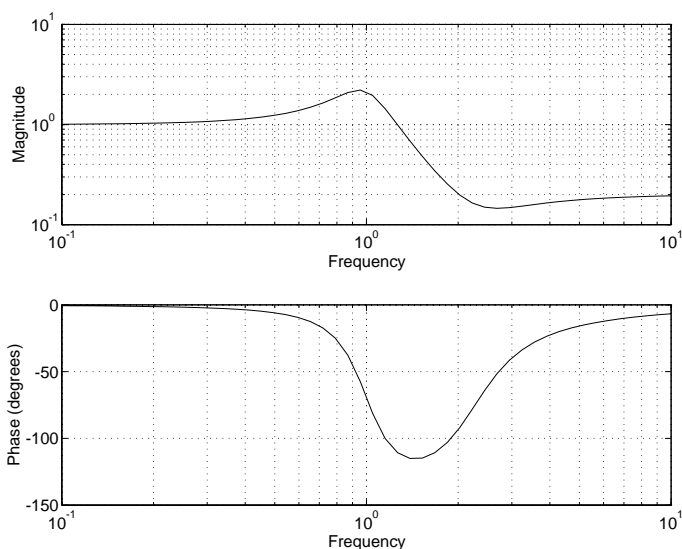
`freqs` works only for real input systems and positive frequencies.

Example

Find and graph the frequency response of the transfer function given by

$$H(s) = \frac{0.2s^2 + 0.3s + 1}{s^2 + 0.4s + 1}$$

```
a = [1 0.4 1];
b = [0.2 0.3 1];
w = logspace(-1, 1);
freqs(b, a, w)
```



You can also create the plot with

```
h = freqs(b, a, w);
mag = abs(h);
phase = angle(h);
subplot(2, 1, 1), loglog(w, mag)
subplot(2, 1, 2), semilogx(w, phase)
```

To convert to Hertz, degrees, and decibels, use

```
f = w/(2*pi);
mag = 20*log10(mag);
phase = phase*180/pi;
```

freqs

Algorithm freqs evaluates the polynomials at each frequency point, then divides the numerator response by the denominator response:

```
s = i*w;  
h = polyval(b,s) ./ polyval(a,s);
```

See Also	abs	Absolute value (magnitude).
	angle	Phase angle.
	freqz	Frequency response of digital filters.
	invfreqs	Continuous-time (analog) filter identification from frequency data.
	logspace	Generate logarithmically spaced vectors (see the online <i>MATLAB Function Reference</i>).
	polyval	Polynomial evaluation (see the online <i>MATLAB Function Reference</i>).

Purpose	Frequency spacing for frequency response.	
Syntax	<pre> f = freqspace(n) f = freqspace(n, 'whole') [f1, f2] = freqspace(n) [f1, f2] = freqspace([m n]) [x1, y1] = freqspace(n, 'meshgrid') [x1, y1] = freqspace([m n], 'meshgrid') </pre>	
Description	<p>freqspace returns the implied frequency range for equally spaced frequency responses. This is useful when creating frequency vectors for use with freqz.</p> <p>$f = \text{freqspace}(n)$ returns the frequency vector f assuming n evenly spaced points around the unit circle. For n even or odd, f is $(0: 2/n: 1)$. For n even, freqspace returns $(n + 2)/2$ points. For N odd, it returns $(n + 1)/2$ points.</p> <p>$f = \text{freqspace}(n, 'whole')$ returns n evenly spaced points around the whole unit circle. In this case, f is $0: 2/n: 2*(n-1)/n$.</p> <p>$[f1, f2] = \text{freqspace}(n)$ returns the two-dimensional frequency vectors $f1$ and $f2$ for an n-by-n matrix. For n odd, both $f1$ and $f2$ are $[-1 + 1/n: 2/n: 1-1/n]$. For n even, both $f1$ and $f2$ are $[-1: 2/n: 1-2/n]$.</p> <p>$[f1, f2] = \text{freqspace}([m\ n])$ returns the two-dimensional frequency vectors $f1$ and $f2$ for an m-by-n matrix.</p> <p>$[x1, y1] = \text{freqspace}(n, 'meshgrid')$ and</p> <p>$[x1, y1] = \text{freqspace}([m\ n], 'meshgrid')$ are equivalent to</p> <pre> [f1, f2] = freqspace(...); [x1, y1] = meshgrid(f1, f2); </pre> <p>See the online <i>MATLAB Function Reference</i> for details on the meshgrid function.</p>	
See Also	<pre> freqz invfreqz </pre>	<p>Frequency response of digital filters.</p> <p>Discrete-time filter identification from frequency data.</p>

freqz

Purpose Frequency response of digital filters.

Syntax

```
[h, w] = freqz(b, a, n)
[h, f] = freqz(b, a, n, Fs)
[h, w] = freqz(b, a, n, 'whole')
[h, f] = freqz(b, a, n, 'whole', Fs)
h = freqz(b, a, w)
h = freqz(b, a, f, Fs)
freqz(b, a)
```

Description `freqz` returns the complex frequency response $H(e^{jw})$ of a digital filter, given the numerator and denominator coefficients in vectors `b` and `a`.

`[h, w] = freqz(b, a, n)` returns the n -point complex frequency response of the digital filter

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}}$$

given the coefficient vectors `b` and `a`. `freqz` returns both `h`, the complex frequency response, and `w`, a vector containing the n frequency points. `freqz` evaluates the frequency response at n points equally spaced around the upper half of the unit circle, so `w` contains n points between 0 and π .

It is best, although not necessary, to choose a value for n that is an exact power of two, because this allows fast computation using an FFT algorithm. If you do not specify a value for n , it defaults to 512.

`[h, f] = freqz(b, a, n, Fs)` specifies a positive sampling frequency `Fs`, in Hertz. It returns a vector `f` containing the actual frequency points between 0 and `Fs/2` at which it calculated the frequency response. `f` is of length n .

`[h, w] = freqz(b, a, n, 'whole')` and

`[h, f] = freqz(b, a, n, 'whole', Fs)` use n points around the whole unit circle (from 0 to 2π , or from 0 to `Fs`).

`h = freqz(b, a, w)` returns the frequency response at the frequencies in vector `w`. These frequencies must be between 0 and 2π .

`h = freqz(b, a, f, Fs)` returns the frequency response at the frequencies in vector `f`, where the elements of `f` are between 0 and `Fs`.

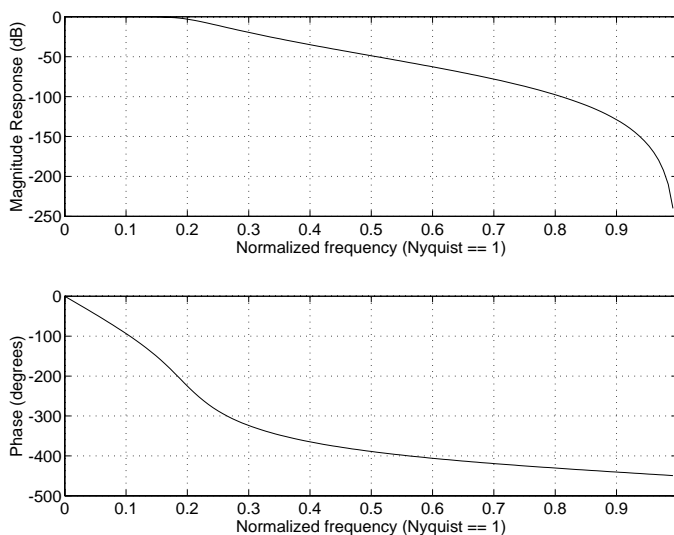
`freqz` with no output arguments plots the magnitude and phase response versus frequency in the current figure window.

`freqz` works for both real and complex input systems.

Example

Plot the magnitude and phase response of a Butterworth filter:

```
[b, a] = butter(5, 0.2);
freqz(b, a, 128)
```



Algorithm

`freqz` uses an FFT algorithm when argument `n` is present. It computes the frequency response as the ratio of the transformed numerator and denominator coefficients, padded with zeros to the desired length:

$$h = \text{fft}(b, n) ./ \text{fft}(a, n)$$

If `n` is not a power of two, the FFT algorithm is not as efficient and may cause long computation times.

When a frequency vector `w` or `f` is present, or if `n` is less than $\max(\text{length}(b), \text{length}(a))$, `freqz` evaluates the polynomials at each

frequency point using Horner's method of polynomial evaluation and then divides the numerator response by the denominator response.

See Also

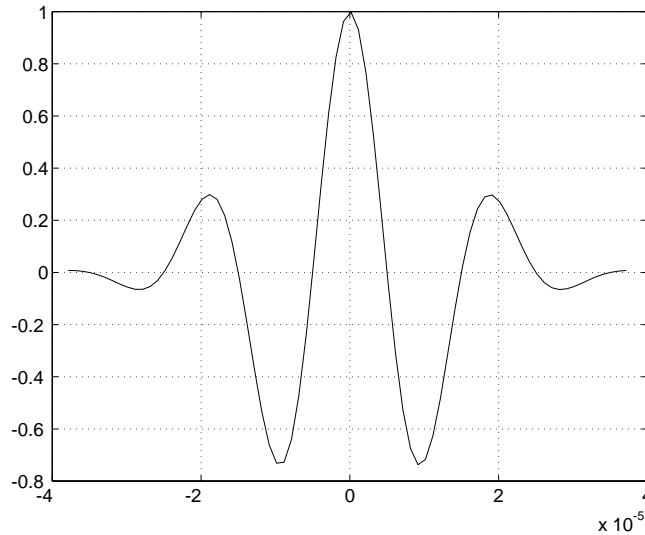
<code>abs</code>	Absolute value (magnitude).
<code>angle</code>	Phase angle.
<code>fft</code>	One-dimensional fast Fourier transform.
<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
<code>freqs</code>	Frequency response of analog filters.
<code>impz</code>	Impulse response of digital filters.
<code>invfreqz</code>	Discrete-time filter identification from frequency data.
<code>logspace</code>	Generate logarithmically spaced vectors (see the online <i>MATLAB Function Reference</i>).

Purpose	Gaussian-modulated sinusoidal pulse generator.
Syntax	<pre>yi = gauspuls(t, fc, bw) yi = gauspuls(t, fc, bw, bwr) [yi, yq] = gauspuls(...) [yi, yq, ye] = gauspuls(...) tc = gauspuls('cutoff', fc, bw, bwr, tpe)</pre>
Description	<p><code>gauspuls</code> generates Gaussian-modulated sinusoidal pulses.</p> <p><code>yi = gauspuls(t, fc, bw)</code> returns a unity-amplitude Gaussian RF pulse at the times indicated in array <code>t</code>, with a center frequency <code>fc</code> in Hertz and a fractional bandwidth <code>bw</code>, which must be greater than 0. The default value for <code>fc</code> is 1000 Hz and for <code>bw</code> is 0.5.</p> <p><code>yi = gauspuls(t, fc, bw, bwr)</code> returns a unity-amplitude Gaussian RF pulse with a fractional bandwidth of <code>bw</code> as measured at a level of <code>bwr</code> dB with respect to the normalized signal peak. The fractional bandwidth reference level <code>bwr</code> must be less than 0, because it indicates a reference level less than the peak (unity) envelope amplitude. The default value for <code>bwr</code> is -6 dB.</p> <p><code>[yi, yq] = gauspuls(...)</code> returns both the in-phase and quadrature pulses.</p> <p><code>[yi, yq, ye] = gauspuls(...)</code> returns the RF signal envelope.</p> <p><code>tc = gauspuls('cutoff', fc, bw, bwr, tpe)</code> returns the cutoff time <code>tc</code> (greater than or equal to 0) at which the trailing pulse envelope falls below <code>tpe</code> dB with respect to the peak envelope amplitude. The trailing pulse envelope level <code>tpe</code> must be less than 0, because it indicates a reference level less than the peak (unity) envelope amplitude. The default value for <code>tpe</code> is -60 dB.</p>
Remarks	Default values are substituted for empty or omitted trailing input arguments.

Example

Plot a 50 kHz Gaussian RF pulse with 60% bandwidth, sampled at a rate of 1 MHz. Truncate the pulse where the envelope falls 40 dB below the peak:

```
tc = gauspuls('cutoff', 50e3, 0.6, [], -40);  
t = -tc : 1e-6 : tc;  
yi = gauspuls(t, 50e3, 0.6);  
plot(t, yi)
```



See Also

<code>chirp</code>	Swept-frequency cosine generator.
<code>cos</code>	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
<code>diric</code>	Dirichlet or periodic sinc function.
<code>pulstran</code>	Pulse train generator.
<code>rectpuls</code>	Sampled aperiodic rectangle generator.
<code>sawtooth</code>	Sawtooth or triangle wave generator.
<code>sin</code>	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
<code>sinc</code>	Sinc or $\sin(\pi t)/\pi t$ function.
<code>square</code>	Square wave generator.
<code>tripuls</code>	Sampled aperiodic triangle generator.

Purpose Average filter delay (group delay).

Syntax

```
[gd, w] = grpdel ay(b, a, n)
[gd, f] = grpdel ay(b, a, n, Fs)
[gd, w] = grpdel ay(b, a, n, 'whole')
[gd, f] = grpdel ay(b, a, n, 'whole', Fs)
gd = grpdel ay(b, a, w)
gd = grpdel ay(b, a, f, Fs)
grpdel ay(b, a)
```

Description The *group delay* of a filter is a measure of the average delay of the filter as a function of frequency. It is the negative first derivative of the phase response of the filter. If the complex frequency response of a filter is $H(e^{j\omega})$, then the group delay is

$$\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega}$$

where ω is frequency and θ is the phase angle of $H(e^{j\omega})$.

`[gd, w] = grpdel ay(b, a, n)` returns the n -point group delay, $\tau_g(\omega)$, of the digital filter

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}}$$

given the numerator and denominator coefficients in vectors `b` and `a`. `grpdel ay` returns both `gd`, the group delay, and `w`, a vector containing the n frequency points in radians. `grpdel ay` evaluates the group delay at n points equally spaced around the upper half of the unit circle, so `w` contains n points between 0 and π . A value for n that is an exact power of two allows fast computation using an FFT algorithm.

`[gd, f] = grpdel ay(b, a, n, Fs)` specifies a positive sampling frequency `Fs` in Hertz. It returns a length n vector `f` containing the actual frequency points at which the group delay is calculated, also in Hertz. `f` contains n points between 0 and `Fs/2`.

`[gd, w] = grpdelay(b, a, n, 'whole')` and

`[gd, f] = grpdelay(b, a, n, 'whole', Fs)` use `n` points around the whole unit circle (from 0 to 2π , or from 0 to F_s).

`gd = grpdelay(b, a, w)` and

`gd = grpdelay(b, a, f, Fs)` return the group delay evaluated at the points in `w` (in radians) or `f` (in Hertz), respectively, where F_s is the sampling frequency in Hertz.

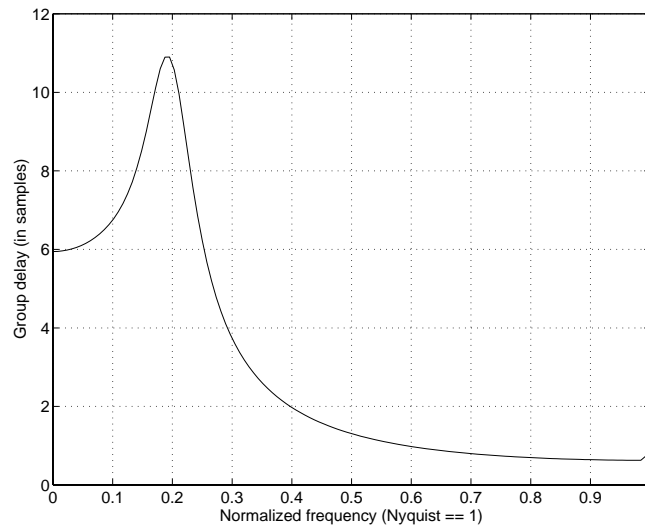
`grpdelay` with no output arguments plots the group delay versus frequency in the current figure window.

`grpdelay` works for both real and complex input systems.

Examples

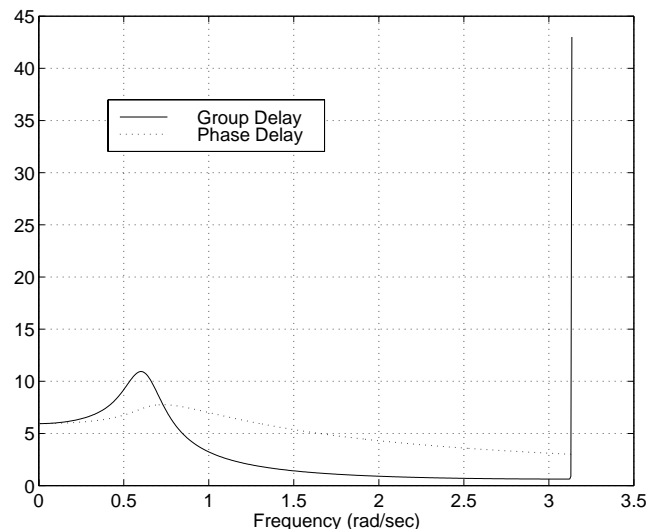
Plot the group delay of Butterworth filter $b(z)/a(z)$:

```
[b, a] = butter(6, 0.2);  
grpdelay(b, a, 128)
```



Plot both the group and phase delays of a system on the same graph:

```
gd = grpdelay(b, a, 512);
gd(1) = []; % avoid NaNs
[h, w] = freqz(b, a, 512); h(1) = []; w(1) = [];
pd = -unwrap(angle(h)) ./ w;
plot(w, gd, w, pd, 'r')
```



Algorithm

grpdelay multiplies the filter coefficients by a unit ramp. After Fourier transformation, this process corresponds to differentiation.

See Also

cceps	Complex cepstral analysis.
fft	One-dimensional fast Fourier transform.
freqz	Frequency response of digital filters.
hilbert	Hilbert transform.
icceps	Inverse complex cepstrum.
rcceps	Real cepstrum and minimum phase reconstruction.

hamming

Purpose	Hamming window.	
Syntax	$w = \text{hamming}(n)$ $w = \text{hamming}(n, \text{sflag})$	
Description	$w = \text{hamming}(n)$ returns an n -point symmetrically sampled Hamming window in the column vector w . n should be a nonnegative integer. The coefficients of a Hamming window are $w[k+1] = 0.54 - 0.46 \cos\left(2\pi \frac{k}{n-1}\right), \quad k = 0, \dots, n-1$ $w = \text{hamming}(n, \text{sflag})$ returns an n -point Hamming window using the window sampling specified by sflag , which can be either 'periodic' or 'symmetric' (the default). When 'periodic' is specified, hamming computes a length $n+1$ window and returns the first n points.	
Diagnostics	An error message is displayed when incorrect arguments are used: Order cannot be less than zero. Sampling must be either 'symmetric' or 'periodic'. A warning message is displayed for noninteger n : Warning: Rounding order to nearest integer.	
See Also	bartlett blackman boxcar chebwin hanning kaiser triang	Bartlett window. Blackman window. Rectangular window. Chebyshev window. Hanning window. Kaiser window. Triangular window.
References	[1] Oppenheim, A.V., and R.W. Schaffer, <i>Discrete-Time Signal Processing</i> . Englewood Cliffs, NJ: Prentice-Hall, 1989.	

Purpose Hanning window.

Syntax
`w = hanning(n)`
`w = hanning(n, sflag)`

Description `w = hanning(n)` returns an n -point symmetrically sampled Hanning window in the column vector `w`. n should be a nonnegative integer. The coefficients of a Hanning window are

$$w[k] = 0.5 \left(1 - \cos \left(2\pi \frac{k}{n+1} \right) \right), \quad k = 1, \dots, n$$

`w = hanning(n, sflag)` returns an n -point Hanning window using the window sampling specified by `sflag`, which can be either `'periodic'` or `'symmetric'` (the default). When `'periodic'` is specified, `hanning` computes a length $n+1$ window and returns the first n points.

Diagnostics An error message is displayed when incorrect arguments are used:

Order cannot be less than zero.
 Sampling must be either `'symmetric'` or `'periodic'`.

A warning message is displayed for noninteger n :

Warning: Rounding order to nearest integer.

See Also

<code>bartlett</code>	Bartlett window.
<code>blackman</code>	Blackman window.
<code>boxcar</code>	Rectangular window.
<code>chebwin</code>	Chebyshev window.
<code>hamming</code>	Hamming window.
<code>kaiser</code>	Kaiser window.
<code>triang</code>	Triangular window.

References [1] Oppenheim, A.V., and R.W. Schaffer, *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

hilbert

Purpose Hilbert transform.

Syntax `y = hilbert(x)`

Description `y = hilbert(x)` returns a complex helical sequence, sometimes called the *analytic signal*, from a real data sequence. The analytic signal has a real part, which is the original data, and an imaginary part, which contains the Hilbert transform. The imaginary part is a version of the original real sequence with a 90° phase shift. Sines are therefore transformed to cosines and vice versa. The Hilbert transformed series has the same amplitude and frequency content as the original real data and includes phase information that depends on the phase of the original data.

If `x` is a matrix, `y = hilbert(x)` operates columnwise on the matrix, finding the Hilbert transform of each column.

The Hilbert transform is useful in calculating instantaneous attributes of a time series, especially the amplitude and frequency. The instantaneous amplitude is the amplitude of the complex Hilbert transform; the instantaneous frequency is the time rate of change of the instantaneous phase angle. For a pure sinusoid, the instantaneous amplitude and frequency are constant. The instantaneous phase, however, is a sawtooth, reflecting the way in which the local phase angle varies linearly over a single cycle. For mixtures of sinusoids, the attributes are short term, or local, averages spanning no more than two or three points.

Reference [1] describes the Kolmogorov method for minimum phase reconstruction, which involves taking the Hilbert transform of the logarithm of the spectrum of a time series. The toolbox function `rceps` performs this reconstruction.

Algorithm The analytic signal for a sequence `x` has a *one-sided Fourier transform*, that is, negative frequencies are 0. To approximate the analytic signal, `hilbert` calculates the FFT of the input sequence, replaces those FFT coefficients that correspond to negative frequencies with zeros, and calculates the inverse FFT of the result.

In detail, `hilbert` uses a four-step algorithm:

- 1 It calculates the FFT of the input sequence, storing the result in a vector `y`. Before transforming, it zero pads the input sequence so its length `n` is the closest power of two, if necessary. This ensures the most efficient FFT computation.
- 2 It creates a vector `h` whose elements `h(i)` have the values
 - 1 for `i = 1, (n/2) + 1`
 - 2 for `i = 2, 3, ..., (n/2)`
 - 0 for `i = (n/2) + 2, ... , n`
- 3 It calculates the element-wise product of `y` and `h`.
- 4 It calculates the inverse FFT of the sequence obtained in step 3 and returns the first `n` elements of the result.

If the input data `x` is a matrix, `hilbert` operates in a similar manner, extending each step above to handle the matrix case.

See Also	<code>fft</code>	One-dimensional fast Fourier transform.
	<code>ifft</code>	One-dimensional inverse fast Fourier transform.
	<code>rceps</code>	Real cepstrum and minimum phase reconstruction.

References	[1] Claerbout, J.F. <i>Fundamentals of Geophysical Data Processing</i> . New York: McGraw-Hill, 1976. Pgs. 59-62.
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icceps

Purpose	Inverse complex cepstrum.	
Syntax	<code>x = icceps(xhat, nd)</code>	
Description	<code>x = icceps(xhat, nd)</code> returns the inverse complex cepstrum of the (assumed real) sequence <code>xhat</code> , removing <code>nd</code> samples of delay. If <code>xhat</code> was obtained with <code>cceps(x)</code> , then the amount of delay that was added to <code>x</code> was the element of <code>round(unwrap(angle(fft(x)))/pi)</code> corresponding to π radians.	
See Also	<code>cceps</code>	Complex cepstral analysis.
	<code>hilbert</code>	Hilbert transform.
	<code>rcceps</code>	Real cepstrum and minimum phase reconstruction.
	<code>unwrap</code>	Unwrap phase angles.
References	[1] Oppenheim, A.V., and R.W. Schafer. <i>Digital Signal Processing</i> . Englewood Cliffs, NJ: Prentice Hall, 1975.	

Purpose Inverse discrete cosine transform.

Syntax
`x = idct(y)`
`x = idct(y, n)`

Description The inverse discrete cosine transform reconstructs a sequence from its discrete cosine transform (DCT) coefficients. The `idct` function is the inverse of the `dct` function.

`x = idct(y)` returns the inverse discrete cosine transform of `y`

$$x(n) = \sum_{k=1}^N w(k) y(k) \cos \frac{\pi(2n-1)(k-1)}{2N}, \quad n = 1, \dots, N$$

where

$$w(k) = \begin{cases} \frac{1}{\sqrt{N}}, & k = 1 \\ \sqrt{\frac{2}{N}}, & 2 \leq k \leq N \end{cases}$$

and $N = \text{length}(x)$, which is the same as $\text{length}(y)$. The series is indexed from $n = 1$ and $k = 1$ instead of the usual $n = 0$ and $k = 0$ because MATLAB vectors run from 1 to N instead of from 0 to $N-1$.

`x = idct(y, n)` appends zeros or truncates the vector `y` to length `n` before transforming.

If `y` is a matrix, `idct` transforms its columns.

See Also

<code>dct</code>	Discrete cosine transform (DCT).
<code>dct2</code>	Two-dimensional DCT (see <i>Image Processing Toolbox User's Guide</i>).
<code>idct2</code>	Two-dimensional inverse DCT (see <i>Image Processing Toolbox User's Guide</i>).
<code>ifft</code>	One-dimensional inverse fast Fourier transform.

References

- [1] Jain, A.K. *Fundamentals of Digital Image Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.
- [2] Pennebaker, W.B., and J.L. Mitchell. *JPEG Still Image Data Compression Standard*. New York, NY: Van Nostrand Reinhold, 1993. Chapter 4.

Purpose	One-dimensional inverse fast Fourier transform.	
Syntax	$y = \text{ifft}(x)$ $y = \text{ifft}(x, n)$	
Description	<p><code>ifft</code> computes the inverse Fourier transform of a vector or array. This function implements the inverse transform given by</p> $x(n+1) = (1/N) \sum_{k=0}^{N-1} X(k+1) W_N^{-kn}$ <p>where $W_N = e^{-j(2\pi/N)}$ and $N = \text{length}(x)$. Note that the series is indexed as $n+1$ and $k+1$ instead of the usual n and k because MATLAB vectors run from 1 to N instead of from 0 to $N-1$.</p> <p>$y = \text{ifft}(x)$ is the inverse Fourier transform of vector x. If x is an array, y is the inverse FFT of each column of the matrix.</p> <p>$y = \text{ifft}(x, n)$ is the n-point inverse FFT. If the length of x is less than n, <code>ifft</code> pads x with trailing zeros to length n. If the length of x is greater than n, <code>ifft</code> truncates the sequence x. When x is an array, <code>ifft</code> adjusts the length of the columns in the same manner.</p> <p><code>ifft</code> is part of the standard MATLAB environment.</p>	
Algorithm	<code>ifft</code> is an M-file. The algorithm for <code>ifft</code> is the same as that for <code>fft</code> , except for a sign change and a scale factor of $n = \text{length}(x)$. The execution time is fastest when n is a power of two and slowest when n is a large prime.	
See Also	<code>fft</code> <code>fft2</code> <code>fftshift</code> <code>ifft2</code>	One-dimensional fast Fourier transform. Two-dimensional fast Fourier transform. Rearrange the outputs of <code>fft</code> and <code>fft2</code> . Two-dimensional inverse fast Fourier transform.

ifft2

Purpose	Two-dimensional inverse fast Fourier transform.	
Syntax	$Y = \text{ifft2}(X)$ $Y = \text{ifft2}(X, m, n)$	
Description	<p>$Y = \text{ifft2}(X)$ returns the two-dimensional inverse fast Fourier transform (FFT) of the array X. If X is a vector, Y has the same orientation as X.</p> <p>$Y = \text{ifft2}(X, m, n)$ truncates or zero pads X, if necessary, to create an m-by-n array before performing the inverse FFT. The result Y is also m-by-n.</p> <p>For any X, $\text{ifft2}(\text{fft2}(X))$ equals X to within roundoff error. If X is real, $\text{ifft2}(\text{fft2}(X))$ may have small imaginary parts.</p> <p>ifft is part of the standard MATLAB environment.</p>	
Algorithm	<p>The algorithm for ifft2 is the same as that for fft2, except for a sign change and scale factors of $[m\ n] = \text{size}(X)$. The execution time is fastest when m and n are powers of two and slowest when they are large primes.</p> <p>ifft2 is part of the standard MATLAB environment.</p>	
See Also	fft	One-dimensional fast Fourier transform.
	fft2	Two-dimensional fast Fourier transform.
	fftn	N -dimensional fast Fourier transform (see the online <i>MATLAB Function Reference</i>).
	fftshift	Rearrange the outputs of fft and fft2 .
	ifft	One-dimensional inverse fast Fourier transform.
	ifftn	N -dimensional inverse fast Fourier transform (see the online <i>MATLAB Function Reference</i>).

Purpose	Impulse invariance method of analog-to-digital filter conversion.
Syntax	<pre>[bz, az] = i mpi nvar (b, a, Fs) [bz, az] = i mpi nvar (b, a) [bz, az] = i mpi nvar (b, a, Fs, tol)</pre>
Description	<p><code>[bz, az] = i mpi nvar (b, a, Fs)</code> creates a digital filter with numerator and denominator coefficients <code>bz</code> and <code>az</code>, respectively, whose impulse response is equal to the impulse response of the analog filter with coefficients <code>b</code> and <code>a</code>, scaled by $1/F_s$.</p> <p><code>[bz, az] = i mpi nvar (b, a)</code> uses the default value of 1 Hz for F_s.</p> <p><code>[bz, az] = i mpi nvar (b, a, Fs, tol)</code> uses the tolerance specified by <code>tol</code> to determine whether poles are repeated. A larger tolerance increases the likelihood that <code>i mpi nvar</code> will consider nearby poles to be repeated. The default is 0.001, or 0.1% of a pole's magnitude. Note that the accuracy of the pole values is still limited to the accuracy obtainable by the <code>roots</code> function.</p>
Example	<p>Convert an analog lowpass filter to a digital filter using <code>i mpi nvar</code> with a sampling frequency of 10 Hz:</p> <pre>[b, a] = butter (4, 0.3, ' s ') ; [bz, az] = i mpi nvar (b, a, 10)</pre> <p><code>bz =</code></p> <pre>1. 0e- 006 * - 0. 0000 0. 1324 0. 5192 0. 1273 0</pre> <p><code>az =</code></p> <pre>1. 0000 - 3. 9216 5. 7679 - 3. 7709 0. 9246</pre>

impinvar

Algorithm `impinvar` performs the impulse-invariant method of analog-to-digital transfer function conversion discussed in reference [1]:

- 1 It finds the partial fraction expansion of the system represented by `b` and `a`.
- 2 It replaces the poles `p` by the poles $\exp(p/Fs)$.
- 3 It finds the transfer function coefficients of the system from the residues from step 1 and the poles from step 2.

See Also	<code>bi1inear</code>	Map variables using bilinear transformation.
	<code>lp2bp</code>	Lowpass to bandpass analog filter transformation.
	<code>lp2bs</code>	Lowpass to bandstop analog filter transformation.
	<code>lp2hp</code>	Lowpass to highpass analog filter transformation.
	<code>lp2lp</code>	Lowpass to lowpass analog filter transformation.

References [1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Pgs. 206-209.

Purpose Impulse response of digital filters.

Syntax

```
[ h, t ] = i mpz( b, a )
[ h, t ] = i mpz( b, a, n )
[ h, t ] = i mpz( b, a, n, Fs )
i mpz( b, a )
i mpz( . . . )
```

Description `[h, t] = i mpz(b, a)` computes the impulse response of the filter with numerator coefficients `b` and denominator coefficients `a`. `i mpz` chooses the number of samples and returns the response in column vector `h` and times (or sample intervals) in column vector `t` (where `t = (0: n-1)'` and `n` is the computed impulse response length).

`[h, t] = i mpz(b, a, n)` computes `n` samples of the impulse response. If `n` is a vector of integers, `i mpz` computes the impulse response at those integer locations where 0 is the starting point of the filter.

`[h, t] = i mpz(b, a, n, Fs)` computes `n` samples and scales `t` so that samples are spaced `1/Fs` units apart. `Fs` is 1 by default.

`[h, t] = i mpz(b, a, [], Fs)` chooses the number of samples for you and scales `t` so that samples are spaced `1/Fs` units apart.

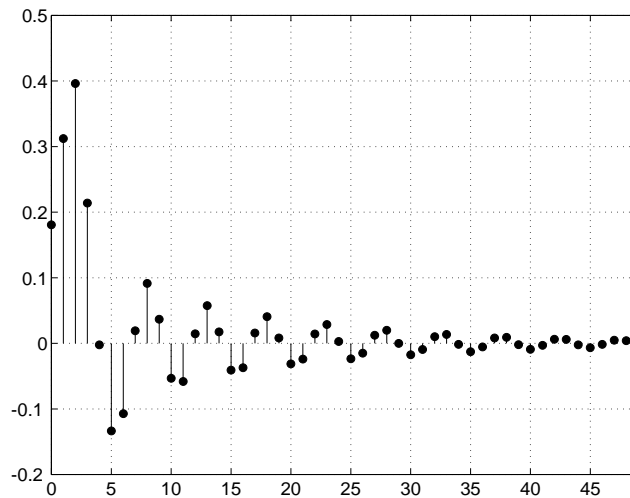
`i mpz` with no output arguments plots the impulse response in the current figure window using `stem(t, h)`.

`i mpz` works for both real and complex input systems.

Example

Plot the first 50 samples of the impulse response of a fourth-order lowpass elliptic filter with cutoff frequency of 0.4 times the Nyquist frequency:

```
[b, a] = ellip(4, 0.5, 20, 0.4);
impz(b, a, 50)
```



Algorithm

`impz` filters a length n impulse sequence using

```
filter(b, a, [1 zeros(1, n-1)])
```

To compute n in the auto-length case, `impz` either uses $n = \text{length}(b)$ for the FIR case or first finds the poles using $p = \text{roots}(a)$, if $\text{length}(a)$ is greater than 1.

If the filter is unstable, n is chosen to be the point at which the term from the largest pole reaches 10^6 times its original value.

If the filter is stable, n is chosen to be the point at which the term due to the largest amplitude pole is $5 \cdot 10^{-5}$ of its original amplitude.

If the filter is oscillatory (poles on the unit circle only), `impz` computes five periods of the slowest oscillation.

If the filter has both oscillatory and damped terms, n is chosen to equal five periods of the slowest oscillation or the point at which the term due to the

largest (nonunity) amplitude pole is 5×10^{-5} of its original amplitude, whichever is greater.

impz also allows for delay in the numerator polynomial, which it adds to the resulting n.

See Also

impulse

Unit impulse response (see *Control System Toolbox User's Guide*).

stem

Plot discrete sequence data (see the online *MATLAB Function Reference*).

interp

Purpose

Increase sampling rate by an integer factor (interpolation).

Syntax

```
y = interp(x, r)
y = interp(x, r, l, al pha)
[y, b] = interp(x, r, l, al pha)
```

Description

Interpolation increases the original sampling rate for a sequence to a higher rate. `interp` performs lowpass interpolation by inserting zeros into the original sequence and then applying a special lowpass filter.

`y = interp(x, r)` increases the sampling rate of `x` by a factor of `r`. The interpolated vector `y` is `r` times longer than the original input `x`.

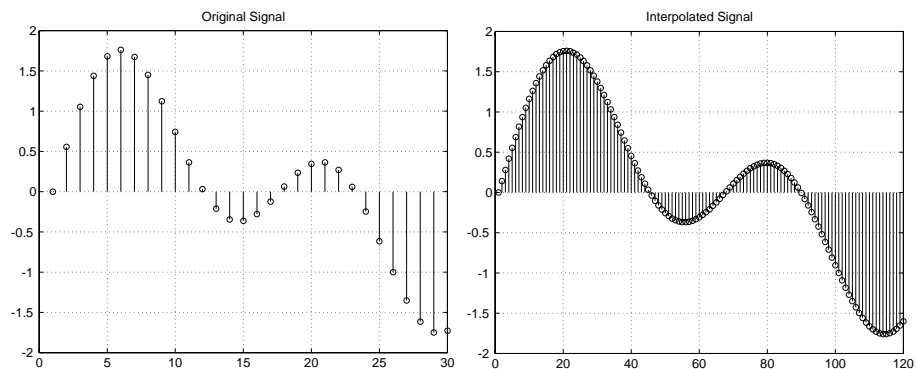
`y = interp(x, r, l, al pha)` specifies `l` (filter length) and `al pha` (cut-off frequency). The default value for `l` is 4 and the default value for `al pha` is 0.5.

`[y, b] = interp(x, r, l, al pha)` returns vector `b` containing the filter coefficients used for the interpolation.

Example

Interpolate a signal by a factor of four:

```
t = 0:0.001:1; % time vector
x = sin(2*pi*30*t) + sin(2*pi*60*t);
y = interp(x, 4);
stem(x(1:30))
figure
stem(y(1:120))
```



Algorithm

`interp` uses the lowpass interpolation Algorithm 8.1 described in [1]:

- 1 It expands the input vector to the correct length by inserting zeros between the original data values.
- 2 It designs a special symmetric FIR filter that allows the original data to pass through unchanged and interpolates between so that the mean-square errors between the interpolated points and their ideal values are minimized.
- 3 It applies the filter to the input vector to produce the interpolated output vector.

The length of the FIR lowpass interpolating filter is $2 \cdot l \cdot r + 1$. The number of original sample values used for interpolation is $2 \cdot l$. Ordinarily, l should be less than or equal to 10. The original signal is assumed to be band limited with normalized cutoff frequency $0 \leq \alpha \leq 1$, where 1 is half the original sampling frequency (the Nyquist frequency). The default value for l is 4 and the default value for α is 0.5.

Diagnostics

If r is not an integer, `interp` gives the following error message:

Resampling rate R must be an integer.

See Also

<code>decimate</code>	Decrease the sampling rate for a sequence (decimation).
<code>interp1</code>	One-dimensional data interpolation (table lookup) (see the online <i>MATLAB Function Reference</i>).
<code>resample</code>	Change sampling rate by any factor.
<code>spline</code>	Cubic spline interpolation (see the online <i>MATLAB Function Reference</i>).
<code>upfirdn</code>	Upsample, apply an FIR filter, and downsample.

References

[1] IEEE. *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979. Algorithm 8.1.

intfilt

Purpose Interpolation FIR filter design.

Syntax
`b = intfilt(r, l, alpha)`
`b = intfilt(r, n, 'Lagrange')`

Description `b = intfilt(r, l, alpha)` designs a linear phase FIR filter that performs ideal bandlimited interpolation using the nearest $2 \times l$ nonzero samples, when used on a sequence interleaved with $r-1$ consecutive zeros every r samples. It assumes an original bandlimitedness of α times the Nyquist frequency. The returned filter is identical to that used by `interp`.

`b = intfilt(r, n, 'Lagrange')` or `b = intfilt(r, n, 'l')` designs an FIR filter that performs n th-order Lagrange polynomial interpolation on a sequence interleaved with $r-1$ consecutive zeros every r samples. `b` has length $(n + 1) \times r$ for n even, and length $(n + 1) \times r - 1$ for n odd.

Both types of filters are basically lowpass and are intended for interpolation and decimation.

Examples Design a digital interpolation filter to upsample a signal by four, using the bandlimited method:

```
alpha = 0.5; % "bandlimitedness" factor
h1 = intfilt(4, 2, alpha); % bandlimited interpolation
```

The filter `h1` works best when the original signal is bandlimited to α times the Nyquist frequency. Create a bandlimited noise signal:

```
randn('seed', 0)
x = filter(fir1(40, 0.5), 1, randn(200, 1)); % bandlimit
```

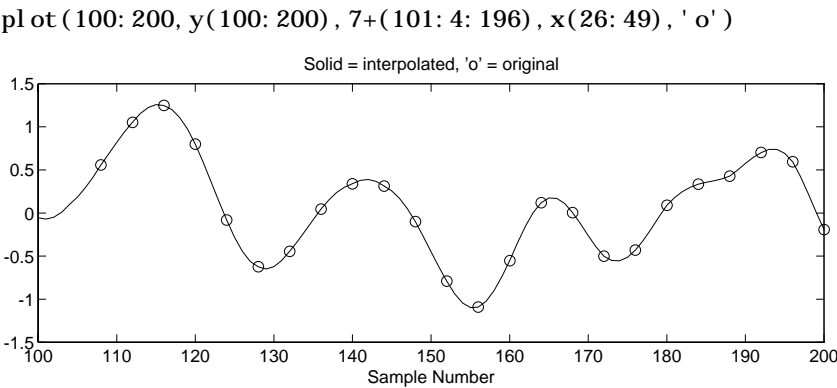
Now zero pad the signal with three zeros between every sample. The resulting sequence is four times the length of `x`:

```
xr = reshape([x zeros(length(x), 3)]', 4*length(x), 1);
```

Interpolate using the `filter` command:

```
y = filter(h1, 1, xr);
```


y is an interpolated version of x, delayed by seven samples (the group-delay of the filter). Zoom in on a section to see this:



intfilt's other type of filter performs Lagrange polynomial interpolation of the original signal. For example, first-order polynomial interpolation is just linear interpolation, which is accomplished with a triangular filter:

```
h2 = intfilt(4, 1, 'l')    % Lagrange interpolation
h2 =
    0.2500    0.5000    0.7500    1.0000    0.7500    0.5000    0.2500
```

Algorithm The bandlimited method uses fir1s to design an interpolation FIR equivalent to that presented in [1]. The polynomial method uses Lagrange's polynomial interpolation formula on equally spaced samples to construct the appropriate filter.

See Also	decimate	Decrease the sampling rate for a sequence (decimation).
	interp	Increase sampling rate by an integer factor (interpolation).
	resample	Change sampling rate by any factor.

References [1] Oetken, Parks, and Schüßler. "New Results in the Design of Digital Interpolators." *IEEE Trans. Acoust., Speech, Signal Processing*. Vol. ASSP-23 (June 1975). Pgs. 301-309.

Purpose Continuous-time (analog) filter identification from frequency data.

Syntax

```
[b, a] = invfreqs(h, w, nb, na)
[b, a] = invfreqs(h, w, nb, na, wt)
[b, a] = invfreqs(h, w, nb, na, wt, iter)
[b, a] = invfreqs(h, w, nb, na, wt, iter, tol)
[b, a] = invfreqs(h, w, nb, na, wt, iter, tol, 'trace')
[b, a] = invfreqs(h, w, 'complex', nb, na, ...)
```

Description `invfreqs` is the inverse operation of `freqs`; it finds a continuous-time transfer function that corresponds to a given complex frequency response. From a laboratory analysis standpoint, `invfreqs` is useful in converting magnitude and phase data into transfer functions.

`[b, a] = invfreqs(h, w, nb, na)` returns the real numerator and denominator coefficient vectors `b` and `a` of the transfer function

$$H(s) = \frac{B(s)}{A(s)} = \frac{b(1)s^{nb} + b(2)s^{(nb-1)} + \dots + b(nb+1)}{a(1)s^{na} + a(2)s^{(na-1)} + \dots + a(na+1)}$$

whose complex frequency response is given in vector `h` at the frequency points specified in vector `w`. Scalars `nb` and `na` specify the desired orders of the numerator and denominator polynomials.

Frequency is specified in radians between 0 and π , and the length of `h` must be the same as the length of `w`. `invfreqs` uses `conj(h)` at $-w$ to ensure the proper frequency domain symmetry for a real filter.

`[b, a] = invfreqs(h, w, nb, na, wt)` weights the fit-errors versus frequency. `wt` is a vector of weighting factors the same length as `w`.

`invfreqs(h, w, nb, na, wt, iter)` and

`invfreqs(h, w, nb, na, wt, iter, tol)` provide a superior algorithm that guarantees stability of the resulting linear system and searches for the best fit using a numerical, iterative scheme. The `iter` parameter tells `invfreqs` to end the iteration when the solution has converged, or after `iter` iterations, whichever comes first. `invfreqs` defines convergence as occurring when the

norm of the (modified) gradient vector is less than `tol`. `tol` is an optional parameter that defaults to 0.01. To obtain a weight vector of all ones, use

```
invfreqs(h, w, nb, na, [], iter, tol)
```

`invfreqs(h, w, nb, na, wt, iter, tol, 'trace')` displays a textual progress report of the iteration.

`invfreqs(h, w, 'complex', nb, na, ...)` creates a complex filter. In this case no symmetry is enforced, and the frequency is specified in radians between $-\pi$ and π .

Remarks

When building higher order models using high frequencies, it is important to scale the frequencies, dividing by a factor such as half the highest frequency present in `w`, so as to obtain well conditioned values of `a` and `b`. This corresponds to a rescaling of time.

Examples

Convert a simple transfer function to frequency response data and then back to the original filter coefficients:

```
a = [1 2 3 2 1 4]; b = [1 2 3 2 3];
[h, w] = freqs(b, a, 64);
[bb, aa] = invfreqs(h, w, 4, 5)
```

```
bb =
```

```
1.0000    2.0000    3.0000    2.0000    3.0000
```

```
aa =
```

```
1.0000    2.0000    3.0000    2.0000    1.0000    4.0000
```

Notice that bb and aa are equivalent to b and a, respectively. However, aa has poles in the left half-plane and thus the system is unstable. Use invfreqs's iterative algorithm to find a stable approximation to the system:

```
[bbb, aaa] = invfreqs(h, w, 4, 5, [], 30)
```

```
bbb =
```

```
0.6816    2.1015    2.6694    0.9113   -0.1218
```

```
aaa =
```

```
1.0000    3.4676    7.4060    6.2102    2.5413    0.0001
```

Suppose you have two vectors, mag and phase, that contain magnitude and phase data gathered in a laboratory, and a third vector w of frequencies. You can convert the data into a continuous-time transfer function using invfreqs:

```
[b, a] = invfreqs(mag.*exp(j.*phase), w, 2, 3);
```

Algorithm

By default, invfreqs uses an equation error method to identify the best model from the data. This finds b and a in

$$\min_{b,a} \sum_{k=1}^n wt(k) |h(k)A(w(k)) - B(w(k))|^2$$

by creating a system of linear equations and solving them with MATLAB's \ operator. Here $A(w(k))$ and $B(w(k))$ are the Fourier transforms of the polynomials a and b, respectively, at the frequency $w(k)$, and n is the number of frequency points (the length of h and w). This algorithm is based on Levi [1]. Several variants have been suggested in the literature, where the weighting function wt gives less attention to high frequencies.

The superior ("output-error") algorithm uses the damped Gauss-Newton method for iterative search [2], with the output of the first algorithm as the initial estimate. This solves the direct problem of minimizing the weighted sum of the squared error between the actual and the desired frequency response points:

$$\min_{b,a} \sum_{k=1}^n wt(k) \left| h(k) - \frac{B(w(k))}{A(w(k))} \right|^2$$

See Also

freqs	Frequency response of analog filters.
freqz	Frequency response of digital filters.
invfreqz	Discrete-time filter identification from frequency data.
prony	Prony's method for time domain IIR filter design.

References

- [1] Levi, E.C. "Complex-Curve Fitting." *IRE Trans. on Automatic Control*. Vol. AC-4 (1959). Pgs. 37-44.
- [2] Dennis, J.E., Jr., and R.B. Schnabel. *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. Englewood Cliffs, NJ: Prentice Hall, 1983.

invfreqz

Purpose Discrete-time filter identification from frequency data.

Syntax

```
[b, a] = invfreqz(h, w, nb, na)
[b, a] = invfreqz(h, w, nb, na, wt)
[b, a] = invfreqz(h, w, nb, na, wt, iter)
[b, a] = invfreqz(h, w, nb, na, wt, iter, tol)
[b, a] = invfreqz(h, w, nb, na, wt, iter, tol, 'trace')
[b, a] = invfreqz(h, w, 'complex', nb, na, ...)
```

Description `invfreqz` is the inverse operation of `freqz`; it finds a discrete-time transfer function that corresponds to a given complex frequency response. From a laboratory analysis standpoint, `invfreqz` can be used to convert magnitude and phase data into transfer functions.

`[b, a] = invfreqz(h, w, nb, na)` returns the real numerator and denominator coefficients in vectors `b` and `a` of the transfer function

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}}$$

whose complex frequency response is given in vector `h` at the frequency points specified in vector `w`. Scalars `nb` and `na` specify the desired orders of the numerator and denominator polynomials.

Frequency is specified in radians between 0 and π , and the length of `h` must be the same as the length of `w`. `invfreqz` uses `conj(h)` at $-w$ to ensure the proper frequency domain symmetry for a real filter.

`[b, a] = invfreqz(h, w, nb, na, wt)` weights the fit-errors versus frequency. `wt` is a vector of weighting factors the same length as `w`.

`invfreqz(h, w, nb, na, wt, iter)` and

`invfreqz(h, w, nb, na, wt, iter, tol)` provide a superior algorithm that guarantees stability of the resulting linear system and searches for the best fit using a numerical, iterative scheme. The `iter` parameter tells `invfreqz` to end the iteration when the solution has converged, or after `iter` iterations, whichever comes first. `invfreqz` defines convergence as occurring when the

norm of the (modified) gradient vector is less than `tol`. `tol` is an optional parameter that defaults to 0.01. To obtain a weight vector of all ones, use

```
invfreqz(h, w, nb, na, [], iter, tol)
```

`invfreqz(h, w, nb, na, wt, iter, tol, 'trace')` displays a textual progress report of the iteration.

`invfreqz(h, w, 'complex', nb, na, ...)` creates a complex filter. In this case no symmetry is enforced, and the frequency is specified in radians between $-\pi$ and π .

Example

Convert a simple transfer function to frequency response data and then back to the original filter coefficients:

```
a = [1 2 3 2 1 4]; b = [1 2 3 2 3];
[h, w] = freqz(b, a, 64);
[bb, aa] = invfreqz(h, w, 4, 5)
```

```
bb =
```

```
1.0000    2.0000    3.0000    2.0000    3.0000
```

```
aa =
```

```
1.0000    2.0000    3.0000    2.0000    1.0000    4.0000
```

Notice that `bb` and `aa` are equivalent to `b` and `a`, respectively. However, `aa` has poles outside the unit circle and thus the system is unstable. Use `invfreqz`'s iterative algorithm to find a stable approximation to the system:

```
[bbb, aaa] = invfreqz(h, w, 4, 5, [], 30)
```

```
bbb =
```

```
0.2427    0.2788    0.0069    0.0971    0.1980
```

```
aaa =
```

```
1.0000   -0.8944    0.6954    0.9997   -0.8933    0.6949
```

Algorithm By default, `invfreqz` uses an equation error method to identify the best model from the data. This finds `b` and `a` in

$$\min_{b,a} \sum_{k=1}^n wt(k) |h(k)A(w(k)) - B(w(k))|^2$$

by creating a system of linear equations and solving them with MATLAB's `\` operator. Here $A(w(k))$ and $B(w(k))$ are the Fourier transforms of the polynomials `a` and `b`, respectively, at the frequency $w(k)$, and n is the number of frequency points (the length of `h` and `w`). This algorithm is based on Levi [1].

The superior ("output-error") algorithm uses the damped Gauss-Newton method for iterative search [2], with the output of the first algorithm as the initial estimate. This solves the direct problem of minimizing the weighted sum of the squared error between the actual and the desired frequency response points:

$$\min_{b,a} \sum_{k=1}^n wt(k) \left| h(k) - \frac{B(w(k))}{A(w(k))} \right|^2$$

See Also	<code>freqs</code>	Frequency response of analog filters.
	<code>freqz</code>	Frequency response of digital filters.
	<code>invfreqs</code>	Continuous-time (analog) filter identification from frequency data.
	<code>prony</code>	Prony's method for time domain IIR filter design.

References

[1] Levi, E.C. "Complex-Curve Fitting." *IRE Trans. on Automatic Control*. Vol. AC-4 (1959). Pgs. 37-44.

[2] Dennis, J.E., Jr., and R.B. Schnabel. *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*. Englewood Cliffs, NJ: Prentice Hall, 1983.

Purpose	Kaiser window.																	
Syntax	<code>w = kaiser(n, beta)</code>																	
Description	<p><code>w = kaiser(n, beta)</code> returns an n-point Kaiser (I_0 - sinh) window in the column vector <code>w</code>. <code>beta</code> is the Kaiser window β parameter that affects the sidelobe attenuation of the Fourier transform of the window.</p> <p>To obtain a Kaiser window that designs an FIR filter with sidelobe height $-\alpha$ dB, use the following β:</p> $\beta = \begin{cases} 0.1102(\alpha - 8.7), & \alpha > 50 \\ 0.5842(\alpha - 21)^{0.4} + 0.07886(\alpha - 21), & 50 \geq \alpha \geq 21 \\ 0, & \alpha < 21 \end{cases}$ <p>Increasing <code>beta</code> widens the mainlobe and decreases the amplitude of the sidelobes (increases the attenuation).</p>																	
See Also	<table><tr><td><code>bartlett</code></td><td>Bartlett window.</td></tr><tr><td><code>blackman</code></td><td>Blackman window.</td></tr><tr><td><code>boxcar</code></td><td>Rectangular window.</td></tr><tr><td><code>chebwin</code></td><td>Chebyshev window.</td></tr><tr><td><code>hamming</code></td><td>Hamming window.</td></tr><tr><td><code>hanning</code></td><td>Hanning window.</td></tr><tr><td><code>kaiserord</code></td><td>Estimate parameters for <code>fir1</code> with Kaiser window.</td></tr><tr><td><code>triang</code></td><td>Triangular window.</td></tr></table>	<code>bartlett</code>	Bartlett window.	<code>blackman</code>	Blackman window.	<code>boxcar</code>	Rectangular window.	<code>chebwin</code>	Chebyshev window.	<code>hamming</code>	Hamming window.	<code>hanning</code>	Hanning window.	<code>kaiserord</code>	Estimate parameters for <code>fir1</code> with Kaiser window.	<code>triang</code>	Triangular window.	
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<code>triang</code>	Triangular window.																	
References	<p>[1] Kaiser, J.F. "Nonrecursive Digital Filter Design Using the I_0 - sinh Window Function." <i>Proc. 1974 IEEE Symp. Circuits and Syst.</i> (April 1974). Pgs. 20-23.</p> <p>[2] IEEE. <i>Digital Signal Processing II</i>. IEEE Press. New York: John Wiley & Sons, 1975.</p>																	

kaiserord

Purpose Estimate parameters for an FIR filter design with Kaiser window.

Syntax

```
[n, Wn, beta, ftype] = kaiserord(f, a, dev)
[n, Wn, beta, ftype] = kaiserord(f, a, dev, Fs)
c = kaiserord(f, a, dev, Fs, 'cell')
```

Description `kaiserord` returns a filter order `n` and `beta` parameter to specify a Kaiser window for use with the `fir1` function. Given a set of specifications in the frequency domain, `kaiserord` estimates the minimum FIR filter order that will approximately meet the specifications. `kaiserord` converts the given filter specifications into passband and stopband ripples and converts cutoff frequencies into the form needed for windowed FIR filter design.

NOTE If the band ripples are specified as unequal, the smallest one is used, since the Kaiser window method is constrained to give filters with equal ripple heights in all the passbands and stopbands.

`[n, Wn, beta, ftype] = kaiserord(f, a, dev)` finds the approximate order `n`, normalized frequency band edges `Wn`, and weights that meet input specifications `f`, `a`, and `dev`. `f` is a vector of band edges and `a` is a vector specifying the desired amplitude on the bands defined by `f`. The length of `f` is twice the length of `a`, minus 2. Together, `f` and `a` define a desired piecewise constant response function. `dev` is a vector the same size as `a` that specifies the maximum allowable error or deviation between the frequency response of the output filter and its desired amplitude, for each band.

`fir1` can use the resulting order `n`, frequency vector `Wn`, multiband magnitude type `ftype`, and the Kaiser window parameter `beta`. The `ftype` string is intended for use with `fir1`; it is equal to 'hi gh' for a highpass filter and 'stop' for a bandstop filter. For multiband filters, it can be equal to 'dc-0' when the first band is a stopband (starting at $f=0$) or 'dc-1' when the first band is a passband.

To design a filter `b` that approximately meets the specifications given by `kaiser` parameters `f`, `a`, and `dev`:

```
b = fir1(n, Wn, kaiser(n+1, beta), ftype, 'noscale')
```

`[n, Wn, beta, ftype] = kaiserord(f, a, dev, Fs)` specifies a sampling frequency `Fs`. If not present, `Fs` defaults to 2 Hz, implying a Nyquist frequency of 1 Hz. You can therefore specify band edges scaled to a particular application's sampling frequency.

`c = kaiserord(f, a, dev, Fs, 'cell')` is a cell-array whose elements are the parameters to `filter`.

NOTE In some cases, `kaiserord` underestimates or overestimates the order `n`. If the filter does not meet the specifications, try a higher order such as `n+1`, `n+2`, and so on, or a lower order.

Results are inaccurate if the cutoff frequencies are near 0 or the Nyquist frequency, or if `dev` is large (greater than 10%).

Algorithm

`kaiserord` uses empirically derived formulas for estimating the orders of lowpass filters, as well as differentiators and Hilbert transformers. Estimates for multiband filters (such as bandpass filters) are derived from the lowpass design formulas.

The design formulas that underlie the Kaiser window and its application to FIR filter design are

$$\beta = \begin{cases} 0.1102(\alpha - 8.7), & \alpha > 50 \\ 0.5842(\alpha - 21)^{0.4} + 0.07886(\alpha - 21), & 50 \geq \alpha \geq 21 \\ 0, & \alpha < 21 \end{cases}$$

where $\alpha = -20\log_{10}\delta$ is the stopband attenuation expressed in decibels (recall that $\delta_p = \delta_s$ is required). The design formula is:

$$n = \frac{\alpha - 7.95}{2.285(\Delta\omega)}$$

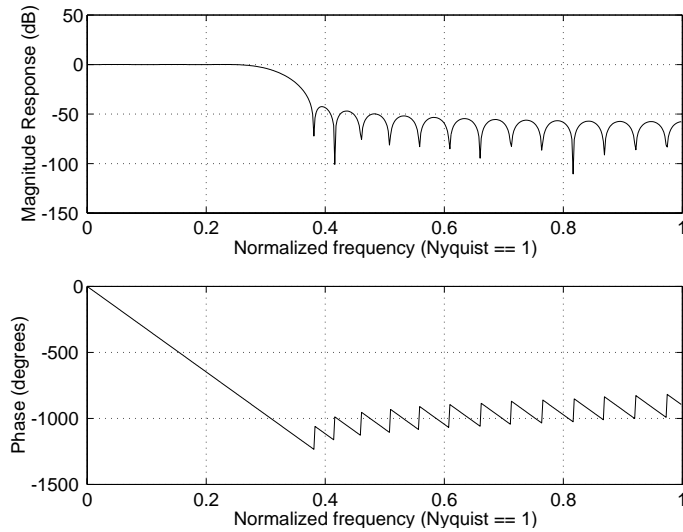
where n is the filter order and $\Delta\omega$ is the width of the smallest transition region.

kaiserord

Examples

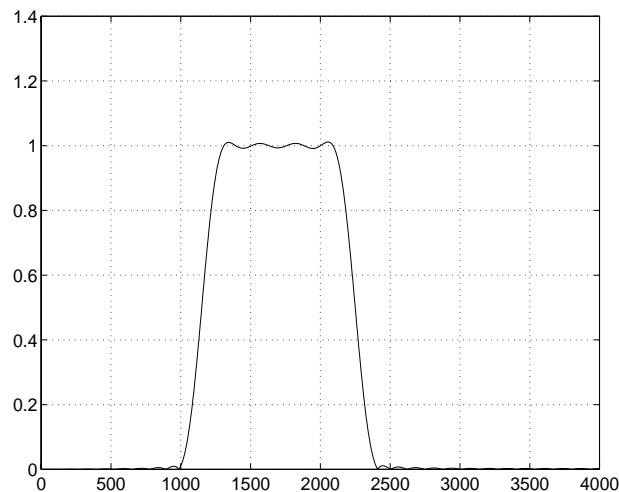
Design a lowpass filter with passband from 0 to 1 kHz and stopband from 1500 Hz to 4 kHz. Specify passband ripple of 5% and stopband attenuation of 40 dB:

```
fsamp = 8000;  
fcuts = [1000 1500];  
mags = [1 0];  
devs = [0.05 0.01];  
[n, Wn, beta, ftype] = kaiserord(fcuts, mags, devs, fsamp);  
hh = fir1(n, Wn, ftype, kaiser(n+1, beta), 'noscale');  
freqz(hh)
```



Design an odd-length bandpass filter (note that odd length means even order, so the input to `fir1` must be an even integer):

```
fsamp = 8000;
fcuts = [1000 1300 2410];
mags = [0 1 0];
devs = [0.01 0.05 0.01];
[n, Wn, beta, ftype] = kaiserord(fcuts, mags, devs, fsamp);
n = n + rem(n, 2);
hh = fir1(n, Wn, ftype, kaiser(n+1, beta), 'noscale');
[H, f] = freqz(hh, 1, 1024, fsamp);
plot(f, abs(H)), grid on
```



Design a lowpass filter with a passband cutoff of 1500 Hz, a stopband cutoff of 2000 Hz, passband ripple of 0.01, stopband ripple of 0.1, and a sampling frequency of 8000 Hz:

```
[n, Wn, beta, ftype] = kaiserord([1500 2000], [1 0], [0.01 0.1], 8000);
b = fir1(n, Wn, ftype, kaiser(n+1, beta), 'noscale');
```

This is equivalent to

```
c = kaiserord([1500 2000], [1 0], [0.01 0.1], 8000, 'cell');
b = fir1(c{:});
```

Remarks Be careful to distinguish between the meanings of filter length and filter order. The filter *length* is the number of impulse response samples in the FIR filter. Generally, the impulse response is indexed from $n = 0$ to $n = L-1$, where L is the filter length. The filter *order* is the highest power in a z -transform representation of the filter. For an FIR transfer function, this representation is a polynomial in z , where the highest power is z^{L-1} and the lowest power is z^0 . The filter order is 1 less than the length ($L-1$) and is also equal to the number of zeros of the z polynomial.

Diagnostics The lengths of the frequency, magnitude, and deviation vectors must match; otherwise, `kaiserord` gives the following error message:

Requires M and DEV to be the same length.
Length of F must be 2*length(M)-2.

When magnitudes are not specified correctly, `kaiserord` gives the following error messages:

Stopbands must be zero.
All passbands must have the same height.

If the band edges are not strictly increasing, `kaiserord` gives the following error message:

Bandedges must be strictly increasing.

See Also	<code>fir1</code>	Window-based finite impulse response filter design—standard response.
	<code>kaiser</code>	Kaiser window.
	<code>remezord</code>	Parks-McClellan optimal FIR filter order estimation.

References

[1] Kaiser, J.F. "Nonrecursive Digital Filter Design Using the I_0 - sinh Window Function." *Proc. 1974 IEEE Symp. Circuits and Syst.* (April 1974). Pgs. 20-23.

[2] IEEE. *Digital Signal Processing II*. IEEE Press. New York: John Wiley & Sons, 1975. Pgs. 123-126.

Purpose	Lattice filter to transfer function conversion.	
Syntax	<pre>[num, den] = latc2tf(k, v) [num, den] = latc2tf(k, 'iir') num = latc2tf(k, 'fir') num = latc2tf(k)</pre>	
Description	<p><code>[num, den] = latc2tf(k, v)</code> finds the transfer function numerator <code>num</code> and denominator <code>den</code> from the IIR lattice coefficients <code>k</code> and ladder coefficients <code>v</code>.</p> <p><code>[num, den] = latc2tf(k, 'iir')</code> assumes that <code>k</code> is associated with an all-pole IIR lattice filter.</p> <p><code>num = latc2tf(k, 'fir')</code> and</p> <p><code>num = latc2tf(k)</code> find the transfer function numerators from the FIR lattice coefficients specified by <code>k</code>.</p>	
See Also	<code>latcfilt</code>	Lattice and lattice-ladder filter implementation.
	<code>tf2latc</code>	Transfer function to lattice filter conversion.

latcfilt

Purpose	Lattice and lattice-ladder filter implementation.	
Syntax	$[f, g] = \text{latcfilt}(k, x)$ $[f, g] = \text{latcfilt}(k, v, x)$ $[f, g] = \text{latcfilt}(k, 1, x)$	
Description	<p>$[f, g] = \text{latcfilt}(k, x)$ filters x with the FIR lattice coefficients in vector k. f is the forward lattice filter result and g is the backward filter result.</p> <p>If k and x are vectors, the result is a (signal) vector.</p> <p>Matrix arguments are permitted under the following rules:</p> <ul style="list-style-type: none">• If x is a matrix and k is a vector, each column of x is processed through the lattice filter specified by k.• If x is a vector and k is a matrix, each column of k is used to filter x, and a signal matrix is returned.• If x and k are both matrices with the same number of columns, then the i-th column of k is used to filter the i-th column of x. A signal matrix is returned. <p>$[f, g] = \text{latcfilt}(k, v, x)$ filters x with the IIR lattice coefficients k and ladder coefficients v. k and v must be vectors, while x may be a signal matrix.</p> <p>$[f, g] = \text{latcfilt}(k, 1, x)$ filters x with the IIR all-pole lattice specified by k. k and x may be vectors or matrices according to the rules given for the FIR lattice.</p>	
See Also	<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
	<code>latc2tf</code>	Lattice filter to transfer function conversion.
	<code>tf2latc</code>	Transfer function to lattice filter conversion.

Purpose Levinson-Durbin recursion.

Syntax `a = levinson(r, n)`

Description The Levinson-Durbin recursion is an algorithm for finding an all-pole IIR filter with a prescribed deterministic autocorrelation sequence. It has applications in filter design, coding, and spectral estimation. The filter that `levinson` produces is minimum phase.

`a = levinson(r, n)` finds the coefficients of an n th-order autoregressive linear process which has `r` as its autocorrelation sequence. `r` is a real or complex deterministic autocorrelation sequence (a vector), and `n` is the order of denominator polynomial $A(z)$; that is, `a` = [1 $a(2)$... $a(n+1)$]. The filter coefficients are ordered in descending powers of z :

$$H(z) = \frac{1}{A(z)} = \frac{1}{1 + a(2)z^{-1} + \dots + a(n+1)z^{-n}}$$

Algorithm `levinson` solves the symmetric Toeplitz system of linear equations:

$$\begin{bmatrix} R(1) & R(2)^* & \dots & R(n)^* \\ R(2) & R(1) & \dots & R(n-1)^* \\ \vdots & \vdots & \ddots & \vdots \\ R(n) & R(n-1) & \dots & R(1) \end{bmatrix} \begin{bmatrix} a(2) \\ a(3) \\ \vdots \\ a(n+1) \end{bmatrix} = \begin{bmatrix} -R(2) \\ -R(3) \\ \vdots \\ -R(n+1) \end{bmatrix}$$

where `r` = [$R(1)$... $R(n+1)$] is the input autocorrelation vector, and $R(j)^*$ denotes the complex conjugate of $R(j)$. The algorithm requires $O(n^2)$ flops and is thus much more efficient than the MATLAB `\` command for large n . However, the `levinson` function uses `\` for low orders to provide the fastest possible execution.

See Also

<code>lpc</code>	Linear prediction coefficients.
<code>prony</code>	Prony's method for time domain IIR filter design.
<code>stmcb</code>	Linear model using Steiglitz-McBride iteration.

References [1] Ljung, L. *System Identification: Theory for the User*. Englewood Cliffs, NJ: Prentice Hall, 1987. Pgs. 278-280.

Purpose Lowpass to bandpass analog filter transformation.

Syntax $[bt, at] = lp2bp(b, a, Wo, Bw)$
 $[At, Bt, Ct, Dt] = lp2bp(A, B, C, D, Wo, Bw)$

Description `lp2bp` transforms analog lowpass filter prototypes with a cutoff frequency of 1 rad/sec into bandpass filters with desired bandwidth and center frequency. The transformation is one step in the digital filter design process for the `butter`, `cheby1`, `cheby2`, and `ellip` functions.

`lp2bp` can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

Transfer Function Form (Polynomial)

$[bt, at] = lp2bp(b, a, Wo, Bw)$ transforms an analog lowpass filter prototype given by polynomial coefficients into a bandpass filter with center frequency Wo and bandwidth Bw . Row vectors b and a specify the coefficients of the numerator and denominator of the prototype in descending powers of s :

$$\frac{b(s)}{a(s)} = \frac{b(1)s^{nn} + \dots + b(nn)s + b(nn+1)}{a(1)s^{nd} + \dots + a(nd)s + a(nd+1)}$$

Scalars Wo and Bw specify the center frequency and bandwidth in units of radians/second. For a filter with lower band edge $w1$ and upper band edge $w2$, use $Wo = \sqrt{w1*w2}$ and $Bw = w2-w1$.

`lp2bp` returns the frequency transformed filter in row vectors bt and at .

State-Space Form

$[At, Bt, Ct, Dt] = lp2bp(A, B, C, D, Wo, Bw)$ converts the continuous-time state-space lowpass filter prototype in matrices A, B, C, D :

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

into a bandpass filter with center frequency ω_0 and bandwidth B_w . For a filter with lower band edge ω_1 and upper band edge ω_2 , use $\omega_0 = \sqrt{\omega_1 \omega_2}$ and $B_w = \omega_2 - \omega_1$.

The bandpass filter is returned in matrices A_t , B_t , C_t , D_t .

Algorithm

lp2bp is a highly accurate state-space formulation of the classic analog filter frequency transformation. Consider the state-space system:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

where u is the input, x is the state vector, and y is the output. The Laplace transform of the first equation is

$$sx = Ax + Bu$$

Now if a bandpass filter is to have center frequency ω_0 and bandwidth B_w , the standard s -domain transformation is

$$s = Q(p^2 + 1)/p$$

where $Q = \omega_0/B_w$ and $p = s/\omega_0$. Substituting this for s in the Laplace transformed state-space equation, and considering the operator p as d/dt :

$$Q\ddot{x} + Q\dot{x} = A\dot{x} + B\dot{u}$$

or

$$Q\ddot{x} - A\dot{x} - B\dot{u} = -Qx$$

Now define

$$Q\dot{\omega} = -Qx$$

which, when substituted, leads to

$$Q\dot{x} = Ax + Q\omega + Bu$$

The last two equations give equations of state. Write them in standard form and multiply the differential equations by ω_0 to recover the time/frequency scaling represented by p and find state matrices for the bandpass filter:

```
Q = Wo/Bw; [ma, na] = size(A);  
At = Wo*[A/Q eye(ma, na); -eye(ma, na) zeros(ma, na)];  
Bt = Wo*[B/Q; zeros(ma, nb)];  
Ct = [C zeros(mc, ma)];  
Dt = d;
```

If the input to `lp2bp` is in transfer function form, the function transforms it into state-space form before applying this algorithm.

See Also

<code>bi l i near</code>	Map variables using bilinear transformation.
<code>i mpi nvar</code>	Impulse invariance method of analog-to-digital filter conversion.
<code>l p2bs</code>	Lowpass to bandstop analog filter transformation.
<code>l p2hp</code>	Lowpass to highpass analog filter transformation.
<code>l p2l p</code>	Lowpass to lowpass analog filter transformation.

Purpose Lowpass to bandstop analog filter transformation.

Syntax
 $[bt, at] = lp2bs(b, a, Wo, Bw)$
 $[At, Bt, Ct, Dt] = lp2bs(A, B, C, D, Wo, Bw)$

Description $lp2bs$ transforms analog lowpass filter prototypes with a cutoff frequency of 1 rad/sec into bandstop filters with desired bandwidth and center frequency. The transformation is one step in the digital filter design process for the `butter`, `cheby1`, `cheby2`, and `ellip` functions.

$lp2bs$ can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

Transfer Function Form (Polynomial)

$[bt, at] = lp2bs(b, a, Wo, Bw)$ transforms an analog lowpass filter prototype given by polynomial coefficients into a bandstop filter with center frequency Wo and bandwidth Bw . Row vectors b and a specify the coefficients of the numerator and denominator of the prototype in descending powers of s :

$$\frac{b(s)}{a(s)} = \frac{b(1)s^{nn} + \dots + b(nn)s + b(nn+1)}{a(1)s^{nd} + \dots + a(nd)s + a(nd+1)}$$

Scalars Wo and Bw specify the center frequency and bandwidth in units of radians/second. For a filter with lower band edge $w1$ and upper band edge $w2$, use $Wo = \sqrt{w1*w2}$ and $Bw = w2-w1$.

$lp2bs$ returns the frequency transformed filter in row vectors bt and at .

State-Space Form

$[At, Bt, Ct, Dt] = lp2bs(A, B, C, D, Wo, Bw)$ converts the continuous-time state-space lowpass filter prototype in matrices A, B, C, D :

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

into a bandstop filter with center frequency ω_0 and bandwidth B_w . For a filter with lower band edge ω_1 and upper band edge ω_2 , use $\omega_0 = \sqrt{\omega_1 \omega_2}$ and $B_w = \omega_2 - \omega_1$.

The bandstop filter is returned in matrices A_t , B_t , C_t , D_t .

Algorithm

`lp2bs` is a highly accurate state-space formulation of the classic analog filter frequency transformation. If a bandstop filter is to have center frequency ω_0 and bandwidth B_w , the standard s -domain transformation is

$$s = \frac{p}{Q(p^2 + 1)}$$

where $Q = \omega_0/B_w$ and $p = s/\omega_0$. The state-space version of this transformation is

```
Q = Wo/Bw;
At = [ Wo/Q*inv(A)  Wo*eye(ma); -Wo*eye(ma)  zeros(ma) ];
Bt = -[ Wo/Q*(A B);  zeros(ma, nb) ];
Ct = [ C/A  zeros(mc, ma) ];
Dt = D - C/A*B;
```

See `lp2bp` for a derivation of the bandpass version of this transformation.

See Also

<code>bi l i near</code>	Map variables using bilinear transformation.
<code>i mpi nvar</code>	Impulse invariance method of analog-to-digital filter conversion.
<code>l p2bp</code>	Lowpass to bandpass analog filter transformation.
<code>l p2hp</code>	Lowpass to highpass analog filter transformation.
<code>l p2l p</code>	Lowpass to lowpass analog filter transformation.

Purpose Lowpass to highpass analog filter transformation.

Syntax [bt, at] = lp2hp(b, a, Wo)
[At, Bt, Ct, Dt] = lp2hp(A, B, C, D, Wo)

Description lp2hp transforms analog lowpass filter prototypes with a cutoff frequency of 1 rad/sec into highpass filters with desired cutoff frequency. The transformation is one step in the digital filter design process for the butter, cheby1, cheby2, and ellip functions.

The lp2hp function can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

Transfer Function Form (Polynomial)

[bt, at] = lp2hp(b, a, Wo) transforms an analog lowpass filter prototype given by polynomial coefficients into a highpass filter with cutoff frequency Wo. Row vectors b and a specify the coefficients of the numerator and denominator of the prototype in descending powers of s :

$$\frac{b(s)}{a(s)} = \frac{b(1)s^{nn} + \dots + b(nn)s + b(nn+1)}{a(1)s^{nd} + \dots + a(nd)s + a(nd+1)}$$

Scalar Wo specifies the cutoff frequency in units of radians/second. The frequency transformed filter is returned in row vectors bt and at.

State-Space Form

[At, Bt, Ct, Dt] = lp2hp(A, B, C, D, Wo) converts the continuous-time state-space lowpass filter prototype in matrices A, B, C, D:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

into a highpass filter with cutoff frequency Wo. The highpass filter is returned in matrices At, Bt, Ct, Dt.

Algorithm

lp2hp is a highly accurate state-space formulation of the classic analog filter frequency transformation. If a highpass filter is to have cutoff frequency ω_0 , the standard s -domain transformation is

$$s = \frac{\omega_0}{p}$$

The state-space version of this transformation is

$$\begin{aligned} A_t &= W_0 * \text{inv}(A); \\ B_t &= -W_0 * (A \setminus B); \\ C_t &= C/A; \\ D_t &= D - C/A * B; \end{aligned}$$

See lp2bp for a derivation of the bandpass version of this transformation.

See Also

bi l i near	Map variables using bilinear transformation.
i m p i nvar	Impulse invariance method of analog-to-digital filter conversion.
l p2bp	Lowpass to bandpass analog filter transformation.
l p2bs	Lowpass to bandstop analog filter transformation.
l p2l p	Lowpass to lowpass analog filter transformation.

Purpose Lowpass to lowpass analog filter transformation.

Syntax [bt, at] = lp2lp(b, a, Wo)
[At, Bt, Ct, Dt] = lp2lp(A, B, C, D, Wo)

Description lp2lp transforms an analog lowpass filter prototype with a cutoff frequency of 1 rad/sec into a lowpass filter with any specified cutoff frequency. The transformation is one step in the digital filter design process for the butter, cheby1, cheby2, and ellip functions.

The lp2lp function can perform the transformation on two different linear system representations: transfer function form and state-space form. In both cases, the input system must be an analog filter prototype.

Transfer Function Form (Polynomial)

[bt, at] = lp2lp(b, a, Wo) transforms an analog lowpass filter prototype given by polynomial coefficients into a lowpass filter with cutoff frequency Wo. Row vectors b and a specify the coefficients of the numerator and denominator of the prototype in descending powers of s :

$$\frac{b(s)}{a(s)} = \frac{b(1)s^{nn} + \dots + b(nn)s + b(nn+1)}{a(1)s^{nd} + \dots + a(nd)s + a(nd+1)}$$

Scalar Wo specifies the cutoff frequency in units of radians/second. lp2lp returns the frequency transformed filter in row vectors bt and at.

State-Space Form

[At, Bt, Ct, Dt] = lp2lp(A, B, C, D, Wo) converts the continuous-time state-space lowpass filter prototype in matrices A, B, C, D:

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}$$

into a lowpass filter with cutoff frequency Wo. lp2lp returns the lowpass filter in matrices At, Bt, Ct, Dt.

Algorithm lp2lp is a highly accurate state-space formulation of the classic analog filter frequency transformation. If a lowpass filter is to have cutoff frequency ω_o , the standard s -domain transformation is

$$s = p / \omega_o$$

The state-space version of this transformation is

$$\begin{aligned} A_t &= W_o * A; \\ B_t &= W_o * B; \\ C_t &= C; \\ D_t &= D; \end{aligned}$$

See lp2bp for a derivation of the bandpass version of this transformation.

See Also	bi l i near	Map variables using bilinear transformation.
	i mpi nvar	Impulse invariance method of analog-to-digital filter conversion.
	l p2bp	Lowpass to bandpass analog filter transformation.
	l p2bs	Lowpass to bandstop analog filter transformation.
	l p2hp	Lowpass to highpass analog filter transformation.

Purpose Linear prediction coefficients.

Syntax `[a, g] = lpc(x, n)`

Description Linear prediction models each sample of a signal as a linear combination of previous samples, that is, as the output of an all-pole IIR filter. It has applications in filter design, speech coding, spectral analysis, and system identification.

`[a, g] = lpc(x, n)` finds the coefficients and gain of an n th-order auto-regressive linear process that models the time series x as

$$\hat{x}(k) = -a(2)x(k-1) - a(3)x(k-2) - \cdots - a(n+1)x(k-n)$$

x is the real input time series (a vector), and n is the order of the denominator polynomial $a(z)$; that is, $a = [1 \ a(2) \ \dots \ a(n+1)]$. The filter coefficients are ordered in descending powers of z .

If n is unspecified, `lpc` uses as a default $n = \text{length}(x) - 1$.

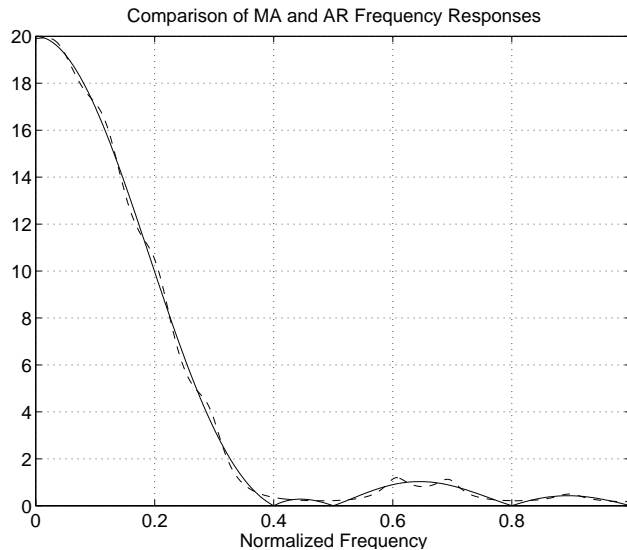
If x is a matrix containing a separate signal in each column, `lpc` returns a model estimate for each column in the rows of a and a row vector of gains g .

Example Model a nonrecursive (FIR) filter with an all-pole IIR filter using `lpc`:

```
x = [1:4 4:-1:1];
[a, g] = lpc(x, 15);
[H, w] = freqz(x, 1, 512); [H1, w] = freqz(g, a, 512);
```

Plot the FIR response with a solid line and the IIR response with a dashed line:

```
plot(w/pi, abs(H), w/pi, abs(H1), '- -')
```



Algorithm

lpc uses the autocorrelation method of autoregressive (AR) modeling to find the filter coefficients. This technique is also called the Yule-Walker AR method of spectral estimation. The filter generated is stable. However, the generated filter might not model the process exactly even if the data sequence is truly an AR process of the correct order. This is because the autocorrelation method implicitly windows the data, that is, it assumes that signal samples beyond the length of x are 0.

lpc computes the least-squares solution to

$$Xa \approx b$$

where

$$X = \begin{bmatrix} x(1) & 0 & \cdots & 0 \\ x(2) & x(1) & \ddots & \vdots \\ \vdots & x(2) & \ddots & 0 \\ x(m) & \vdots & \ddots & x(1) \\ 0 & x(m) & \ddots & x(2) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & x(m) \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ a(2) \\ \vdots \\ a(n+1) \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and m is the length of x . Solving the least-squares problem via the normal equations

$$X^H X a = X^H b$$

leads to the Yule-Walker equations

$$\begin{bmatrix} r(1) & r(2)^* & \cdots & r(n)^* \\ r(2) & r(1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & r(2)^* \\ r(n) & \cdots & r(2) & r(1) \end{bmatrix} \begin{bmatrix} a(2) \\ a(3) \\ \vdots \\ a(n+1) \end{bmatrix} = \begin{bmatrix} -r(2) \\ -r(3) \\ \vdots \\ -r(n+1) \end{bmatrix}$$

where $r = [r(1) \ r(2) \ \dots \ r(n+1)]$ is an autocorrelation estimate for x computed using `xcorr`. The Yule-Walker equations are solved in $O(n^2)$ flops by the Levinson-Durbin algorithm (see `levinson`).

The filter gain is given by

$$g = \sqrt{r^H a}$$

so that

$$x(1) + x(2)z^{-1} + \cdots + x(m)z^{-m+1} \approx \frac{g}{1 + a(2)z^{-1} + \cdots + a(n+1)z^{-n}}$$

in the least-squares sense.

See Also	ar	Compute autoregressive models of signals (see <i>System Identification Toolbox User's Guide</i>).
	levinson	Levinson-Durbin recursion.
	prony	Prony's method for time domain IIR filter design.
	pyul ear	Power spectrum estimate using Yule-Walker AR method.
	stmcb	Linear model using Steiglitz-McBride iteration.

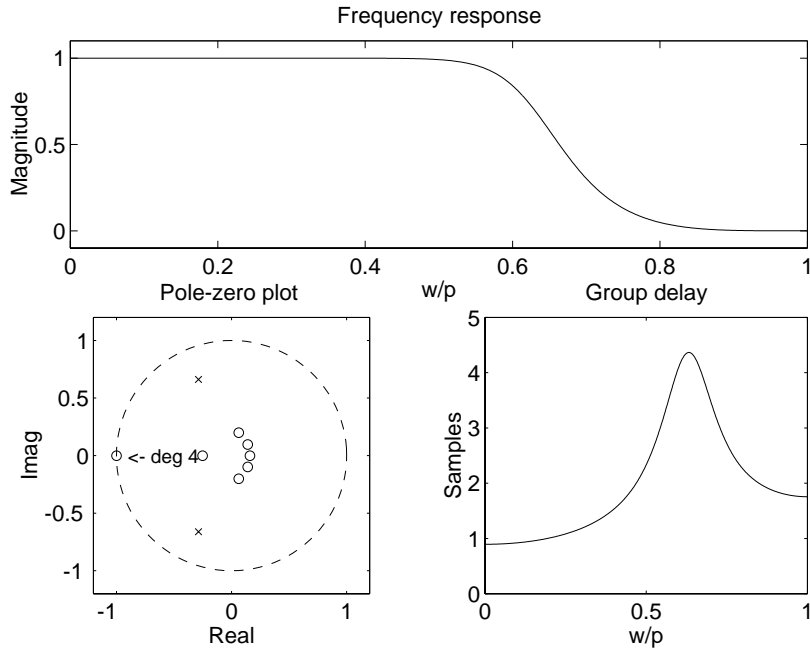
References [1] Jackson, L.B. *Digital Filters and Signal Processing*. Second Ed. Boston: Kluwer Academic Publishers, 1989. Pgs. 255-257.

Purpose	Generalized digital Butterworth filter design.
Syntax	<pre>[b, a,] = maxflat (nb, na, Wn) b = maxflat (nb, 'sym', Wn) [b, a, b1, b2] = maxflat (nb, na, Wn) [. . .] = maxflat (nb, na, Wn, 'design_flag')</pre>
Description	<p><code>[b, a,] = maxflat (nb, na, Wn)</code> is a lowpass Butterworth filter with numerator and denominator coefficients <code>b</code> and <code>a</code> of orders <code>nb</code> and <code>na</code> respectively. <code>Wn</code> is the cutoff frequency at which the magnitude response of the filter is equal to $1/\sqrt{2}$ (approx. -3 dB). <code>Wn</code> must be between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency).</p> <p><code>b = maxflat (nb, 'sym', Wn)</code> is a symmetric FIR Butterworth filter. <code>nb</code> must be even, and <code>Wn</code> is restricted to a subinterval of [0,1]. The function raises an error if <code>Wn</code> is specified outside of this subinterval.</p> <p><code>[b, a, b1, b2] = maxflat (nb, na, Wn)</code> returns two polynomials <code>b1</code> and <code>b2</code> whose product is equal to the numerator polynomial <code>b</code> (that is, <code>b = conv(b1, b2)</code>). <code>b1</code> contains all the zeros at $z = -1$, and <code>b2</code> contains all the other zeros.</p> <p><code>[. . .] = maxflat (nb, na, Wn, 'design_flag')</code> enables you to monitor the filter design, where <i>design_flag</i> is</p> <ul style="list-style-type: none"> • <code>trace</code>, for a textual display of the design table used in the design • <code>plots</code>, for plots of the filter's magnitude, group delay, and zeros and poles • <code>both</code>, for both the textual display and plots

maxflat

Examples

```
nb = 10; na = 2; Wn = 0.2*pi;  
[b, a, b1, b2] = maxflat(nb, na, Wn, 'plots')
```



Algorithm

The method consists of the use of formulae, polynomial root finding, and a transformation of polynomial roots.

See Also

<code>butter</code>	Butterworth analog and digital filter design.
<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
<code>freqz</code>	Frequency response of digital filters.

References

[1] Selesnick, I.W., and C.S. Burrus. "Generalized Digital Butterworth Filter Design." *Proceedings of the IEEE Int. Conf. Acoust., Speech, Signal Processing*. Vol. 3 (May 1996).

Purpose	One-dimensional median filtering.							
Syntax	<pre>y = medfilt1(x, n) y = medfilt1(x, n, blksize)</pre>							
Description	<p><code>y = medfilt1(x, n)</code> applies an order n, one-dimensional median filter to vector x. y is the same length as x; the function treats the signal as if it is 0 beyond the end points.</p> <p>For n odd, $y(k)$ is the median of $x(k-(n-1)/2:k+(n-1)/2)$.</p> <p>For n even, $y(k)$ is the median of $x(k-(n/2), x(k-(n/2)+1), \dots, x(k+(n/2)-1)$. In this case, <code>medfilt1</code> sorts the numbers, then takes the average of the $(n-1)/2$ and $((n-1)/2)+1$ elements.</p> <p>The default for n is 3.</p> <p><code>y = medfilt1(x, n, blksize)</code> uses a for-loop to compute <code>blksize</code> (block size) output samples at a time. Use <code>blksize << length(x)</code> if you are low on memory, since <code>medfilt1</code> uses a working matrix of size n-by-<code>blksize</code>. By default, <code>blksize = length(x)</code>; this is the fastest execution if you have sufficient memory.</p> <p>If x is a matrix, <code>medfilt1</code> median filters its columns using</p> <pre>y(:, i) = medfilt1(x(:, i), n, blksize)</pre> <p>in a loop over the columns of x.</p>							
See Also	<table><tr><td><code>filter</code></td><td>Filter data with a recursive (IIR) or nonrecursive (FIR) filter.</td></tr><tr><td><code>medfilt2</code></td><td>Two-dimensional median filtering (see <i>Image Processing Toolbox User's Guide</i>).</td></tr><tr><td><code>median</code></td><td>Median value (see the online <i>MATLAB Function Reference</i>).</td></tr></table>	<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.	<code>medfilt2</code>	Two-dimensional median filtering (see <i>Image Processing Toolbox User's Guide</i>).	<code>median</code>	Median value (see the online <i>MATLAB Function Reference</i>).	
<code>filter</code>	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.							
<code>medfilt2</code>	Two-dimensional median filtering (see <i>Image Processing Toolbox User's Guide</i>).							
<code>median</code>	Median value (see the online <i>MATLAB Function Reference</i>).							
References	<p>[1] Pratt, W.K. <i>Digital Image Processing</i>. New York: John Wiley & Sons, 1978. Pgs. 330-333.</p>							

modulate

Purpose Modulation for communications simulation.

Syntax

```
y = modulate(x, Fc, Fs, 'method')  
y = modulate(x, Fc, Fs, 'method', opt)  
[y, t] = modulate(x, Fc, Fs)
```

Description `y = modulate(x, Fc, Fs, 'method')` and

`y = modulate(x, Fc, Fs, 'method', opt)` modulate the real message signal `x` with a carrier frequency `Fc` and sampling frequency `Fs`, using one of the options listed below for `method`. Note that some methods accept an option, `opt`.

`amdsb-sc` **Amplitude modulation, double sideband, suppressed carrier.**

or Multiplies `x` by a sinusoid of frequency `Fc`:

`am`

$$y = x \cdot \cos(2\pi \cdot Fc \cdot t)$$

`amdsb-tc` **Amplitude modulation, double sideband, transmitted carrier.**
Subtracts scalar `opt` from `x` and multiplies the result by a sinusoid of frequency `Fc`:

$$y = (x - opt) \cdot \cos(2\pi \cdot Fc \cdot t)$$

If the `opt` parameter is not present, `modulate` uses a default of `min(min(x))` so that the message signal `(x-opt)` is entirely non-negative and has a minimum value of 0.

`amssb` **Amplitude modulation, single sideband.** Multiplies `x` by a sinusoid of frequency `Fc` and adds the result to the Hilbert transform of `x` multiplied by a phase shifted sinusoid of frequency `Fc`:

$$y = x \cdot \cos(2\pi \cdot Fc \cdot t) + i \cdot \text{mag}(\text{hilbert}(x)) \cdot \sin(2\pi \cdot Fc \cdot t)$$

This effectively removes the upper sideband.

- fm** **Frequency modulation.** Creates a sinusoid with instantaneous frequency that varies with the message signal x :
- $$y = \cos(2\pi * F_c * t + \text{opt} * \text{cumsum}(x))$$
- cumsum is a rectangular approximation to the integral of x .
 modulate uses opt as the constant of frequency modulation. If opt is not present, modulate uses a default of
- $$\text{opt} = (F_c / F_s) * 2\pi / (\max(\max(x)))$$
- so the maximum frequency excursion from F_c is F_c Hz.
- pm** **Phase modulation.** Creates a sinusoid of frequency F_c whose phase varies with the message signal x :
- $$y = \cos(2\pi * F_c * t + \text{opt} * x)$$
- modulate uses opt as the constant of phase modulation. If opt is not present, modulate uses a default of
- $$\text{opt} = \pi / (\max(\max(x)))$$
- so the maximum phase excursion is π radians.
- pwm** **Pulse-width modulation.** Creates a pulse-width modulated signal from the pulse widths in x . The elements of x must be between 0 and 1, specifying the width of each pulse in fractions of a period. The pulses start at the beginning of each period, that is, they are left justified.
- $$\text{modulate}(x, F_c, F_s, 'pwm', 'centered')$$
- yields pulses centered at the beginning of each period. y is length $\text{length}(x) * F_s / F_c$.
- ptm** **Pulse time modulation.** Creates a pulse time modulated signal from the pulse times in x . The elements of x must be between 0 and 1, specifying the left edge of each pulse in fractions of a period. opt is a scalar between 0 and 1 that specifies the length of each pulse in fractions of a period. The default for opt is 0.1. y is length $\text{length}(x) * F_s / F_c$.
- qam** **Quadrature amplitude modulation.** Creates a quadrature amplitude modulated signal from signals x and opt :
- $$y = x * \cos(2\pi * F_c * t) + \text{opt} * \sin(2\pi * F_c * t)$$
- opt must be the same size as x .

modulate

If you do not specify *method*, then `modulate` assumes `am`. Except for the `pwm` and `ptm` cases, `y` is the same size as `x`.

If `x` is an array, `modulate` modulates its columns.

`[y, t] = modulate(x, Fc, Fs)` returns the internal time vector `t` that `modulate` uses in its computations.

See Also

`demod`

Demodulation for communications simulation.

`vco`

Voltage controlled oscillator.

Purpose Power spectrum estimate using the Burg method.

Syntax

```
Pxx = pburg(x, p, nfft)
[Pxx, freq] = pburg(x, p, nfft, Fs)
[Pxx, freq, a] = pburg(... )
pburg(... )
```

Description pburg estimates the power spectral density (PSD) of the signal vector $x[n]$ using the Burg method. This method fits an autoregressive (AR) model to the lattice reflection coefficients estimated from the signal. Since it represents the spectrum by an all-pole model, the correct choice of the model order p is crucial. pburg returns the same results as pyul ear for large signal lengths.

$P_{xx} = \text{pburg}(x, p, nfft)$ returns P_{xx} , the power spectrum estimate. x is the input signal, p is the model order for the all-pole filter, and $nfft$ is the FFT length (defaults to 256 if not specified).

$[P_{xx}, \text{freq}] = \text{pburg}(x, p, nfft, F_s)$ returns P_{xx} , the power spectrum estimate, and freq , a vector of frequencies at which the PSD was estimated. x is the input signal, p is the model order for the all-pole filter, and $nfft$ is the FFT length (defaults to 256 if not specified). F_s specifies the signal's sampling frequency, which is used to scale the output frequency vector freq . If the input signal is real-valued, freq ranges from 0 to $F_s/2$. If the input signal is complex, freq ranges from 0 to F_s . F_s defaults to 2 if not specified.

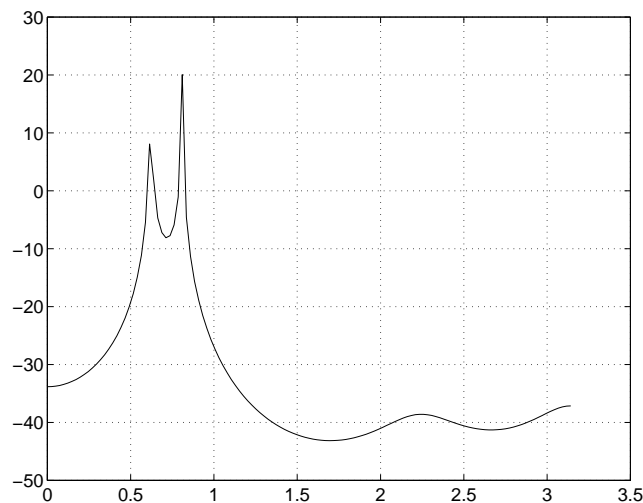
$[P_{xx}, \text{freq}, a] = \text{pburg}(\dots)$ returns vector a of filter coefficients for the all-pole filter model.

$\text{pburg}(\dots)$ plots the power spectral density in the first available figure window.

Example This example analyzes a sequence $x[n]$, assuming that two real signals are present in the signal subspace. In this case, the model order must be four or larger, because each real sinusoid is the sum of two complex exponentials.

Experience shows that taking a larger model order than the minimum seems to work better.

```
% Create xx as a signal vector.  
  
nn = 0:199;  
randn('seed', 0)  
xx = cos(0.257*pi*nn) + sin(0.2*pi*nn) + 0.01*randn(size(nn));  
[PP, ff, aa] = pburg(xx, 7); % 7th order model  
  
plot(ff*pi, 10*log10(PP)), grid % Plot the pole locations.
```



Diagnostics

The first input argument must be a full vector, otherwise pburg generates the following error message:

```
Input signal cannot be sparse.
```

If you specify an empty matrix for the second argument, pburg generates the following error message:

```
Model order must be given, empty not allowed.
```

See Also

lpc	Linear prediction coefficients.
pmtm	Power spectrum estimate using the multitaper method (MTM).
pmusic	Power spectrum estimate using MUSIC eigenvector method.
prony	Prony's method for time domain IIR filter design.
psd	Estimate the power spectral density (PSD) of a signal using Welch's method.
pyul ear	Power spectrum estimate using Yule-Walker AR method.

References

- [1] Marple, S.L. *Digital Spectral Analysis*. Englewood Cliffs, NJ: Prentice Hall, 1987. Chapter 7.

Purpose

Power spectrum estimate using the multitaper method (MTM).

Syntax

```
Pxx = pmtm(x)
Pxx = pmtm(x, nw)
Pxx = pmtm(x, nw, nfft)
[Pxx, f] = pmtm(x, nw, nfft, Fs)
[Pxx, f] = pmtm(x, nw, nfft, Fs, 'method')
[Pxx, Pxxc, f] = pmtm(x, nw, nfft, Fs, 'method')
[Pxx, Pxxc, f] = pmtm(x, nw, nfft, Fs, 'method', p)
[Pxx, Pxxc, f] = pmtm(x, e, v, nfft, Fs, 'method', p)
[Pxx, Pxxc, f] = pmtm(x, dpss_params, nfft, Fs, 'method', p)
```

Description

`pmtm` estimates the power spectral density (PSD) of the real time series x using the multitaper method (MTM), described in [1].

`Pxx = pmtm(x, nw)` estimates the PSD using `nw` as the time-bandwidth product for the discrete prolate spheroidal sequences (Slepian sequences) that are used as data windows. The default for `nw` is 4; other typical choices are 2, 5/2, 3, 7/2. The number of sequences used to form `Pxx` is 2^{nw-1} .

`Pxx = pmtm(x, nw, nfft)` defines the frequency grid as length `nfft`. When x is real, `Pxx` is length $(nfft/2+1)$ for `nfft` even and $(nfft+1)/2$ for `nfft` odd; when x is complex, `Pxx` is length `nfft`. The default for `nfft` is 256 or the next power of 2 greater than the length of x , whichever is larger.

`[Pxx, f] = pmtm(x, nw, nfft, Fs)` returns `f`, the vector of frequencies at which the PSD is estimated, for the sampling frequency `Fs`. The default for `Fs` is 2 Hz.

`[Pxx, f] = pmtm(x, nw, nfft, Fs, 'method')` specifies the algorithm used for combining the individual spectral estimates, where `method` is

- `adapt`, to specify Thomson's adaptive nonlinear combination (default)
- `unity`, to specify a linear combination with unity weights
- `eigen`, to specify a linear combination with eigenvalue weights

`[Pxx, Pxxc, f] = pmtm(x, nw, nfft, Fs, 'method')` returns `Pxxc`, the 95% confidence interval for `Pxx`, and

`[Pxx, Pxxc, f] = pmtm(x, nw, nfft, Fs, 'method', p)` returns `Pxxc`, the $p \times 100\%$ confidence interval for `Pxx`, where p is a scalar between 0 and 1. Confidence intervals are computed using a chi-squared approach, where `Pxxc(:, 1)` is the lower bound and `Pxxc(:, 2)` is the upper bound of the confidence interval.

`[Pxx, Pxxc, f] = pmtm(x, e, v, nfft, Fs, 'method', p)` returns the PSD estimate `Pxx`, the confidence interval `Pxxc`, and the frequency vector `f` from the data tapers in `e` and their concentrations `v`.

`[Pxx, Pxxc, f] = pmtm(x, dpss_params, nfft, Fs, 'method', p)` returns the PSD estimate `Pxx`, the confidence interval `Pxxc`, and the frequency vector `f` from the data tapers computed using `dpss` with parameters from the cell array `dpss_params`, whose first element is the *second* input to `dpss`. The first `dpss` parameter (n) is determined by the length of `x`. For example, `pmtm(x, {3.5, 'trace'}, 512, Fs)` calculates the Slepian sequences for $nw = 3.5$, and displays the method that `dpss` uses. See `dpss` for other options.

Remarks

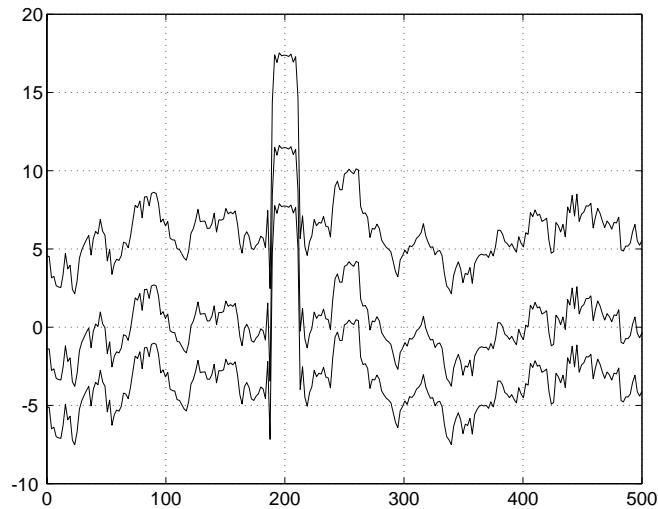
`pmtm` with no output arguments plots the PSD in the current or next available figure, with confidence intervals.

To use default parameters for any argument in an expression, insert an empty matrix `[]`. For example, `pmtm(x, [], [], 1000)` uses defaults for the second and third elements, in this case, `nw` and `nfft`.

Example

This example analyzes a sinusoid in white noise:

```
Fs = 1000; t = 0:1/Fs:0.3;  
x = cos(2*pi*t*200) + randn(size(t));  
[Pxx, Pxxc, f] = pmtm(x, 3.5, 512, Fs, [], 0.99);  
plot(f, 10*log10([Pxx Pxxc]))
```



See Also

dpss	Discrete prolate spheroidal sequences (Slepian sequences).
pburg	Power spectrum estimate using the Burg method.
pmusic	Power spectrum estimate using MUSIC eigenvector method.
psd	Estimate the power spectral density (PSD) of a signal using Welch's method.
pyulear	Power spectrum estimate using Yule-Walker AR method.

References

[1] Percival, D.B., and A.T. Walden. *Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques*. Cambridge: Cambridge University Press, 1993.

[2] Thomson, D.J. "Spectrum estimation and harmonic analysis." In *Proceedings of the IEEE*. Vol. 70 (1982). Pgs. 1055-1096.

Purpose Power spectrum estimate using MUSIC eigenvector method.

Syntax

```
[Pxx, f] = pmusic(x, p)
[Pxx, f] = pmusic(x, [p thresh])
[Pxx, f] = pmusic(x, [p thresh], nfft, Fs, window, noverlap)
[Pxx, f] = pmusic(x, ..., 'corr')
[Pxx, f] = pmusic(x, ..., 'ev')
[Pxx, f, evecs, sval s] = pmusic(x, ...)
```

Description `pmusic` estimates the power spectral density (PSD) of a signal or correlation matrix using Schmidt's eigen-analysis method [1]. The name MUSIC is an acronym for MULTiple SIGNAL Classification. The *eigenvector method*, which uses eigenvalue weighting, is also supported [2]. The calling syntax is similar to that of `psd`, which also performs spectrum estimation. `psd` uses the classical FFT-based approach while `pmusic` performs eigen-analysis of the signal's correlation matrix.

`[Pxx, f] = pmusic(x, p)` and

`[Pxx, f] = pmusic(x, [p thresh])` return `Pxx`, the power spectrum estimate, and `f`, a vector of frequencies at which the PSD is estimated. `x` is the input signal, where

- A row or column vector represents one observation of the process output (for example, one “signal”)
- A rectangular (possibly square) array assumes that each column of `x` is a separate observation of the process output (for example, each column is one output of an array of sensors, as in array processing)
- A square matrix, given the trailing argument 'corr', represents a correlation matrix

The second argument is a one- or two-element vector, either `p` or `[p thresh]`. If only `p` is specified, the signal subspace dimension is `p`. If `[p thresh]` is specified, `thresh` is multiplied by λ_{\min} , the smallest eigenvalue; eigenvalues below the threshold $\lambda_{\min} * \text{thresh}$ are assigned to the noise subspace. In this case, `p` is the maximum dimension of the signal subspace.

CAUTION `pmusic` must assign eigenvectors to the noise and signal subspaces, but this is very difficult to do in practice. The two parameters `p` and `thresh` are provided for flexibility and control.

`[Pxx, f] = pmusic(x, [p thresh], nfft, Fs, window, noverlap)` specifies the FFT length `nfft` (default is 256) and the sampling frequency for the signal `Fs` (default is 2). If `Fs` is specified, the output frequency vector `f` is scaled by this value. If the input signal is real-valued, the frequency range is 0 to $F_s/2$; for the complex case, it is 0 to F_s . `window` is a scalar specifying the rectangular window length, or a vector giving the actual window coefficients. `noverlap`, used in conjunction with `window`, is a scalar that gives the number of points by which to overlap successive windows.

`[Pxx, f] = pmusic(x, ..., 'corr')` forces `x` to be taken as a correlation matrix. In this case, the arguments `window` and `noverlap` are ignored.

`[Pxx, f] = pmusic(x, ..., 'ev')` selects the eigenvector variant of the MUSIC estimator. See the “Algorithm” section below for an explanation of how this is different from the MUSIC method.

`[Pxx, f, evecs, sval s] = pmusic(x, ...)` returns two additional arguments. `evecs` is a matrix of eigenvectors spanning the noise subspace (one per column). `sval s` is either a vector of singular values (squared) from `svd` or a vector of eigenvalues of the correlation matrix when the `'corr'` option is present.

Remarks

The input `x` can be a vector or a matrix. `x` can be interpreted as signal data or as a correlation matrix, in one of three ways:

- `x` is a vector of signal values (row or column). In this case, the dimension of the eigenvectors must be given. This is done either by taking the default value of $2 \times p$ or by specifying a window length using `window`.
- `x` is a rectangular (m -by- n , possibly square) matrix. In this case, each column of `x` is a separate observation signal that enters into the SVD analysis, n is

the number of observations, and the dimension of the eigenvectors is equal to m , the length of a column.

- x is a square matrix and the trailing 'corr' is present. x is treated as a correlation matrix. In this case, the matrix must have only real, nonnegative eigenvalues.

The inputs p and thresh can determine the number of noise eigenvectors in one of three ways:

- If $\text{thresh} < 1$, or if it is unspecified, the number of eigenvectors spanning the signal subspace will be equal to p . p must be an integer satisfying $0 \leq p < n$, where n is the dimension of the eigenvectors. This dimension n is the column length in the data matrix case, the matrix size in the correlation matrix case, or the window length for signal vectors. The value of thresh is unused.
- If $p \geq n$, thresh must be at least 1. thresh is used as the multiplier to determine an absolute threshold for splitting the eigenvalues between the signal and noise subspaces:

$$\lambda_k \leq (\text{thresh}) \min\{\lambda_k\} \Rightarrow \{\lambda_k, v_k\} \text{ belong to noise subspace}$$

If $\text{thresh} < 1$, there will be no noise eigenvectors. This case is not allowed and gives the following error message:

Noise subspace dimension cannot be zero.

- When $p < n$ and $\text{thresh} \geq 1$, p specifies the maximum number of signal eigenvectors. However, the threshold test specified by thresh can also take eigenvectors from the signal subspace and assign them to the noise subspace.

Examples

This example analyzes a signal vector xx , assuming that two real signals are present in the signal subspace. In this case, the dimension of the signal subspace is 4 because each real sinusoid is the sum of two complex exponentials:

```
nn = 0:199;
xx = cos(0.257*pi*nn) + sin(0.2*pi*nn) + 0.01*randn(size(nn));
[PP, ff] = pmusic(xx, 4);
```

This example analyzes the same signal vector xx with an eigenvalue cutoff of 10% above the minimum. Setting $p = \text{Inf}$ forces the signal/noise subspace

decision to be based on thresh. Use eigenvectors of dimension 7 and a sampling frequency Fs of 8 kHz:

```
[PP, ff] = pmusic(xx, [Inf, 1, 1], [], 8000, 7); % window length = 7
```

With the third and fourth outputs, by plotting the zeros of the noise-eigenvector polynomials, it is possible to create a “Root-MUSIC” algorithm, as the following zplane plot illustrates:

```
[PP, ff, v_noise] = pmusic(xx, 4);
for kk = 1:size(v_noise, 2)
    rr(:, kk) = roots(v_noise(:, kk));
end
zplane(rr)
```

Assume that RR is a square correlation matrix (for example, 7-by-7):

```
RR = toeplitz(cos(0.1*pi*[0:6])) + 0.1*eye(7);
[PP, ff] = pmusic(RR, 4, 'corr');
```

Make an observation matrix xx that is rectangular (100-by-7):

```
xx = reshape(cos(0.257*pi*(0:699)), 7, 100) + 0.1*randn(7, 100);
[PP, ff] = pmusic(xx, 4);
```

Use the same signal, but let pmusic form the 100-by-7 data matrix using its window and overlap inputs. In addition, use a longer FFT:

```
yy = xx(:);
[PP, ff] = pmusic(yy, 4, 512, [], 7, 0);
```

If we set $p = 0$, all the eigenvectors are assigned to the noise subspace. 'ev' specifies the eigenvector weighting. This turns out to be equivalent to MVDL (Capon's MLM):

```
[PP, ff] = pmusic(RR, 0, 'ev', 'corr');
```

Algorithm

The MUSIC estimate is given by the formula

$$P_{music}(f) = \frac{1}{\mathbf{e}^H(f) \left(\sum_{k=p+1}^N \mathbf{v}_k \mathbf{v}_k^H \right) \mathbf{e}(f)} = \frac{1}{\sum_{k=p+1}^N \left| \mathbf{v}_k^H \mathbf{e}(f) \right|^2}$$

where N is the dimension of the eigenvectors and \mathbf{v}_k is the k -th eigenvector of the correlation matrix of the input signal. The integer p is the dimension of the signal subspace, so the eigenvectors \mathbf{v}_k used in the sum correspond to the smallest eigenvalues and also span the noise subspace. The vector $\mathbf{e}(f)$ consists of complex exponentials, so the inner product

$$\mathbf{v}_k^H \mathbf{e}(f)$$

amounts to a Fourier transform. The second form is preferred for computation because the FFT is computed for each \mathbf{v}_k and then the squared magnitudes are summed.

In the eigenvector method, the summation is weighted by the eigenvalues λ_k of the correlation matrix:

$$P_{ev}(f) = \frac{1}{\sum_{k=p+1}^N |\mathbf{v}_k^H \mathbf{e}(f)|^2 / \lambda_k}$$

The function relies on the svd matrix decomposition in the signal case, and it uses the `ei g` function for analyzing the correlation matrix. If SVD is used, the correlation matrix is never explicitly computed, but the singular values are the λ_k .

Diagnostics

There must be at least one output argument and at least two inputs; otherwise, `pmusic` stops and gives one of the following error messages:

- Must have at least 1 output argument.
- Must have at least 2 input arguments.

The first argument must be a full matrix, otherwise `pmusic` gives the following error message:

- Input signal or correlation cannot be sparse.

If the second argument was entered as an empty matrix, or if it has more than two elements, or if it has negative or non-integer elements, `pmusic` gives one of the following error messages:

P cannot be empty.
 Second input must have only 1 or 2 elements.
 P must be an integer.
 Second input must contain non-negative entries.

If the value of `p` is too large with respect to the eigenvector dimension, and `thresh` is less than 1, no eigenvectors can be assigned to the noise subspace and the algorithm fails. In this case, `pmusic` gives the following error message:

Noise subspace dimension cannot be zero.

If the '`corr`' parameter is used, then the first input must be a square correlation matrix. If it is not, `pmusic` gives the following error message:

Correlation matrix (R) is not square.

The correlation matrix is then checked for validity; if it fails, `pmusic` gives the following error message:

Correlation matrix (R) has negative or complex eigenvalue.

See Also

<code>lpc</code>	Linear prediction coefficients.
<code>pburg</code>	Power spectrum estimate using the Burg method.
<code>pmtm</code>	Power spectrum estimate using the multitaper method (MTM).
<code>prony</code>	Prony's method for time domain IIR filter design.
<code>psd</code>	Estimate the power spectral density (PSD) of a signal using Welch's method.
<code>pyul ear</code>	Power spectrum estimate using Yule-Walker AR method.

References

- [1] Schmidt, R.O. "Multiple Emitter Location and Signal Parameter Estimation." *IEEE Trans. Antennas Propagation*. Vol. AP-34 (March 1986). Pgs. 276-280.
- [2] Marple, S.L. *Digital Spectral Analysis*. Englewood Cliffs, NJ: Prentice Hall, 1987. Pgs. 373-378.

poly2rc

Purpose Reflection coefficients from polynomial coefficients.

Syntax `k = poly2rc(a)`

Description `k = poly2rc(a)` finds the reflection coefficients of the lattice structure of the discrete filter `a`. `a` must be real, and `a(1)` cannot be 0. `k` is a row vector of size `length(a)-1`.

A simple, fast way to check if `a` has all of its roots inside the unit circle is to check if each of the elements of `k` have magnitude less than 1:

```
stable = all(abs(poly2rc(a)) < 1)
```

Example Consider an IIR filter given by

```
a = [ 1.0000    0.6149    0.9899    0.0000    0.0031   -0.0082];
```

Its reflection coefficient representation is

```
k = poly2rc(a)
```

```
k =  
    0.3090    0.9800    0.0031    0.0082   -0.0082
```

Limitations If `abs(k(i)) == 1` for any `i`, finding the reflection coefficients is an ill-conditioned problem. `poly2rc` will return some NaNs and provide a warning message in this case.

Algorithm `poly2rc` implements the recursive relationship:

$$k(n) = a_n(n)$$
$$a_{n-1}(m) = \frac{a_n(m) - k(n)a_n(n-m)}{1 - k(n)^2}, \quad m = 1, 2, \dots, n-1$$

This relationship is based on Levinson's recursion [1]. To implement it, `poly2rc` loops through `a` in reverse order after discarding its first element. For each loop iteration `i`, the function

- 1 sets `k(i)` equal to `a(i)`
- 2 applies the second relationship above to elements 1 through `i` of the vector `a`:

$$a = (a - k(i) * \text{flip}(\text{r}(a))) / (1 - k(i)^2);$$

See Also

<code>latc2tf</code>	Lattice filter to transfer function conversion.
<code>latcfilter</code>	Lattice and lattice-ladder filter implementation.
<code>rc2poly</code>	Polynomial coefficients from reflection coefficients.
<code>tf2latc</code>	Transfer function to lattice filter conversion.

References

- [1] McClellan, J. "Parametric Signal Modeling." *Advanced Topics in Signal Processing*. Oppenheim, A.V., and J.S. Lim, ed. Englewood Cliffs, NJ: Prentice Hall, 1988. Pgs. 20-21.

polystab

Purpose Stabilize polynomial.

Syntax `b = polystab(a)`

Description `polystab` stabilizes a polynomial with respect to the unit circle; it reflects roots with magnitudes greater than 1 inside the unit circle.

`b = polystab(a)` where `a` is a vector of polynomial coefficients normally in the z -domain:

$$a(z) = a(1) + a(2)z^{-1} + \cdots + a(na+1)z^{-na}$$

returns a row vector containing the stabilized polynomial.

Example `polystab` can convert a linear-phase filter into a minimum-phase filter with the same magnitude response:

```
h = fir1(25, 0.4);  
hmin = polystab(h) * norm(h) / norm(polystab(h));
```

Algorithm `polystab` finds the roots of the polynomial and maps those roots found outside the unit circle to the inside of the unit circle:

```
v = roots(a);  
vs = 0.5 * (sign(abs(v)-1) + 1);  
v = (1-vs) .* v + vs ./ conj(v);  
b = a(1) * poly(v);
```

See Also `roots` Polynomial roots (see the online *MATLAB Function Reference*).

Purpose Prony's method for time domain IIR filter design.

Syntax `[b, a] = prony(h, nb, na)`

Description Prony's method is an algorithm for finding an IIR filter with a prescribed time domain impulse response. It has applications in filter design, exponential signal modeling, and system identification (parametric modeling).

`[b, a] = prony(h, nb, na)` finds a filter with numerator order `nb`, denominator order `na`, and the time domain impulse response in `h`. `prony` returns the filter coefficients in row vectors `b` and `a`, of length `nb + 1` and `na + 1`, respectively. The filter coefficients are in descending powers of z .

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \dots + a(na+1)z^{-na}}$$

Example Recover the coefficients of a Butterworth filter from its impulse response:

```
[ b, a ] = butter(4, 0.2)
```

```
b =
```

```
0.0048    0.0193    0.0289    0.0193    0.0048
```

```
a =
```

```
1.0000   -2.3695    2.3140   -1.0547    0.1874
```

```
h = filter(b, a, [1 zeros(1, 25)]);
```

```
[bb, aa] = prony(h, 4, 4)
```

```
bb =
```

```
0.0048    0.0193    0.0289    0.0193    0.0048
```

```
ab =
```

```
1.0000   -2.3695    2.3140   -1.0547    0.1874
```

Algorithm `prony` implements the method described in reference [1]. This method uses a variation of the covariance method of AR modeling to find the denominator coefficients `a` and then finds the numerator coefficients `b` for which the impulse response of the output filter matches exactly the first `nb + 1` samples of `x`. The

filter is not necessarily stable, but potentially can recover the coefficients exactly if the data sequence is truly an autoregressive moving average (ARMA) process of the correct order.

See Also	butter	Butterworth analog and digital filter design.
	cheby1	Chebyshev type I filter design (passband ripple).
	cheby2	Chebyshev type II filter design (stopband ripple).
	ellip	Elliptic (Cauer) filter design.
	invfreqz	Discrete-time filter identification from frequency data.
	levinson	Levinson-Durbin recursion.
	lpc	Linear prediction coefficients.
	stmcb	Linear model using Steiglitz-McBride iteration.

References [1] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Pgs. 226-228.

Purpose	Estimate the power spectral density (PSD) of a signal using Welch's method.
Syntax	<pre> Pxx = psd(x) Pxx = psd(x, nfft) [Pxx, f] = psd(x, nfft, Fs) Pxx = psd(x, nfft, Fs, window) Pxx = psd(x, nfft, Fs, window, noverlap) Pxx = psd(x, ..., 'dfFlag') [Pxx, Pxxc, f] = psd(x, nfft, Fs, window, noverlap, p) psd(x, ...) </pre>
Description	<p><code>Pxx = psd(x)</code> estimates the power spectrum of the sequence <code>x</code> using the Welch method of spectral estimation. <code>Pxx = psd(x)</code> uses the following default values:</p> <ul style="list-style-type: none"> • <code>nfft = min(256, length(x))</code> • <code>Fs = 2</code> • <code>window = hanning(nfft)</code> • <code>nooverlap = 0</code> <p><code>nfft</code> specifies the FFT length that <code>psd</code> uses. This value determines the frequencies at which the power spectrum is estimated. <code>Fs</code> is a scalar that specifies the sampling frequency. <code>window</code> specifies a windowing function and the number of samples <code>psd</code> uses in its sectioning of the <code>x</code> vector. <code>nooverlap</code> is the number of samples by which the sections overlap. Any arguments that you omit from the end of the input parameter list use the default values shown above.</p> <p>If <code>x</code> is real, <code>psd</code> estimates the spectrum at positive frequencies only; in this case, the output <code>Pxx</code> is a column vector of length <code>nfft/2+1</code> for <code>nfft</code> even and <code>(nfft+1)/2</code> for <code>nfft</code> odd. If <code>x</code> is complex, <code>psd</code> estimates the spectrum at both positive and negative frequencies and <code>Pxx</code> has length <code>nfft</code>.</p> <p><code>Pxx = psd(x, nfft)</code> uses the specified FFT length <code>nfft</code> in estimating the power spectrum for <code>x</code>. Specify <code>nfft</code> as a power of 2 for fastest execution.</p> <p><code>[Pxx, f] = psd(x, nfft, Fs)</code> returns a vector <code>f</code> of frequencies at which the function evaluates the PSD. <code>f</code> is the same size as <code>Pxx</code>, so <code>plot(f, Pxx)</code> plots the power spectrum versus properly scaled frequency. <code>Fs</code> has no effect on the output <code>Pxx</code>; it is a frequency scaling multiplier.</p>

`Pxx = psd(x, nfft, Fs, window)` specifies a windowing function and the number of samples per section of the `x` vector. If you supply a scalar for `window`, `psd` uses a Hanning window of that length. The length of the window must be less than or equal to `nfft`; `psd` zero pads the sections if the length of the window is less than `nfft`. `psd` returns an error if the length of the window is greater than `nfft`.

`Pxx = psd(x, nfft, Fs, window, noverlap)` overlaps the sections of `x` by `noverlap` samples.

You can use the empty matrix `[]` to specify the default value for any input argument except `x`. For example,

```
psd(x, [], 10000)
```

is equivalent to

```
psd(x)
```

but with a sampling frequency of 10,000 Hz instead of the default of 2 Hz.

`Pxx = psd(x, ..., 'dflag')` specifies a detrend option, where *dflag* is

- `linear`, to remove the best straight-line fit from the pre-windowed sections of `x`
- `mean`, to remove the mean from the pre-windowed sections of `x`
- `none`, for no detrending (default)

The *dflag* parameter must appear last in the list of input arguments. `psd` recognizes a *dflag* string no matter how many intermediate arguments are omitted.

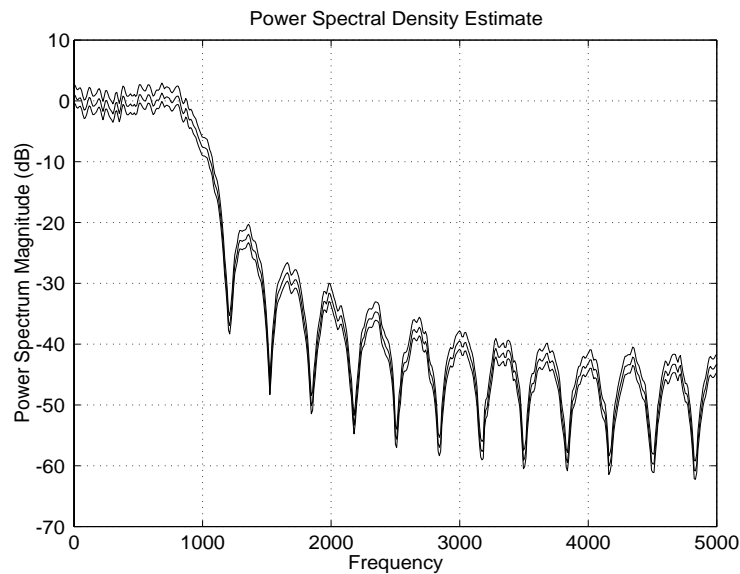
`[Pxx, Pxxc, f] = psd(x, nfft, Fs, window, noverlap, p)` where `p` is a positive scalar between 0 and 1 returns a vector `Pxxc` that contains an estimate of the `p`*100 percent confidence interval for `Pxx`. `Pxxc` is a two-column matrix that is the same length as `Pxx`. The interval `[Pxxc(:, 1), Pxxc(:, 2)]` covers the true PSD with probability `p`. `plot(f, [Pxx Pxxc])` plots the power spectrum inside the `p`*100 percent confidence interval. If unspecified, `p` defaults to 0.95.

`psd(x, ...)` with no output arguments plots the PSD versus frequency in the current figure window. If the `p` parameter is specified, the plot includes the confidence interval.

Example

Generate a colored noise signal and plot its PSD with a confidence interval of 95%. Specify a length 1024 FFT, a 512-point Kaiser window with no overlap, and a sampling frequency of 10 kHz:

```
h = fir1(30, 0.2, boxcar(31)); % design a lowpass filter
r = randn(16384, 1);          % white noise
x = filter(h, 1, r);           % color the noise
psd(x, 1024, 10000, kaiser(512, 5), 0, 0.95)
```



Algorithm `psd` calculates the power spectral density using Welch's method (see references [1] and [2]):

- 1 It applies the window specified by the `window` vector to each successive detrended section of `x`.
- 2 It transforms each section with an `nfft`-point FFT.
- 3 It forms the periodogram of each section by scaling the magnitude squared of each transform.
- 4 It averages the periodograms of the overlapping sections to form `Pxx`, the power spectrum of `x`.

The number of sections that `psd` averages is

$$k = \text{fix}((\text{length}(x) - \text{noverlap}) / (\text{length}(\text{window}) - \text{noverlap}))$$

Diagnostics An appropriate diagnostic message is displayed when incorrect arguments to `psd` are used:

- Requires `window`'s length to be no greater than FFT length.
- Requires `NOVERLAP` to be strictly less than the window length.
- Requires positive integer values for `NFFT` and `NOVERLAP`.
- Requires confidence parameter to be a scalar between 0 and 1.
- Requires vector input.

See Also	<code>cohere</code>	Estimate magnitude squared coherence function between two signals.
	<code>csd</code>	Estimate the cross spectral density (CSD) of two signals.
	<code>pburg</code>	Power spectrum estimate using the Burg method.
	<code>pmtm</code>	Power spectrum estimate using the multitaper method (MTM).
	<code>pmusic</code>	Power spectrum estimate using MUSIC eigenvector method.
	<code>pyulear</code>	Power spectrum estimate using Yule-Walker AR method.
	<code>specgram</code>	Time-dependent frequency analysis (spectrogram).
	<code>tfe</code>	Transfer function estimate from input and output.

References

- [1] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1975. Pgs. 399-419.
- [2] Welch, P.D. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms." *IEEE Trans. Audio Electroacoust.* Vol. AU-15 (June 1967). Pgs. 70-73.
- [3] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 311-312.

pulstran

Purpose Pulse train generator.

Syntax

```
y = pulstran(t, d, 'func')  
y = pulstran(t, d, 'func', p1, p2, ...)  
y = pulstran(t, d, p, Fs)  
y = pulstran(t, d, p)
```

Description `pulstran` generates pulse trains from continuous functions or sampled prototype pulses.

`y = pulstran(t, d, 'func')` generates a pulse train based on samples of a continuous function, `'func'`, where `func` is

- `gauspuls`, for Gaussian-modulated sinusoidal pulse generator
- `rectpuls`, for sampled aperiodic rectangle generator
- `tripuls`, for sampled aperiodic triangle generator

`pulstran` is evaluated `length(d)` times and returns the sum of the evaluations
 $y = \text{func}(t-d(1)) + \text{func}(t-d(2)) + \dots$

The function is evaluated over the range of argument values specified in array `t`, after removing a scalar argument offset taken from the vector `d`. Note that `func` must be a vectorized function that can take an array `t` as an argument.

An optional gain factor may be applied to each delayed evaluation by specifying `d` as a two-column matrix, with the offset defined in column 1 and associated gain in column 2 of `d`. Note that a row vector will be interpreted as specifying delays only.

`pulstran(t, d, 'func', p1, p2, ...)` allows additional parameters to be passed to `'func'` as necessary. For example,

$$\text{func}(t-d(1), p1, p2, \dots) + \text{func}(t-d(2), p1, p2, \dots) + \dots$$

`pulstran(t, d, p, Fs)` generates a pulse train that is the sum of multiple delayed interpolations of the prototype pulse in vector `p`, sampled at the rate `Fs`, where `p` spans the time interval $[0, (\text{length}(p)-1)/Fs]$, and its samples are identically 0 outside this interval. By default, linear interpolation is used for generating delays.

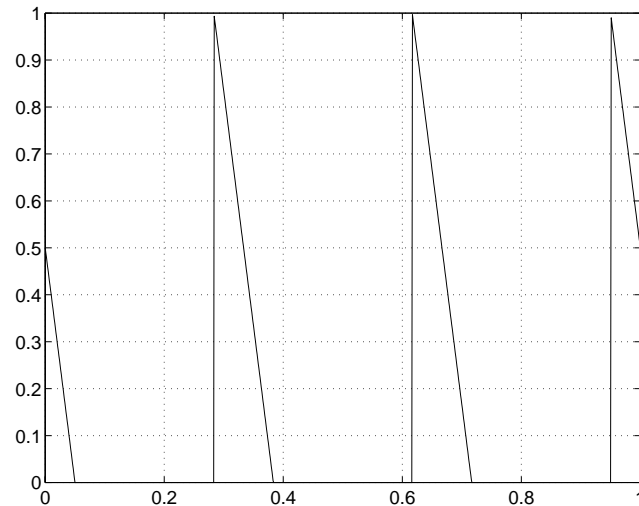
`pulstran(t, d, p)` assumes that the sampling rate F_s is equal to 1 Hz.

`pulstran(..., 'func')` specifies alternative interpolation methods. See `interp1` for a list of available methods.

Examples

This example generates an asymmetric sawtooth waveform with a repetition frequency of 3 Hz and a sawtooth width of 0.1 sec. It has a signal length of 1 sec and a 1 kHz sample rate:

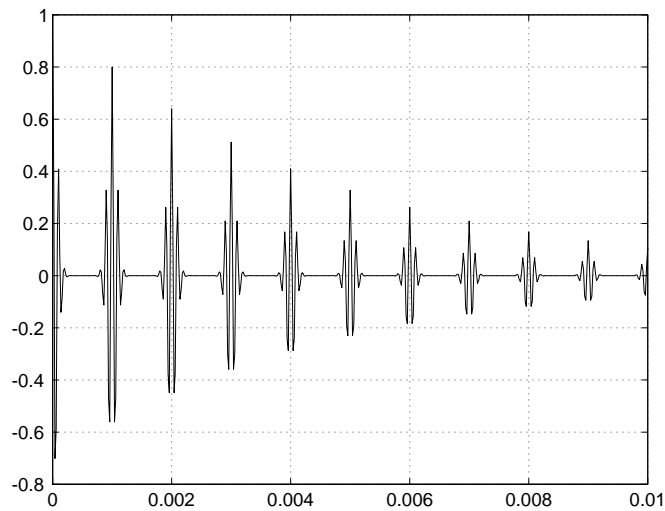
```
t = 0 : 1/1e3 : 1;           % 1 kHz sample freq for 1 sec
d = 0 : 1/3 : 1;             % 3 Hz repetition freq
y = pulstran(t, d, 'tripuls', 0.1, -1);
plot(t, y)
```



This example generates a periodic Gaussian pulse signal at 10 kHz, with 50% bandwidth. The pulse repetition frequency is 1 kHz, sample rate is 50 kHz, and

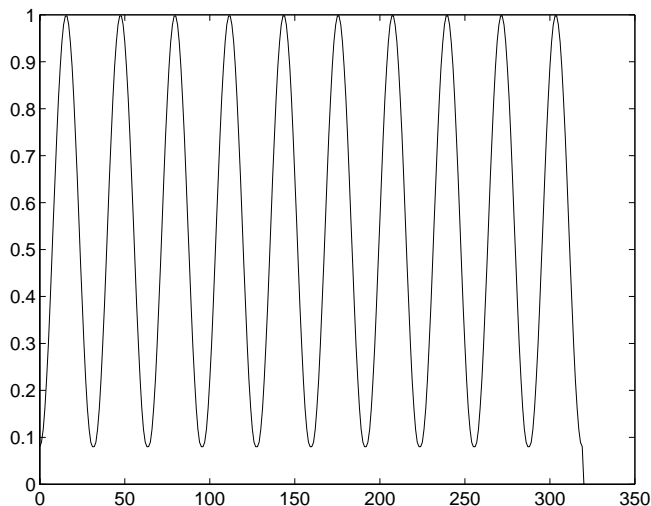
pulse train length is 10 msec. The repetition amplitude should attenuate by 0.8 each time:

```
t = 0 : 1/50E3 : 10e-3;  
d = [0 : 1/1E3 : 10e-3 ; 0.8.^(0:10)]';  
y = pulstran(t, d, 'gauspuls', 10e3, 0.5);  
plot(t, y)
```



This example generates a train of 10 Hamming windows:

```
p = hamming(32);
t = 0:320; d = (0:9)' * 32;
y = pulstran(t, d, p);
plot(t, y)
```



See Also

chirp	Swept-frequency cosine generator.
cos	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
diric	Dirichlet or periodic sinc function.
gauspuls	Gaussian-modulated sinusoidal pulse generator.
rectpuls	Sampled aperiodic rectangle generator.
sawtooth	Sawtooth or triangle wave generator.
sin	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
sinc	Sinc or $\sin(\pi t)/\pi t$ function.
square	Square wave generator.
tripuls	Sampled aperiodic triangle generator.

Purpose Power spectrum estimate using Yule-Walker AR method.

Syntax

```
[Pxx, freq] = pyulear(x, p)
[Pxx, freq] = pyulear(x, p, nfft, Fs, 'corr')
[Pxx, freq, a] = pyulear(x, p, nfft, Fs, 'corr')
pyulear(...)
```

Description `pyulear` estimates the power spectral density (PSD) of the signal vector $x[n]$ or correlation matrix \mathbf{R} using the Yule-Walker AR method. It derives an all-pole model to represent the spectrum, so the correct choice of the model order p is crucial.

`[Pxx, freq] = pyulear(x, p)` returns `Pxx`, the power spectrum estimate, and `freq`, a vector of frequencies at which the PSD was estimated. `x` is the input signal, or the input correlation matrix, where

- A row or column vector represents one signal
- A square, Hermitian symmetric matrix represents a correlation matrix (when 'corr' is used)
- A rectangular array assumes that each column of `x` is a separate “look” at the signal (as in array processing)

`p` is the model order for the all-pole filter.

`[Pxx, freq] = pyulear(x, p, nfft, Fs, 'corr')` specifies the FFT length `nfft` (default is 256) and the sampling frequency for the signal `Fs` (default is 2). If `Fs` is specified, the output frequency vector `freq` is scaled by this value. If the input signal is real-valued, `freq` ranges from 0 to `Fs/2`. If the input signal is complex, `freq` ranges from 0 to `Fs`. 'corr' is a text string to specify a correlation option. Specifying 'corr' forces `x` to be taken as a correlation matrix. 'corr' must appear at the end of the argument list.

`[Pxx, freq, a] = pyulear(x, p, nfft, Fs, 'corr')` returns vector `a` of filter coefficients for the all-pole filter model.

`pyulear(...)` plots the power spectral density in the first available figure window.

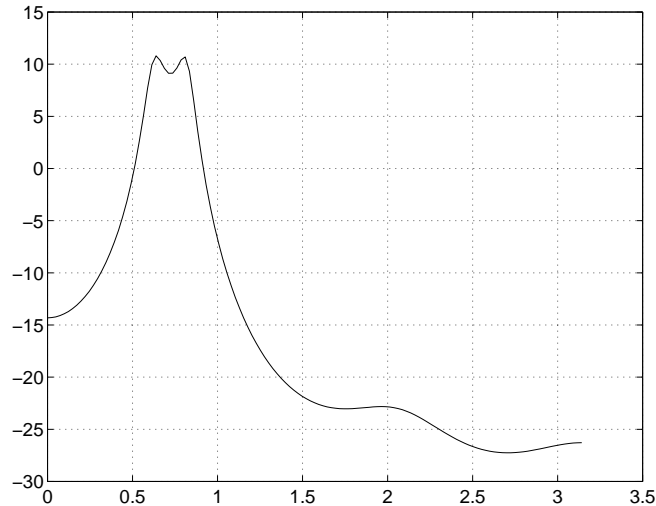
Examples

This example analyzes a sequence $x[n]$, assuming that two real signals are present in the signal subspace. In this case, the model order must be four or larger, because each real sinusoid is the sum of two complex exponentials. Experience shows that taking a larger model order than the minimum seems to work better.

```
% Create xx as a signal vector.

nn = 0:199;
randn('seed', 0)
xx = cos(0.257*pi*nn) + sin(0.2*pi*nn) + 0.01*randn(size(nn));
[PP, ff, aa] = pyulear(xx, 7); % 7th order model

plot(ff*pi, 10*log10(PP)) % Plot the pole locations.
```



The following examples use `xx` as a correlation matrix and a data matrix:

```
% Assume that RR is a square corr. matrix (for example, 7 by 7).  
  
RR = toeplitz(cos(0.1*pi*[0:6])) + 0.1*eye(7);  
[PP, ff] = pyulear(RR, 4, 'corr');  
  
% Create a matrix of 100 observations of a stochastic process:  
% xx is rectangular (7 by 100).  
  
xx = reshape(cos(0.257*pi*(0:699)), 7, 100) + 0.1*randn(7, 100);  
[PP, ff] = pyulear(xx, 4); % default FFT length = 256  
  
% Same (7 by 100) matrix with a longer FFT.  
  
[PP, ff] = pyulear(xx, 4, 512);
```

Algorithm

The Yule-Walker AR estimate is given by the formula:

$$P_{yule}(f) = \frac{1}{|\mathbf{a}^H \mathbf{e}(f)|^2}$$

where the vector of all-pole filter coefficients \mathbf{a} is the solution of the autocorrelation normal equation:

$$\mathbf{R}\mathbf{a} = \mathbf{r}$$

The matrix \mathbf{R} is the autocorrelation matrix, which should be Toeplitz [1]. The elements of the vector \mathbf{r} are also correlations.

Diagnostics

There must be at least one output argument and at least two inputs; otherwise, `pyulear` stops and generates one of the following error messages:

```
Must have at least 1 output argument.  
Must have at least 2 input arguments.
```

The first argument must be a full matrix, otherwise `pyulear` generates the following error message:

```
Input signal or correlation cannot be sparse.
```

If you specify an empty matrix for the second argument, pyul ear generates the following error message:

```
Model order must be given, empty not allowed.
```

If the final argument is the string 'corr', then the first input must be a square correlation matrix that is also Hermitian symmetric, otherwise pyul ear gives the following error messages:

```
Correlation matrix (R) is not square.
```

```
Correlation matrix (R) is not Hermitian symmetric.
```

See Also

lpc	Linear prediction coefficients.
pburg	Power spectrum estimate using the Burg method.
pmtm	Power spectrum estimate using the multitaper method (MTM).
pmusic	Power spectrum estimate using MUSIC eigenvector method.
prony	Prony's method for time domain IIR filter design.
psd	Estimate the power spectral density (PSD) of a signal using Welch's method.

References

[1] Marple, S.L. *Digital Spectral Analysis*. Englewood Cliffs, NJ: Prentice Hall, 1987. Chapter 7.

rc2poly

Purpose	Polynomial coefficients from reflection coefficients.								
Syntax	$a = \text{rc2poly}(k)$								
Description	$a = \text{rc2poly}(k)$ finds the filter coefficients a , with $a(1) = 1$, from the real reflection coefficients k of the lattice structure of a discrete filter. a is row vector of length $k + 1$.								
Example	<p>Consider an IIR filter given by reflection coefficients k:</p> $k = [0.3090 \quad 0.9800 \quad 0.0031 \quad 0.0082 \quad -0.0082];$ <p>Its polynomial representation is</p> $a = \text{rc2poly}(k)$ $a =$ $1.0000 \quad 0.6149 \quad 0.9899 \quad 0.0000 \quad 0.0031 \quad -0.0082$								
Algorithm	<p>rc2poly computes a using Levinson's recursion [1]. The function</p> <ol style="list-style-type: none">1 Sets the output vector a to the first element of k2 Loops through the remaining elements of k For each loop iteration i, $a = [a + a(i-1:-1:1)*k(i) \quad k(i)]$.3 Implements $a = [1 \ a]$								
See Also	<table><tr><td><code>latc2tf</code></td><td>Lattice filter to transfer function conversion.</td></tr><tr><td><code>latcfilt</code></td><td>Lattice and lattice-ladder filter implementation.</td></tr><tr><td><code>poly2rc</code></td><td>Reflection coefficients from polynomial coefficients.</td></tr><tr><td><code>tf2latc</code></td><td>Transfer function to lattice filter conversion.</td></tr></table>	<code>latc2tf</code>	Lattice filter to transfer function conversion.	<code>latcfilt</code>	Lattice and lattice-ladder filter implementation.	<code>poly2rc</code>	Reflection coefficients from polynomial coefficients.	<code>tf2latc</code>	Transfer function to lattice filter conversion.
<code>latc2tf</code>	Lattice filter to transfer function conversion.								
<code>latcfilt</code>	Lattice and lattice-ladder filter implementation.								
<code>poly2rc</code>	Reflection coefficients from polynomial coefficients.								
<code>tf2latc</code>	Transfer function to lattice filter conversion.								
References	[1] McClellan, J. "Parametric Signal Modeling." <i>Advanced Topics in Signal Processing</i> . Oppenheim, A.V., and J.S. Lim, ed. Englewood Cliffs, NJ: Prentice Hall, 1988. Pgs. 20-21.								

Purpose	Real cepstrum and minimum phase reconstruction.	
Syntax	<pre>y = rceps(x) [y, ym] = rceps(x)</pre>	
Description	<p>The <i>real cepstrum</i> is the inverse Fourier transform of the real logarithm of the magnitude of the Fourier transform of a sequence.</p> <p><code>rceps(x)</code> returns the real cepstrum of the real sequence <code>x</code>. The real cepstrum is a real-valued function.</p> <p><code>[y, ym] = rceps(x)</code> returns both the real cepstrum <code>y</code> and a minimum phase reconstructed version <code>ym</code> of the input sequence.</p>	
Algorithm	<p><code>rceps</code> is an M-file implementation of algorithm 7.2 in [2], that is:</p> <pre>y = real (ifft(log(abs(fft(x)))));</pre> <p>Appropriate windowing in the cepstral domain forms the reconstructed minimum phase signal:</p> <pre>w = [1; 2*ones(n/2-1, 1); ones(1 - rem(n, 2), 1); zeros(n/2-1, 1)]; ym = real (ifft(exp(fft(w.*y))));</pre>	
See Also	<pre>cceps</pre> <pre>fft</pre> <pre>hilbert</pre> <pre>icceps</pre> <pre>unwrap</pre>	<p>Complex cepstral analysis.</p> <p>One-dimensional fast Fourier transform.</p> <p>Hilbert transform.</p> <p>Inverse complex cepstrum.</p> <p>Unwrap phase angles.</p>
References	<p>[1] Oppenheim, A.V., and R.W. Schaffer. <i>Digital Signal Processing</i>. Englewood Cliffs, NJ: Prentice Hall, 1975.</p> <p>[2] IEEE. <i>Programs for Digital Signal Processing</i>. IEEE Press. New York: John Wiley & Sons, 1979.</p>	

rectpuls

Purpose Sampled aperiodic rectangle generator.

Syntax `y = rectpuls(t)`
 `y = rectpuls(t, w)`

Description `y = rectpuls(t)` returns a continuous, aperiodic, unity-height rectangular pulse at the sample times indicated in array `t`, centered about `t = 0` and with a default width of 1. Note that the interval of non-zero amplitude is defined to be open on the right, that is, `rectpuls(-0.5) = 1` while `rectpuls(0.5) = 0`.

`y = rectpuls(t, w)` generates a rectangle of width `w`.

`rectpuls` is typically used in conjunction with the pulse train generating function, `pulstran`.

See Also	<code>chirp</code>	Swept-frequency cosine generator.
	<code>cos</code>	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
	<code>diric</code>	Dirichlet or periodic sinc function.
	<code>gauspuls</code>	Gaussian-modulated sinusoidal pulse generator.
	<code>pulstran</code>	Pulse train generator.
	<code>sawtooth</code>	Sawtooth or triangle wave generator.
	<code>sin</code>	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
	<code>sinc</code>	Sinc or $\sin(\pi t)/\pi t$ function.
	<code>square</code>	Square wave generator.
	<code>tripuls</code>	Sampled aperiodic triangle generator.

Purpose

Parks-McClellan optimal FIR filter design.

Syntax

```

b = remez(n, f, a)
b = remez(n, f, a, w)
b = remez(n, f, a, 'ftype')
b = remez(n, f, a, w, 'ftype')
b = remez(..., {lgrid})
b = remez(n, f, 'fresp', w)
b = remez(n, f, 'fresp', w, 'ftype')
b = remez(n, f, {'fresp', p1, p2, ...}, w)
b = remez(n, f, {'fresp', p1, p2, ...}, w, 'ftype')
[b, delta] = remez(...)
[b, delta, opt] = remez(...)

```

Description

`remez` designs a linear-phase FIR filter using the Parks-McClellan algorithm [1]. The Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with an optimal fit between the desired and actual frequency responses. The filters are optimal in the sense that the maximum error between the desired frequency response and the actual frequency response is minimized. Filters designed this way exhibit an equiripple behavior in their frequency responses and hence are sometimes called *equiripple* filters.

`b = remez(n, f, a)` returns row vector `b` containing the $n+1$ coefficients of the order n FIR filter whose frequency-amplitude characteristics match those given by vectors `f` and `a`.

The output filter coefficients (taps) in `b` obey the symmetry relation

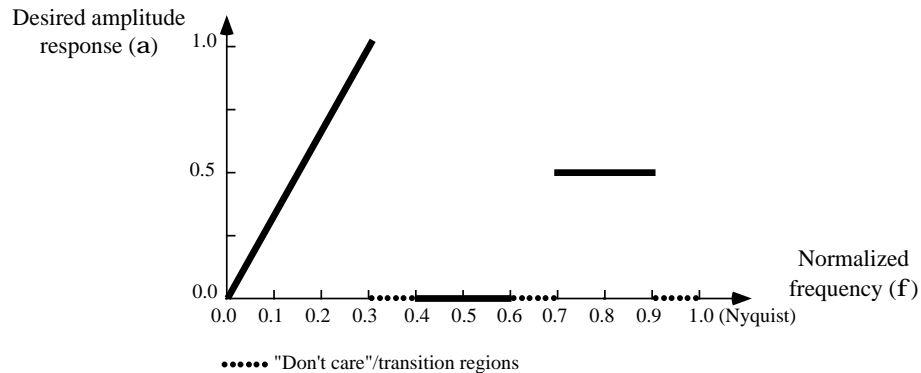
$$b(k) = b(n+2-k), \quad k = 1, \dots, n+1$$

Vectors f and a specify the frequency-magnitude characteristics of the filter:

- f is a vector of pairs of frequency points, specified in the range between 0 and 1, where 1 corresponds to half the sampling frequency (the Nyquist frequency). The frequencies must be in increasing order.
- a is a vector containing the desired amplitudes at the points specified in f .
The desired amplitude at frequencies between pairs of points $(f(k), f(k+1))$ for k odd is the line segment connecting the points $(f(k), a(k))$ and $(f(k+1), a(k+1))$.
The desired amplitude at frequencies between pairs of points $(f(k), f(k+1))$ for k even is unspecified. The areas between such points are transition or “don’t care” regions.
- f and a must be the same length. The length must be an even number.

The relationship between the f and a vectors in defining a desired frequency response is shown below:

```
f = [0 .3 .4 .6 .7 .9]
a = [0 1 0 0 .5 .5]
```



`remez(n, f, a, w)` uses the weights in vector w to weight the fit in each frequency band. The length of w is half the length of f and a , so there is exactly one weight per band.

`b = remez(n, f, a, 'ftype')` and

`b = remez(n, f, a, w, 'ftype')` specify a filter type, where *ftype* is

- `hilbert`, for linear-phase filters with odd symmetry (type III and type IV)
The output coefficients in `b` obey the relation $b(k) = -b(n+2-k)$, $k = 1, \dots, n+1$. This class of filters includes the Hilbert transformer, which has a desired amplitude of 1 across the entire band.

For example,

```
h = remez(30, [0.1 0.9], [1 1], 'hilbert');
```

designs an approximate FIR Hilbert transformer of length 31.

- `differentiator`, for type III and IV filters, using a special weighting technique

For nonzero amplitude bands, it weights the error by a factor of $1/f$ so that the error at low frequencies is much smaller than at high frequencies. For FIR differentiators, which have an amplitude characteristic proportional to frequency, these filters minimize the maximum relative error (the maximum of the ratio of the error to the desired amplitude).

`b = remez(..., {lgrid})` uses the integer `lgrid` to control the density of the frequency grid, which has roughly $(lgrid \cdot n) / (2 \cdot bw)$ frequency points, where `bw` is the fraction of the total frequency band interval $[0,1]$ covered by `f`.

Increasing `lgrid` often results in filters that are more exactly equiripple, but which take longer to compute. The default value of 16 is the minimum value that should be specified for `lgrid`. Note that the `{lgrid}` argument must be a 1-by-1 cell array.

`b = remez(n, f, 'fresp', w)` returns row vector `b` containing the $n+1$ coefficients of the order `n` FIR filter whose frequency-amplitude characteristics best approximate the response specified by function `fresp`. The function is called from within `remez` with the following syntax:

```
[dh, dw] = fresp(n, f, gf, w)
```

The arguments are similar to those for `remez`:

- `n` is the filter order.
- `f` is the vector of frequency band edges that appear monotonically between 0 and 1, where 1 is the Nyquist frequency.
- `gf` is a vector of grid points that have been linearly interpolated over each specified frequency band by `remez`. `gf` determines the frequency grid at which the response function must be evaluated, and contains the same data returned by `cremez` in the `fgri d` field of the `opt` structure.
- `w` is a vector of real, positive weights, one per band, used during optimization. `w` is optional in the call to `remez`; if not specified, it is set to unity weighting before being passed to '*fresp*'.
- `dh` and `dw` are the desired complex frequency response and band weight vectors, respectively, evaluated at each frequency in grid `gf`.

The predefined frequency response function (*fresp*) that `remez` calls is `remezfrf` in the `signal/private` directory.

`b = remez(n, f, {'fresp', p1, p2, ...}, w)` allows you to specify additional parameters (`p1`, `p2`, etc.) to pass to *fresp*. Note that `b = remez(n, f, a, w)` is a synonym for `b = remez(n, f, {'remezfrf', a}, w)`, where `a` is a vector containing the desired amplitudes at the points specified in `f`.

`b = remez(n, f, 'fresp', w, 'ftype')` and

`b = remez(n, f, {'fresp', p1, p2, ...}, w, 'ftype')` design antisymmetric (odd) rather than symmetric (even) filters, where '*ftype*' is either '*d*' for a differentiator or '*h*' for a Hilbert transformer.

In the absence of a specification for *ftype*, a preliminary call is made to *fresp* to determine the default symmetry property `sym`. This call is made using the syntax:

```
sym = fresp('defaults', {n, f, [], w, p1, p2, ...})
```

The arguments `n`, `f`, `w`, etc., may be used as necessary in determining an appropriate value for `sym`, which `remez` expects to be either '*even*' or '*odd*'. If the *fresp* function does not support this calling syntax, `remez` defaults to even symmetry.

`[b, del ta] = remez(...)` returns the maximum ripple height in del ta.

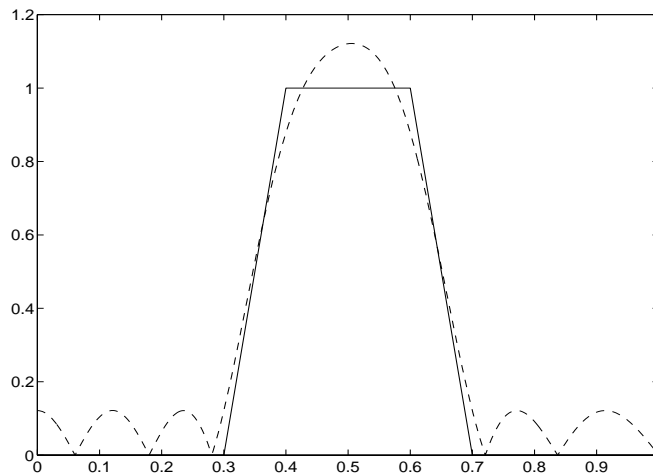
`[b, del ta, opt] = remez(...)` returns a structure, opt, of optional results with the following fields:

opt. fgri d	Frequency grid vector used for the filter design optimization
opt. des	Desired frequency response for each point in opt. fgri d
opt. wt	Weighting for each point in opt. fgri d
opt. H	Actual frequency response for each point in opt. fgri d
opt. error	Error at each point in opt. fgri d (opt. des–opt. H)
opt. i extr	Vector of indices into opt. fgri d for extremal frequencies
opt. fextr	Vector of extremal frequencies

Example

Graph the desired and actual frequency responses of a 17th-order Parks-McClellan bandpass filter:

```
f = [0 0.3 0.4 0.6 0.7 1]; a = [0 0 1 1 0 0];  
b = remez(17, f, a);  
[h, w] = freqz(b, 1, 512);  
plot(f, a, w/pi, abs(h))
```



Algorithm

`remez` is a MEX-file version of the original Fortran code from [1], altered to design arbitrarily long filters with arbitrarily many linear bands.

`remez` designs type I, II, III, and IV linear-phase filters. Type I and Type II are the defaults for n even and n odd, respectively, while Type III (n even) and Type IV (n odd) are obtained with the 'hilbert' and 'differentiator' flags. The different types of filters have different symmetries and certain constraints on their frequency responses (see reference [5] for more details):

Linear Phase Filter type	Filter Order n	Symmetry of Coefficients	Response $H(f)$, $f = 0$	Response $H(f)$, $f = 1$ (Nyquist)
Type I	Even	even:	No restriction	No restriction
Type II	Odd	$b(k) = b(n+2-k)$, $k = 1, \dots, n+1$	No restriction	$H(1) = 0$
Type III	Even	odd:	$H(0) = 0$	$H(1) = 0$
Type IV	Odd	$b(k) = -b(n+2-k)$, $k = 1, \dots, n+1$	$H(0) = 0$	No restriction

Diagnostics

An appropriate diagnostic message is displayed if incorrect arguments are used:

Filter order must be 3 or more.
 There should be one weight per band.
 Frequency and amplitude vectors must be the same length.
 The number of frequency points must be even.
 Frequencies must lie between 0 and 1.
 Frequencies must be specified in bands.
 Frequencies must be nondecreasing.
 Adjacent bands not allowed.

A more serious warning message is

-- Failure to Converge --
 Probable cause is machine rounding error.

In the rare event that you see this message, it is possible that the filter design may still be correct. Verify the design by checking its frequency response.

See Also

butter	Butterworth analog and digital filter design.
cheby1	Chebyshev type I filter design (passband ripple).
cheby2	Chebyshev type II filter design (stopband ripple).
cremez	Complex and nonlinear-phase equiripple FIR filter design
ellip	Elliptic (Cauer) filter design.
fir1	Window-based finite impulse response filter design—standard response.
fir2	Window-based finite impulse response filter design—arbitrary response.
fircls	Constrained least square FIR filter design for multiband filters.
fircls1	Constrained least square filter design for lowpass and highpass linear phase FIR filters.
firls	Least square linear-phase FIR filter design.
firrcos	Raised cosine FIR filter design.
remezord	Parks-McClellan optimal FIR filter order estimation.
yulewalk	Recursive digital filter design.

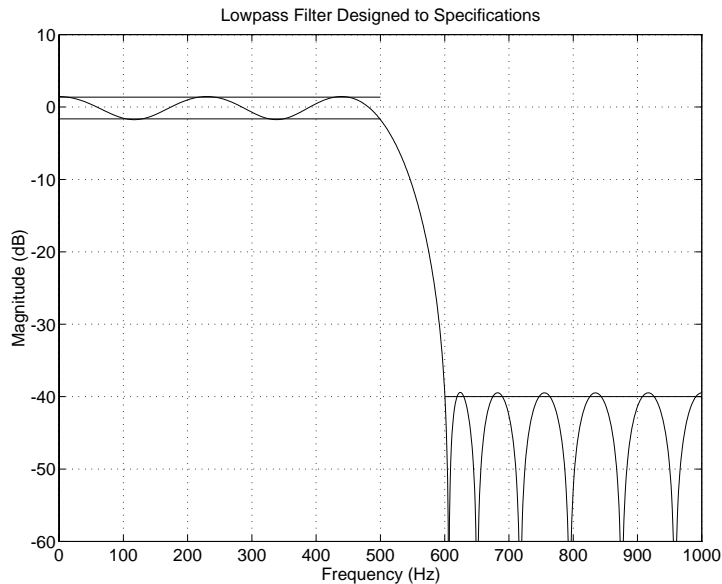
References

- [1] IEEE. *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979. Algorithm 5.1.
- [2] IEEE. *Selected Papers in Digital Signal Processing, II*. IEEE Press. New York: John Wiley & Sons, 1979.
- [3] Parks, T.W., and C.S. Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987. Pg. 83.
- [4] Rabiner, L.R., J.H. McClellan, and T.W. Parks. "FIR Digital Filter Design Techniques Using Weighted Chebyshev Approximations." *Proc. IEEE* 63 (1975).
- [5] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 256-266.

Purpose	Parks-McClellan optimal FIR filter order estimation.
Syntax	<pre>[n, fo, ao, w] = remezord(f, a, dev) [n, fo, ao, w] = remezord(f, a, dev, Fs) c = remezord(f, a, dev, Fs, ' cell ')</pre>
Description	<p><code>[n, fo, ao, w] = remezord(f, a, dev)</code> finds the approximate order, normalized frequency band edges, frequency band amplitudes, and weights that meet input specifications <code>f</code>, <code>a</code>, and <code>dev</code>, to use with the <code>remez</code> command.</p> <ul style="list-style-type: none"> • <code>f</code> is a vector of frequency band edges (between 0 and $F_s/2$, where F_s is the sampling frequency), and <code>a</code> is a vector specifying the desired amplitude on the bands defined by <code>f</code>. The length of <code>f</code> is twice the length of <code>a</code>, minus 2. The desired function is piecewise constant. • <code>dev</code> is a vector the same size as <code>a</code> that specifies the maximum allowable deviation or ripples between the frequency response and the desired amplitude of the output filter, for each band. <p>Use <code>remez</code> with the resulting order <code>n</code>, frequency vector <code>fo</code>, amplitude response vector <code>ao</code>, and weights <code>w</code> to design the filter <code>b</code> which approximately meets the specifications given by <code>remezord</code> input parameters <code>f</code>, <code>a</code>, and <code>dev</code>:</p> <pre>b = remez(n, fo, ao, w)</pre> <p><code>[n, fo, ao, w] = remezord(f, a, dev, Fs)</code> specifies a sampling frequency <code>Fs</code>. <code>Fs</code> defaults to 2 Hz, implying a Nyquist frequency of 1 Hz. You can therefore specify band edges scaled to a particular application's sampling frequency.</p> <p>In some cases <code>remezord</code> underestimates the order <code>n</code>. If the filter does not meet the specifications, try a higher order such as <code>n+1</code> or <code>n+2</code>.</p> <p><code>c = remezord(f, a, dev, Fs, ' cell ')</code> specifies a cell-array whose elements are the parameters to <code>remez</code>.</p>
Examples	Design a minimum-order lowpass filter with a 500 Hz passband cutoff frequency and 600 Hz stopband cutoff frequency, with a sampling frequency of

2000 Hz), at least 40 dB attenuation in the stopband, and less than 3 dB of ripple in the passband:

```
rp = 3;           % passband ripple
rs = 40;          % stopband ripple
Fs = 2000;        % sampling frequency
f = [500 600];    % cutoff frequencies
a = [1 0];        % desired amplitudes
% compute deviations
dev = [ (10^(rp/20)-1)/(10^(rp/20)+1)  10^(-rs/20) ];
[n, fo, ao, w] = remezord(f, a, dev, Fs);
b = remez(n, fo, ao, w);
[h, f] = freqz(b, 1, 1024, Fs);
plot(f, 20*log10(abs(h)))
```



Note that the filter falls slightly short of meeting the specifications. Using $n+1$ in the call to `remez` instead of `n` achieves the desired amplitude characteristics.

Design a lowpass filter with a 1500 Hz passband cutoff frequency and 2000 Hz stopband cutoff frequency, with a sampling frequency of 8000 Hz, a maximum stopband amplitude of 0.1, and a maximum passband error (ripple) of 0.01:

```
[n, fo, ao, w] = remezord( [1500 2000], [1 0], [0.01 0.1], 8000 );
b = remez(n, fo, ao, w);
```

This is equivalent to

```
c = remezord( [1500 2000], [1 0], [0.01 0.1], 8000, 'cell');
b = remez(c{:});
```

NOTE In some cases, remezord underestimates or overestimates the order n. If the filter does not meet the specifications, try a higher order such as n+1 or n+2.

Results are inaccurate if the cutoff frequencies are near 0 or the Nyquist frequency.

Algorithm	remezord uses the algorithm suggested in [1]. This method is inaccurate for band edges close to either 0 or the Nyquist frequency ($F_s/2$).	
Diagnostics	If the input parameter lengths are not consistent, remezord gives the following error messages: Requires M and DEV to be the same length. Length of F must be length(M)-2.	
See Also	buttord	Butterworth filter order selection.
	cheb1ord	Chebyshev type I filter order selection.
	cheb2ord	Chebyshev type II filter order selection.
	ellipord	Elliptic filter order selection.
	kaiserord	Estimate parameters for an FIR filter design with Kaiser window.
	remez	Parks-McClellan optimal FIR filter design.

References

- [1] Rabiner, L.R., and O. Herrmann. "The Predictability of Certain Optimum Finite Impulse Response Digital Filters." *IEEE Trans. on Circuit Theory*. Vol. CT-20, No. 4 (July 1973). Pgs. 401-408.
- [2] Rabiner, L.R., and B. Gold. *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1975. Pgs. 156-157.

Purpose Change sampling rate by any factor.

Syntax

```
y = resample(x, p, q)
y = resample(x, p, q, n)
y = resample(x, p, q, n, beta)
y = resample(x, p, q, b)
[y, b] = resample(x, p, q)
```

Description `y = resample(x, p, q)` resamples the sequence in vector `x` at p/q times the original sampling rate, using a polyphase filter implementation. The length of `y` is equal to $\text{ceil}(\text{length}(x) * p/q)$. `p` and `q` must be positive integers. If `x` is a matrix, `resample` works down the columns of `x`.

`resample` applies an anti-aliasing (lowpass) FIR filter to `x` during the resampling process. It designs the filter using `fir1` with a Kaiser window.

`y = resample(x, p, q, n)` uses `n` terms on either side of the current sample, `x(k)`, to perform the resampling. The length of the FIR filter `resample` uses is proportional to `n`; larger values of `n` provide better accuracy at the expense of more computation time. The default for `n` is 10. If you let `n = 0`, `resample` performs a nearest-neighbor interpolation:

$$y(k) = x(\text{round}((k-1) * q/p) + 1)$$

where $y(k) = 0$ if the index to `x` is greater than $\text{length}(x)$.

`y = resample(x, p, q, n, beta)` uses `beta` as the design parameter for the Kaiser window that `resample` employs in designing the lowpass filter. The default for `beta` is 5.

`y = resample(x, p, q, b)` filters `x` with `b`, a vector of filter coefficients.

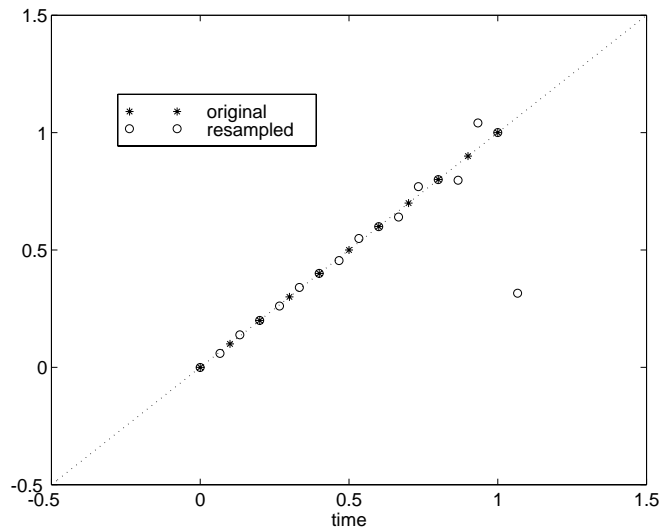
`[y, b] = resample(x, p, q)` returns the vector `b`, which contains the coefficients of the filter applied to `x` during the resampling process.

resample

Examples

Resample a simple linear sequence at $3/2$ the original rate:

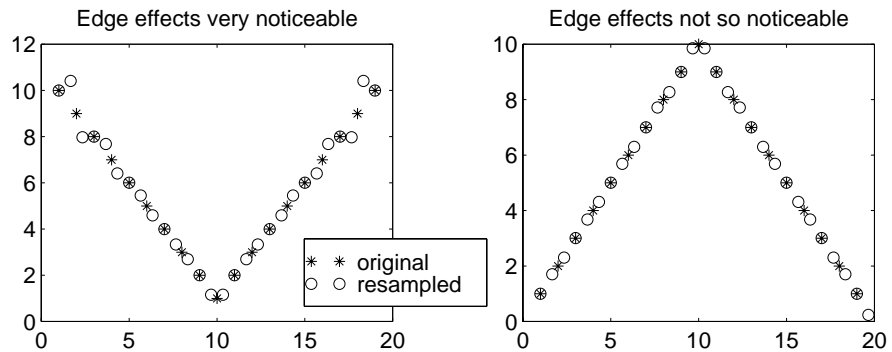
```
Fs1 = 10;           % original sampling frequency in Hz
t1 = 0:1/Fs1:1;     % time vector
x = t1;             % define a linear sequence
y = resample(x,3,2); % now resample it
t2 = (0:(length(y)-1))*2/(3*Fs1); % new time vector
plot(t1,x,'*',t2,y,'o',-0.5:0.01:1.5,-0.5:0.01:1.5,':')
legend('original','resampled')
```



Notice that the last few points of the output y are inaccurate. In its filtering process, `resample` assumes the samples at times before and after the given samples in x are equal to zero. Thus large deviations from zero at the end

points of the sequence x can cause inaccuracies in y at its end points. The following two plots illustrate this side effect of `resample`:

```
x = [1:10 9:-1:1]; y = resample(x, 3, 2);
plot(1:19, x, '*', (0:28)*2/3 + 1, y, 'o')
x = [10:-1:1 2:10]; y = resample(x, 3, 2);
plot(1:19, x, '*', (0:28)*2/3 + 1, y, 'o')
```



Diagnostics

If p or q are not positive integers, `resample` gives the appropriate error message:

P must be a positive integer.
 Q must be a positive integer.

If x is not a vector, `resample` gives the following error message:

Input X must be a vector.

resample

See Also

<code>decimate</code>	Decrease the sampling rate for a sequence (decimation).
<code>fir1</code>	Window-based finite impulse response filter design—standard response.
<code>interp</code>	Increase sampling rate by an integer factor (interpolation).
<code>interp1</code>	One-dimensional data interpolation (table lookup) (see the online <i>MATLAB Function Reference</i>).
<code>intfilt</code>	Interpolation FIR filter design.
<code>kaiser</code>	Kaiser window.
<code>spline</code>	Cubic spline interpolation (see the online <i>MATLAB Function Reference</i>).
<code>upfirdn</code>	Upsample, apply an FIR filter, and downsample.

Purpose z-transform partial-fraction expansion.

Syntax [r, p, k] = resi duez(b, a)
[b, a] = resi duez(r, p, k)

Description resi duez converts a discrete time system, expressed as the ratio of two polynomials, to partial fraction expansion, or residue, form. It also converts the partial fraction expansion back to the original polynomial coefficients.

[r, p, k] = resi duez(b, a) finds the residues, poles, and direct terms of a partial fraction expansion of the ratio of two polynomials, $b(z)$ and $a(z)$. Vectors b and a specify the coefficients of the polynomials of the discrete-time system $b(z)/a(z)$ in descending powers of z .

$$b(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m}$$

$$a(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}$$

If there are no multiple roots and $a > n-1$,

$$\frac{b(z)}{a(z)} = \frac{r(1)}{1 - p(1)z^{-1}} + \cdots + \frac{r(n)}{1 - p(n)z^{-1}} + k(1) + k(2)z^{-1} + \cdots + k(m - n + 1)z^{-(m-n)}$$

The returned column vector r contains the residues, column vector p contains the pole locations, and row vector k contains the direct terms. The number of poles is

$$n = \text{length}(a) - 1 = \text{length}(r) = \text{length}(p)$$

The direct term coefficient vector k is empty if $\text{length}(b) < \text{length}(a)$; otherwise

$$\text{length}(k) = \text{length}(b) - \text{length}(a) + 1$$

If $p(j) = \dots = p(j+s-1)$ is a pole of multiplicity s, then the expansion includes terms of the form

$$\frac{r(j)}{1 - p(j)z^{-1}} + \frac{r(j+1)}{(1 - p(j)z^{-1})^2} + \cdots + \frac{r(j+s-1)}{(1 - p(j)z^{-1})^s}$$

`[b, a] = residuez(r, p, k)` with three input arguments and two output arguments, converts the partial fraction expansion back to polynomials with coefficients in row vectors `b` and `a`.

The `residuez` function in the MATLAB environment is very similar to `residue`. It computes the partial fraction expansion of continuous-time systems in the Laplace domain (see reference [1]), rather than discrete-time systems in the z -domain as does `residue`.

Algorithm

`residuez` applies standard MATLAB functions and partial fraction techniques to find `r`, `p`, and `k` from `b` and `a`. It finds:

- 1 The direct terms `a` using `deconv` (polynomial long division) when $\text{length}(b) > \text{length}(a) - 1$.
- 2 The poles using `p = roots(a)`. `mpoles` finds repeated poles and reorders the poles according to their multiplicities.
- 3 The residue for each nonrepeating pole p_i by multiplying $b(z)/a(z)$ by $1/(1-p_i z^{-1})$ and evaluating the resulting rational function at $z = p_i$.
- 4 The residues for the repeated poles by solving

$$S2 * r2 = h - S1 * r1$$

for `r2` using `\`. `h` is the impulse response of the reduced $b(z)/a(z)$, `S1` is a matrix whose columns are impulse responses of the first-order systems made up of the nonrepeating roots, and `r1` is a column containing the residues for the nonrepeating roots. Each column of matrix `S2` is an impulse response. For each root p_j of multiplicity s_j , `S2` contains s_j columns representing the impulse responses of each of the following systems:

$$\frac{1}{1-p_j z^{-1}}, \frac{1}{(1-p_j z^{-1})^2}, \dots, \frac{1}{(1-p_j z^{-1})^{s_j}}$$

The vector `h` and matrices `S1` and `S2` have `n + xtra` rows, where `n` is the total number of roots and the internal parameter `xtra`, set to 1 by default, determines the degree of overdetermination of the system of equations.

Diagnostics

If `a(1) == 0` while computing the partial fraction decomposition using `[r, p, k] = residuez(b, a)`, `residuez` gives the following error message:

First coefficient in A vector must be nonzero.

If the number of residues r and poles p is not the same, `residuez` gives the following error message:

R and P vectors must be the same size.

See Also

<code>convmtx</code>	Convolution matrix.
<code>deconv</code>	Deconvolution and polynomial division (see the online <i>MATLAB Function Reference</i>).
<code>poly</code>	Polynomial with specified roots (see the online <i>MATLAB Function Reference</i>).
<code>prony</code>	Prony's method for time domain IIR filter design.
<code>residue</code>	Partial fraction expansion (see the online <i>MATLAB Function Reference</i>).
<code>roots</code>	Polynomial roots (see the online <i>MATLAB Function Reference</i>).
<code>ss2tf</code>	State-space to zero-pole-gain conversion.
<code>tf2ss</code>	Transfer function to state-space conversion.
<code>tf2zp</code>	Transfer function to zero-pole-gain conversion.
<code>zp2ss</code>	Zero-pole-gain to state-space conversion.

References

[1] Oppenheim, A.V., and R.W. Schaffer. *Digital Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1975. Pgs. 166-170.

sawtooth

Purpose Sawtooth or triangle wave generator.

Syntax `x = sawtooth(t)`
`x = sawtooth(t, width)`

Description `sawtooth(t)` generates a sawtooth wave with period 2π for the elements of time vector `t`. `sawtooth(t)` is similar to `sin(t)`, but it creates a sawtooth wave with peaks of -1 and 1 instead of a sine wave. The sawtooth wave is defined to be -1 at multiples of 2π and to increase linearly with time with a slope of $1/\pi$ at all other times.

`sawtooth(t, width)` generates a modified triangle wave where `width`, a scalar parameter between 0 and 1, determines the fraction between 0 and 2π at which the maximum occurs. The function increases from -1 to 1 on the interval 0 to $2\pi \cdot \text{width}$, then decreases linearly from 1 to -1 on the interval $2\pi \cdot \text{width}$ to 2π . Thus a parameter of 0.5 specifies a standard triangle wave, symmetric about time instant π with peak-to-peak amplitude of 1. `sawtooth(t, 1)` is equivalent to `sawtooth(t)`.

Diagnostics If the `width` parameter is not a scalar, `sawtooth` gives the following error message:

Requires WIDTH parameter to be a scalar.

See Also

<code>chirp</code>	Swept-frequency cosine generator.
<code>cos</code>	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
<code>diric</code>	Dirichlet or periodic sinc function.
<code>gauspuls</code>	Gaussian-modulated sinusoidal pulse generator.
<code>pulstran</code>	Pulse train generator.
<code>rectpuls</code>	Sampled aperiodic rectangle generator.
<code>sin</code>	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
<code>sinc</code>	Sinc or $\sin(\pi t)/\pi t$ function.
<code>square</code>	Square wave generator.
<code>tripuls</code>	Sampled aperiodic triangle generator.

Purpose Sinc function.

Syntax `y = sinc(x)`

Description `sinc` computes the sinc function of an input vector or array, where the sinc function is

$$\text{sinc}(t) = \begin{cases} 1, & t = 0 \\ \frac{\sin(\pi t)}{\pi t}, & t \neq 0 \end{cases}$$

This function is the continuous inverse Fourier transform of the rectangular pulse of width 2π and height 1:

$$\text{sinc}(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega t} d\omega$$

`y = sinc(x)` returns an array `y` the same size as `x`, whose elements are the `sinc` function of the elements of `x`.

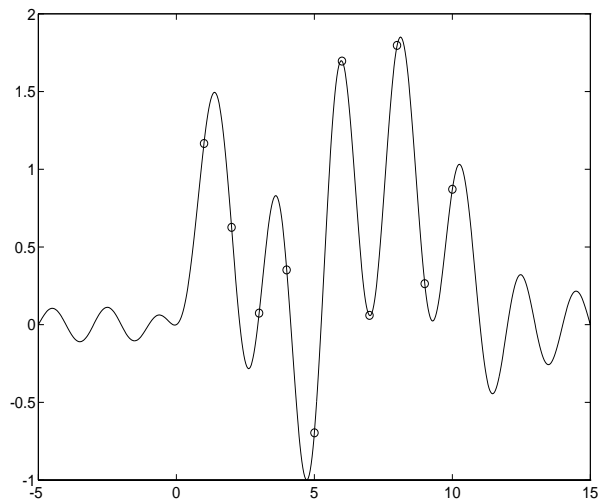
The space of functions bandlimited in the frequency band $\omega \in [-\pi, \pi]$ is spanned by the infinite (yet countable) set of sinc functions shifted by integers. Thus any such bandlimited function $g(t)$ can be reconstructed from its samples at integer spacings:

$$g(t) = \sum_{n=-\infty}^{\infty} g(n) \text{sinc}(t - n)$$

Example

Perform ideal bandlimited interpolation by assuming that the signal to be interpolated is 0 outside of the given time interval and that it has been sampled at exactly the Nyquist frequency:

```
t = (1:10)';           % a column vector of time samples
randn('seed', 0);
x = randn(size(t));    % a column vector of data
% ts is times at which to interpolate data
ts = linspace(-5, 15, 600)';
y = sinc(ts(:, ones(size(t))) - t(:, ones(size(ts))))' * x;
plot(t, x, 'o', ts, y)
```



See Also

chi rp	Swept-frequency cosine generator.
cos	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
di ri c	Dirichlet or periodic sinc function.
gauspul s	Gaussian-modulated sinusoidal pulse generator.
pul stran	Pulse train generator.
rectpul s	Sampled aperiodic rectangle generator.
sawtooth	Sawtooth or triangle wave generator.
si n	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
square	Square wave generator.
tri pul s	Sampled aperiodic triangle generator.

Purpose Second-order section to state-space conversion.

Syntax [A, B, C, D] = sos2ss(sos)

Description sos2ss converts a second-order section representation of a given system to an equivalent state-space representation.

[A, B, C, D] = sos2ss(sos) converts the system sos, in second-order section form, to a single-input, single-output state-space representation:

$$\begin{aligned}x[n+1] &= Ax[n] + Bu[n] \\y[n] &= Cx[n] + Du[n]\end{aligned}$$

The discrete transfer function in second-order section form is given by

$$H(z) = \prod_{k=1}^L H_k(z) = \prod_{k=1}^L \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}}$$

where L is the number of rows in sos. sos is a L -by-6 matrix organized as

$$\text{sos} = \begin{bmatrix} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{bmatrix}$$

The entries of sos must be real for proper conversion to state space. The returned matrix A is size N -by- N , where $N = 2L-1$, B is a length $N-1$ column vector, C is a length $N-1$ row vector, and D is a scalar.

Example Compute the state-space representation of a simple second-order section form system:

```
sos = [ 1   1   1   1   0 -1; -2   3   1   1 10   1];  
[A, B, C, D] = sos2ss(sos)
```

```
A =  
-10.8990   -3.1463         0         0  
  3.1463         0         0         0  
 -9.8990   -2.8284    0.8990    0.3178  
         0         0    0.3178         0
```

```
B =  
  1  
  0  
  1  
  0
```

```
C =  
19.7980    5.6569    1.2020    2.5106
```

```
D =  
-2
```

Algorithm sos2ss first finds the zeros and poles of the second-order sections using roots, then uses zp2ss to find a state-space representation of the system.

See Also	sos2tf	Second-order section to transfer function conversion.
	sos2zp	Second-order section to zero-pole-gain conversion.
	ss2sos	State-space to second-order section conversion.
	zp2sos	Zero-pole-gain to second-order section conversion.

Purpose Second-order section to transfer function conversion.

Syntax [b, a] = sos2tf(sos)

Description sos2tf converts a second-order section representation of a given system to an equivalent transfer function representation.

[b, a] = sos2tf(sos) returns the numerator coefficients b and denominator coefficients a of the transfer function that describes a discrete-time system given by sos in second-order section form. The second-order section format of $H(z)$ is given by

$$H(z) = \prod_{k=1}^L H_k(z) = \prod_{k=1}^L \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}}$$

where L is the number of rows of sos. sos is an L -by-6 matrix which contains the coefficients of each second-order section stored in its rows:

$$\text{sos} = \begin{bmatrix} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{bmatrix}$$

Row vectors b and a contain the numerator and denominator coefficients of $H(z)$ stored in descending powers of z .

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(n+1)z^{-n}}{a(1) + a(2)z^{-1} + \cdots + a(m+1)z^{-m}}$$

Algorithm sos2tf uses the conv function to multiply all of the numerator and denominator second-order polynomials together.

Example

Compute the transfer function representation of a simple second-order section form system:

```
sos = [1  1  1  1  0 -1; -2  3  1  1 10  1];
[b, a] = sos2tf(sos)
```

```
b =
    -2     1     2     4     1
```

```
a =
     1    10     0   -10    -1
```

See Also

<code>sos2ss</code>	Second-order section to state-space conversion.
<code>sos2zp</code>	Second-order section to zero-pole-gain conversion.
<code>ss2sos</code>	State-space to second-order section conversion.
<code>zp2sos</code>	Zero-pole-gain to second-order section conversion.

Purpose Second-order section to zero-pole-gain conversion.

Syntax [z, p, k] = sos2zp(sos)

Description sos2zp converts a second-order section representation of a given system to an equivalent zero-pole-gain representation.

[z, p, k] = sos2zp(sos) returns the zeros z, poles p, and gain k of the system given by sos in second-order section form. The second-order section format of $H(z)$ is given by

$$H(z) = \prod_{k=1}^L H_k(z) = \prod_{k=1}^L \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}}$$

where L is the number of rows of sos. sos is an L -by-6 matrix which contains the coefficients of each second-order section stored in its rows:

$$\text{sos} = \begin{bmatrix} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{bmatrix}$$

Column vectors z and p contain the zeros and poles of the transfer function $H(z)$:

$$H(z) = k \frac{(z - z(1))(z - z(2)) \cdots (z - z(N))}{(p - p(1))(p - p(2)) \cdots (p - p(M))}$$

where the orders N and M are determined by the matrix sos.

Example

Compute the poles, zeros, and gain of a simple system in second-order section form:

```

sos = [ 1  1  1  1  0 -1; -2  3  1  1 10  1];
[z, p, k] = sos2zp(sos)

```

z =

```

-0.5000 + 0.8660i
-0.5000 - 0.8660i
 1.7808
-0.2808

```

p =

```

-1.0000
 1.0000
-9.8990
-0.1010

```

k

```

-2

```

Algorithm

sos2zp finds the roots and poles of each second-order section using the `roots` command. `sos2zp` returns the roots and poles with conjugate pairs in consecutive locations, with the order of the pairs determined by their row in the `sos` matrix. The gain `k` is the product of the gains of the sections:

$$k = \prod_{k=1}^L \frac{b_{0k}}{a_{0k}}$$

See Also

<code>sos2ss</code>	Second-order section to state-space conversion.
<code>sos2tf</code>	Second-order section to transfer function conversion.
<code>ss2sos</code>	State-space to second-order section conversion.
<code>zp2sos</code>	Zero-pole-gain to second-order section conversion.

specgram

Purpose Time-dependent frequency analysis (spectrogram).

Syntax

```
B = specgram(a)
B = specgram(a, nfft)
[B, f] = specgram(a, nfft, Fs)
[B, f, t] = specgram(a, nfft, Fs)
B = specgram(a, nfft, Fs, window)
B = specgram(a, nfft, Fs, window, overlap)
specgram(a)
B = specgram(a, f, Fs, window, overlap)
```

Description `specgram` computes the windowed discrete-time Fourier transform of a signal using a sliding window. The spectrogram is the magnitude of this function.

`B = specgram(a)` calculates the spectrogram for the signal in vector `a`. This syntax uses the default values:

- `nfft = min(256, length(a))`
- `Fs = 2`
- `window = hanning(nfft)`
- `overlap = length(window)/2`

`nfft` specifies the FFT length that `specgram` uses. This value determines the frequencies at which the discrete-time Fourier transform is computed. `Fs` is a scalar that specifies the sampling frequency. `window` specifies a windowing function and the number of samples `specgram` uses in its sectioning of vector `a`. `overlap` is the number of samples by which the sections overlap. Any arguments that you omit from the end of the input parameter list use the default values shown above.

If `a` is real, `specgram` computes the discrete-time Fourier transform at positive frequencies only. If `n` is even, `specgram` returns `nfft/2+1` rows (including the zero and Nyquist frequency terms). If `n` is odd, `specgram` returns `nfft/2` rows. The number of columns in `B` is

$$k = \text{fix}((n - \text{overlap}) / (\text{length}(\text{window}) - \text{overlap}))$$

If `a` is complex, `specgram` computes the discrete-time Fourier transform at both positive and negative frequencies. In this case, `B` is a complex matrix with `nfft`

rows. Time increases linearly across the columns of *B*, starting with sample 1 in column 1. Frequency increases linearly down the rows, starting at 0.

`B = specgram(a, nfft)` uses the specified FFT length *nfft* in its calculations. Specify *nfft* as a power of 2 for fastest execution.

`[B, f] = specgram(a, nfft, Fs)` returns a vector *f* of frequencies at which the function computes the discrete-time Fourier transform. *Fs* has no effect on the output *B*; it is a frequency scaling multiplier.

`[B, f, t] = specgram(a, nfft, Fs)` returns frequency and time vectors *f* and *t* respectively. *t* is a column vector of scaled times, with length equal to the number of columns of *B*. *t(j)* is the earliest time at which the *j*-th window intersects *a*. *t(1)* is always equal to 0.

`B = specgram(a, nfft, Fs, window)` specifies a windowing function and the number of samples per section of the *x* vector. If you supply a scalar for *window*, `specgram` uses a Hanning window of that length. The length of the window must be less than or equal to *nfft*; `specgram` zero pads the sections if the length of the window exceeds *nfft*.

`B = specgram(a, nfft, Fs, window, noverlap)` overlaps the sections of *x* by *noverlap* samples.

You can use the empty matrix `[]` to specify the default value for any input argument. For example,

```
B = specgram(x, [], 10000)
```

is equivalent to

```
B = specgram(x)
```

but with a sampling frequency of 10,000 Hz instead of the default 2 Hz.

`specgram` with no output arguments displays the scaled logarithm of the spectrogram in the current figure window using

```
imagesc(t, f, 20*log10(abs(b))), axis xy, colormap(jet)
```

The `axis xy` mode displays the low-frequency content of the first portion of the signal in the lower-left corner of the axes. `specgram` uses `Fs` to label the axes according to true time and frequency.

`B = specgram(a, f, Fs, window, noverlap)` computes the spectrogram at the frequencies specified in `f`, using either the chirp z -transform (for more than 20 evenly spaced frequencies) or a polyphase decimation filter bank. `f` is a vector of frequencies in Hertz; it must have at least two elements.

Algorithm

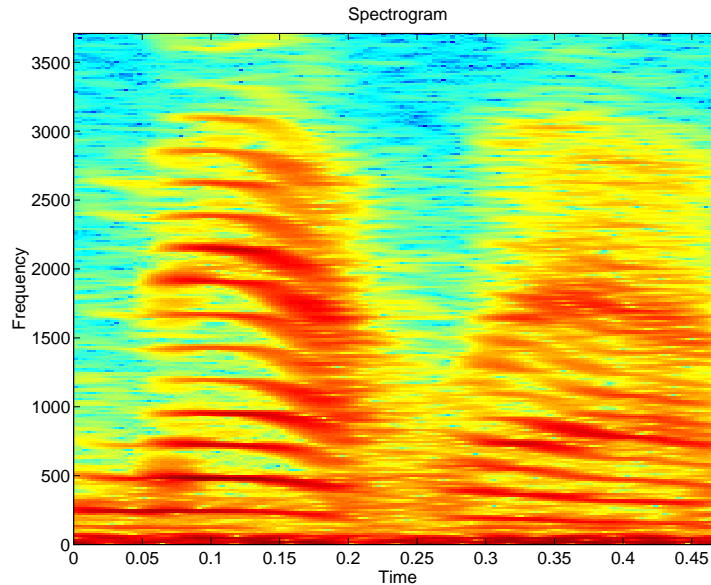
`specgram` calculates the spectrogram for a given signal as follows:

- 1 It splits the signal into overlapping sections and applies the window specified by the `window` parameter to each section.
- 2 It computes the discrete-time Fourier transform of each section with a length `nfft` FFT to produce an estimate of the short-term frequency content of the signal; these transforms make up the columns of `B`. `specgram` zero pads the windowed sections if `nfft > length(window)`, so the quantity $(\text{length}(\text{window}) - \text{noverlap})$ specifies by how many samples `specgram` shifts the window.
- 3 For real input, `specgram` truncates the spectrogram to the first $\text{nfft}/2 + 1$ points for `nfft` even and $(\text{nfft} + 1)/2$ for `nfft` odd.

Example

Plot the spectrogram of a digitized speech signal:

```
load mtlb
specgram(mtlb, 512, Fs, kaiser(500, 5), 475)
```



Diagnostics

An appropriate diagnostic message is displayed when incorrect arguments are used:

- Requires window's length to be no greater than the FFT length.
- Requires NOVERLAP to be strictly less than the window length.
- Requires positive integer values for NFFT and NOVERLAP.
- Requires vector input.

See Also

cohere	Estimate magnitude squared coherence function between two signals.
csd	Estimate the cross spectral density (CSD) of two signals.
psd	Estimate the power spectral density (PSD) of a signal using Welch's method.
tfe	Transfer function estimate from input and output.

References

- [1] Oppenheim, A.V., and R.W. Schafer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 713-718.
- [2] Rabiner, L.R., and R.W. Schafer. *Digital Processing of Speech Signals*. Englewood Cliffs, NJ: Prentice Hall, 1978.

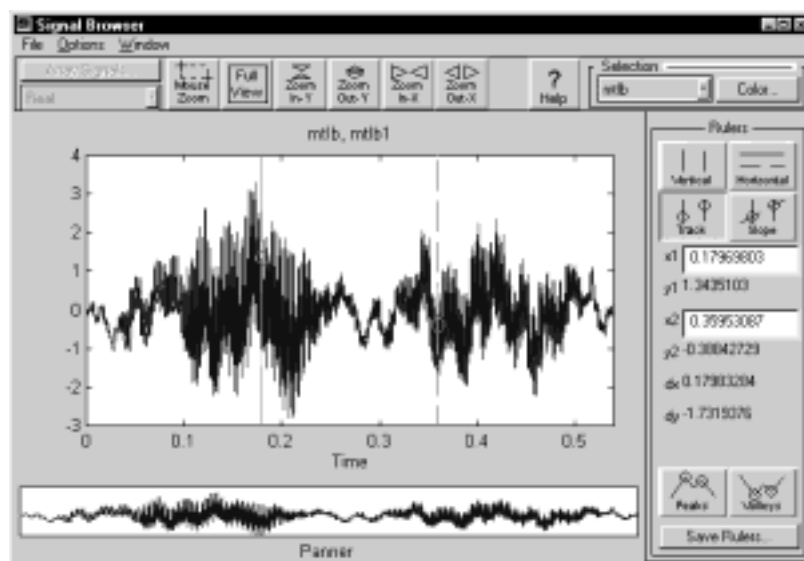
Purpose Interactive digital signal processing tool (SPTool).

Syntax sptool

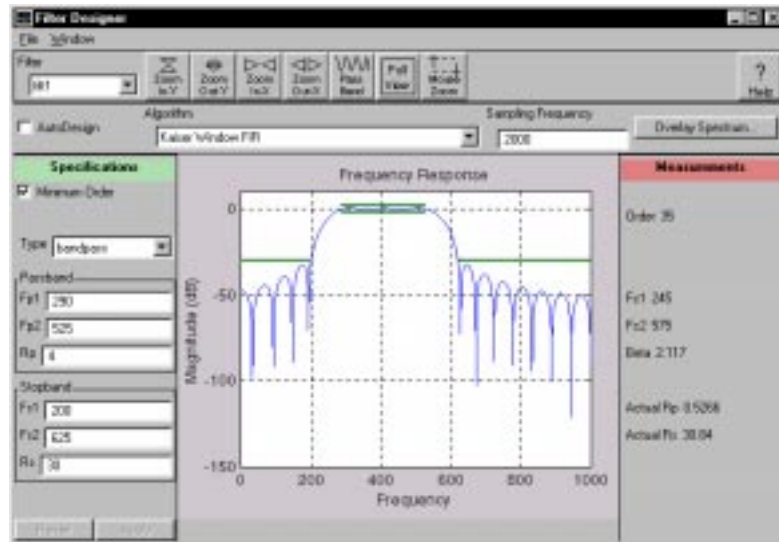
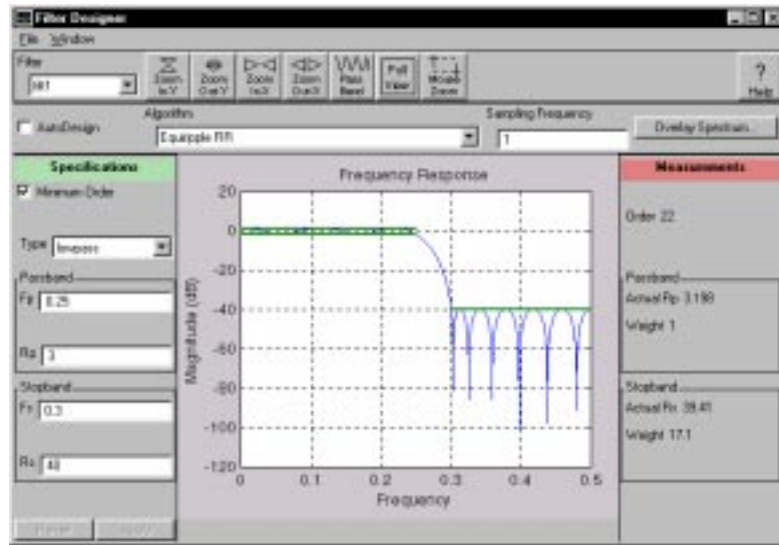
Description The sptool command invokes a suite of graphical user interface (GUI) tools that provides access to many of the signal, filter, and spectral analysis functions in the toolbox in a powerful, easy-to-use interactive signal display and exploration environment.

Using SPTool, you can import, export, and manage signals, filters, and spectra. From SPTool, you can activate its four integrated signal processing tools:

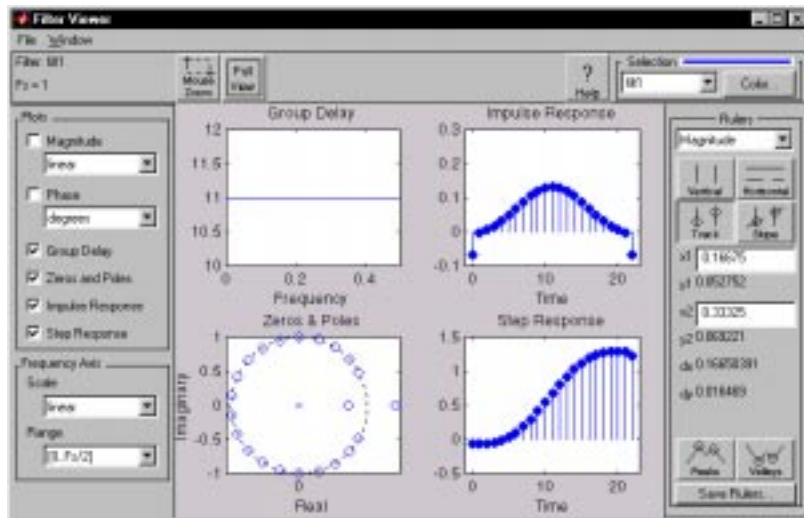
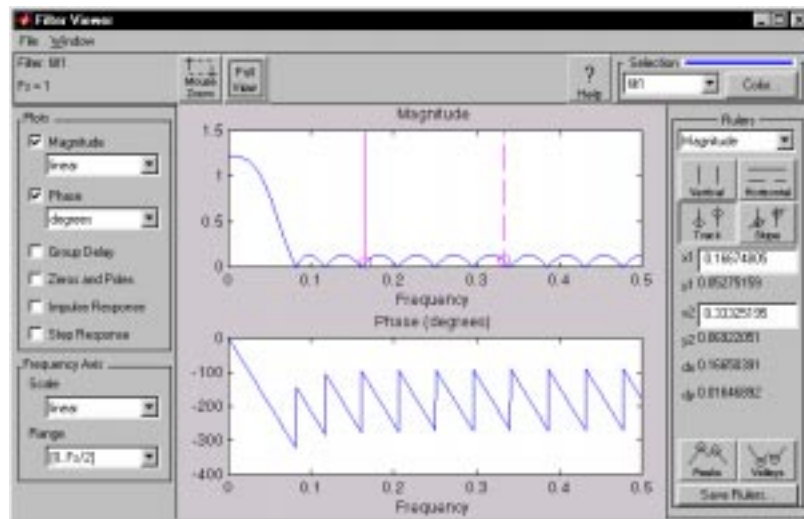
- The *Signal Browser*, for viewing, measuring, and analyzing time-domain information of imported signals:



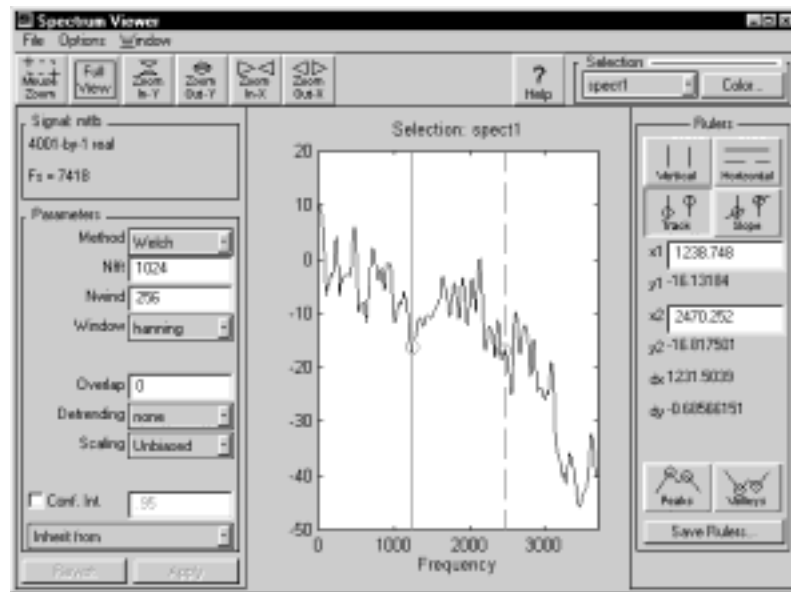
- The *Filter Designer*, for designing and editing FIR and IIR filters of various lengths and types, with standard (lowpass, highpass, bandpass, and bandstop) configurations:



- The *Filter Viewer*, for viewing the characteristics of a designed or imported filter, including its magnitude response, phase response, group delay, pole-zero plot, impulse response, and step response:



- The *Spectrum Viewer*, for graphical analysis of frequency-domain data using a variety of methods of spectral density estimation, including the Burg method, the FFT method, the multitaper method (MTM), the MUSIC eigenvector method, Welch's method (PSD and CSD), and the Yule-Walker AR method:



See Chapter 5, “Interactive Tools” for a full discussion of how to use SPTool.

Purpose	Square wave generator.	
Syntax	$x = \text{square}(t)$ $x = \text{square}(t, \text{duty})$	
Description	$\text{square}(t)$ generates a square wave with period 2π for the elements of time vector t . $\text{square}(t)$ is similar to $\sin(t)$, but it creates a square wave with peaks of ± 1 instead of a sine wave. $\text{square}(t, \text{duty})$ generates a square wave with specified duty cycle, duty . The <i>duty cycle</i> is the percent of the period in which the signal is positive.	
See Also	chirp	Swept-frequency cosine generator.
	\cos	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
	diric	Dirichlet or periodic sinc function.
	gauspuls	Gaussian-modulated sinusoidal pulse generator.
	pulstran	Pulse train generator.
	rectpuls	Sampled aperiodic rectangle generator.
	sawtooth	Sawtooth or triangle wave generator.
	\sin	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
	sinc	Sinc or $\sin(\pi t)/\pi t$ function.
	tripuls	Sampled aperiodic triangle generator.

Purpose State-space to second-order section conversion.

Syntax

```
sos = ss2sos(A, B, C, D)
sos = ss2sos(A, B, C, D, iu)
sos = ss2sos(A, B, C, D, 'order')
sos = ss2sos(A, B, C, D, iu, 'order')
```

Description `ss2sos` converts a state-space representation of a given system to an equivalent second-order section representation.

`sos = ss2sos(A, B, C, D)` finds a matrix `sos` in second-order section form that is equivalent to the state-space system represented by input arguments `A`, `B`, `C` and `D`. The input system must be single input and real. `sos` is an L -by-6 matrix

$$sos = \begin{bmatrix} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{bmatrix}$$

whose rows contain the numerator and denominator coefficients b_{ik} and a_{ik} of the second-order sections of $H(z)$:

$$H(z) = \prod_{k=1}^L H_k(z) = \prod_{k=1}^L \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}}$$

`sos = ss2sos(A, B, C, D, iu)` specifies a scalar `iu` that determines which output of the state-space system `A`, `B`, `C`, `D` is used in the conversion. The default for `iu` is 1.

`sos = ss2sos(A, B, C, D, 'order')` and

`sos = ss2sos(A, B, C, D, iu, 'order')` specify the order of the rows in `sos`, where `order` is

- down, to order the sections so the first row of `sos` contains the poles closest to the unit circle
- up, to order the sections so the first row of `sos` contains the poles farthest from the unit circle (default)

Example

Find a second-order section form of a Butterworth lowpass filter:

```
[A, B, C, D] = butter(5, 0.2);
sos = ss2sos(A, B, C, D)

sos =

    0.2451    0.2454         0    1.0000   -0.5095         0
    0.0647    0.1294    0.0647    1.0000   -1.0966    0.3554
    0.0809    0.1616    0.0807    1.0000   -1.3693    0.6926
```

Algorithm

`ss2sos` uses a four-step algorithm to determine the second-order section representation for an input state-space system:

- 1 It finds the poles and zeros of the system given by A, B, C and D.
- 2 It uses the function `zp2sos`, which first groups the zeros and poles into complex conjugate pairs using the `cplxpair` function. `zp2sos` then forms the second-order sections by matching the pole and zero pairs according to the following rules:
 - a Match the poles closest to the unit circle with the zeros closest to those poles.
 - b Match the poles next closest to the unit circle with the zeros closest to those poles.
 - c Continue until all of the poles and zeros are matched.

`ss2sos` groups real poles into sections with the real poles closest to them in absolute value. The same rule holds for real zeros.

- 3 It orders the sections according to the proximity of the pole pairs to the unit circle. `ss2sos` normally orders the sections with poles closest to the unit circle last in the cascade. You can tell `ss2sos` to order the sections in the reverse order by specifying the 'down' flag.

Putting “high Q” sections at the beginning of the cascade, by specifying the 'down' flag, reduces the sensitivity of the filter response to quantization noise near those poles. Putting “high Q” sections at the end of the cascade (the default) prevents reduction in signal power level early in the cascade. `ss2sos` orders all-zero sections according to the minimum of $|z_i|$ and $|z_i^{-1}|$.

where z_i ($i = 1, 2$) are the zeros in the section. References [1] and [2] provide a detailed discussion of section ordering.

- 4 ss2sos scales the sections so the maximum of the magnitude of the transfer function of the first N sections in cascade is less than 1:

$$\max_{|\omega| \leq \pi} \left| \prod_{i=1}^N H_i(e^{j\omega}) \right| < 1, \quad N = 1, \dots, L-1$$

subject to the constraint that the overall gain, k , stays the same:

$$k = \prod_{k=1}^L \frac{b_{0k}}{a_{0k}}$$

This scaling is an attempt to minimize overflow in some standard fixed point implementations of filtering.

Diagnostics

If there is more than one input to the system, ss2sos gives the following error message:

State-space system must have only one input.

See Also

cplxpair	Group complex numbers into complex conjugate pairs.
sos2ss	Second-order section to state-space conversion.
sos2tf	Second-order section to transfer function conversion.
sos2zp	Second-order section to zero-pole-gain conversion.
zp2sos	Zero-pole-gain to second-order section conversion.

References

[1] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 363-370.

[2] Jackson, L.B. *Digital Filters and Signal Processing*. Second Ed. Boston: Kluwer Academic Publishers, 1989. Pgs. 319-324.

Purpose	State-space to transfer function conversion.	
Syntax	<code>[num, den] = ss2tf(a, b, c, d, i u)</code>	
Description	<p><code>ss2tf</code> converts a state-space representation of a given system to an equivalent transfer function representation.</p> <p><code>[num, den] = ss2tf(a, b, c, d, i u)</code> returns the transfer function</p> $H(s) = \frac{num(s)}{den(s)} = C(sI - A)^{-1} B + D$ <p>of the system</p> $\dot{x} = Ax + Bu$ $y = Cx + Du$ <p>from the <code>i u</code>-th input. Vector <code>den</code> contains the coefficients of the denominator in descending powers of s. The numerator coefficients are returned in array <code>num</code> with as many rows as there are outputs y. The function <code>tf2ss</code> is the inverse of <code>ss2tf</code>. <code>ss2tf</code> also works with systems in discrete time, in which case it returns the z-transform representation.</p>	
Algorithm	<p><code>ss2tf</code> uses <code>poly</code> to find the characteristic polynomial $\det(sI - A)$ and the equality</p> $H(s) = c(sI - A)^{-1} b = \frac{\det(sI - A + bc) - \det(sI - A)}{\det(sI - A)}$	
See Also	<p><code>ss2zp</code> State-space to zero-pole-gain conversion.</p> <p><code>tf2ss</code> Transfer function to state-space conversion.</p> <p><code>tf2zp</code> Transfer function to zero-pole-gain conversion.</p> <p><code>zp2ss</code> Zero-pole-gain to state-space conversion.</p> <p><code>zp2tf</code> Zero-pole-gain to transfer function conversion.</p>	

Purpose State-space to zero-pole-gain conversion.

Syntax `[Z, p, k] = ss2zp(A, B, C, D, i u)`

Description `ss2zp` converts a state-space representation of a given system to an equivalent zero-pole-gain representation. The zeros, poles, and gains of state-space systems represent the transfer function in factored form.

`[Z, p, k] = ss2zp(A, B, C, D, i u)` calculates the transfer function in factored form

$$H(s) = \frac{z(s)}{p(s)} = k \frac{(s - z(1))(s - z(2)) \cdots (s - z(n))}{(s - p(1))(s - p(2)) \cdots (s - p(n))}$$

of the system

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

from the i u-th input. Returned column vector p contains the pole locations of the denominator coefficients of the transfer function. Array Z contains the numerator zeros in its columns, with as many columns as there are outputs y . Column vector k contains the gains for each numerator transfer function.

The function `zp2ss` is the inverse of `ss2zp`. `ss2zp` also works with systems in discrete time, in which case it returns the z -transform representation. The input state-space system must be real.

Example

Here are two ways of finding the zeros, poles, and gains of a system:

```

num = [2 3];
den = [1 0.4 1];
[z, p, k] = tf2zp(num, den)

z =
    -1.5000

p =
    -0.2000 + 0.9798i
    -0.2000 - 0.9798i

k =
     2

[A, B, C, D] = tf2ss(num, den);
[z, p, k] = ss2zp(A, B, C, D, 1)

z =
    -1.5000

p =
    -0.2000 + 0.9798i
    -0.2000 - 0.9798i

k =
     2

```

Algorithm

ss2zp finds the poles from the eigenvalues of the A array. The zeros are the finite solutions to a generalized eigenvalue problem:

$$z = \text{eig}([A \ B; C \ D], \text{diag}([\text{ones}(1, n) \ 0]));$$

In many situations this algorithm produces spurious large, but finite, zeros. ss2zp interprets these large zeros as infinite.

ss2zp finds the gains by solving for the first nonzero Markov parameters.

ss2zp

See Also	pzmap	Pole-zero map of LTI system (<i>Control System Toolbox User's Guide</i>).
	ss2tf	State-space to transfer function conversion.
	tf2ss	Transfer function to state-space conversion.
	zp2ss	Zero-pole-gain to state-space conversion.

References

[1] Laub, A.J., and B.C. Moore. "Calculation of Transmission Zeros Using QZ Techniques." *Automatica* 14 (1978). Pg. 557.

Purpose Linear model using Steiglitz-McBride iteration.

Syntax

```
[ b, a ] = stmcb(x, nb, na)
[ b, a ] = stmcb(x, u, nb, na)
[ b, a ] = stmcb(x, nb, na, ni ter)
[ b, a ] = stmcb(x, u, nb, na, ni ter)
[ b, a ] = stmcb(x, nb, na, ni ter, ai )
[ b, a ] = stmcb(x, u, nb, na, ni ter, ai )
```

Description Steiglitz-McBride iteration is an algorithm for finding an IIR filter with a prescribed time domain impulse response. It has applications in both filter design and system identification (parametric modeling).

[b, a] = stmcb(x, nb, na) finds the coefficients b and a of the system $b(z)/a(z)$ with approximate impulse response x, exactly nb zeros, and exactly na poles.

[b, a] = stmcb(x, u, nb, na) finds the system coefficients b and a of the system that, given u as input, has x as output. x and u must be the same length.

[b, a] = stmcb(x, nb, na, ni ter) and

[b, a] = stmcb(x, u, nb, na, ni ter) use ni ter iterations. The default for ni ter is 5.

[b, a] = stmcb(x, nb, na, ni ter, ai) and

[b, a] = stmcb(x, u, nb, na, ni ter, ai) use the vector ai as the initial estimate of the denominator coefficients. If ai is not specified, stmcb uses the output argument from [b, ai] = prony(x, 0, na) as the vector ai .

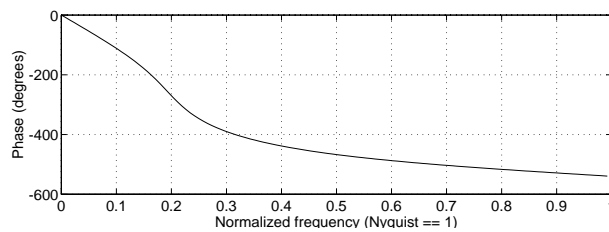
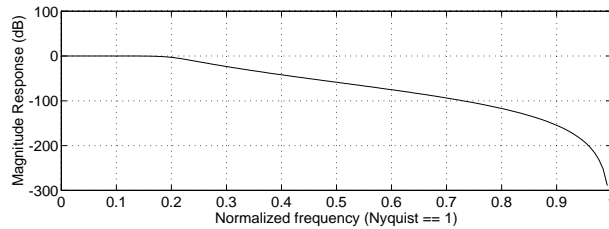
stmcb returns the IIR filter coefficients in length nb+1 and na+1 row vectors b and a. The filter coefficients are ordered in descending powers of z.

$$H(z) = \frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \cdots + b(nb+1)z^{-nb}}{a(1) + a(2)z^{-1} + \cdots + a(na+1)z^{-na}}$$

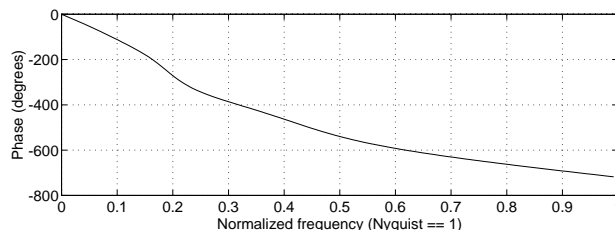
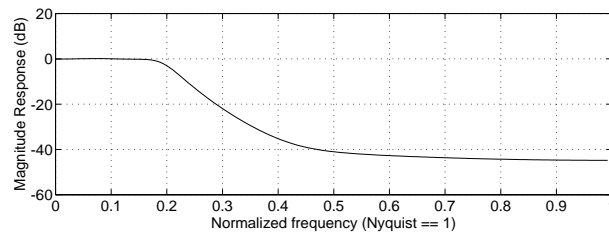
Example

Approximate the impulse response of a Butterworth filter with a system of lower order:

```
[b, a] = butter(6, 0.2);  
h = filter(b, a, [1 zeros(1, 100)]);  
freqz(b, a, 128)
```



```
[bb, aa] = stmcb(h, 4, 4);  
freqz(bb, aa, 128)
```



Algorithm

stmcb attempts to minimize the squared error between the impulse response x' of $b(z)/a(z)$ and the input signal x :

$$\min_{a,b} \sum_{i=0}^{\infty} |x(i) - x'(i)|^2$$

stmcb iterates using two steps:

- 1 It prefilters x and u using $1/a(z)$.
- 2 It solves a system of linear equations for b and a using \backslash .

stmcb repeats this process `ni ter` times. No checking is done to see if the b and a coefficients have converged in fewer than `ni ter` iterations.

Diagnostics

If x and u have different lengths, stmcb gives the following error message:

X and U must have same length.

See Also

<code>levinson</code>	Levinson-Durbin recursion.
<code>lpc</code>	Linear prediction coefficients.
<code>oe</code>	Compute the prediction error estimate of an output-error model (see <i>System Identification Toolbox User's Guide</i>).
<code>prony</code>	Prony's method for time domain IIR filter design.

References

- [1] Steiglitz, K., and L.E. McBride. "A Technique for the Identification of Linear Systems." *IEEE Trans. Automatic Control*. Vol. AC-10 (1965). Pgs. 461-464.
- [2] Ljung, L. *System Identification: Theory for the User*. Englewood Cliffs, NJ: Prentice Hall, 1987. Pg. 297.

strips

Purpose

Strip plot.

Syntax

`strips(x)`
`strips(x, n)`
`strips(x, sd, Fs)`
`strips(x, sd, Fs, scale)`

Description

`strips(x)` plots vector `x` in horizontal strips of length 250. If `x` is a matrix, `strips(x)` plots each column of `x`. The left-most column (column 1) is the top horizontal strip.

`strips(x, n)` plots vector `x` in strips that are each `n` samples long.

`strips(x, sd, Fs)` plots vector `x` in strips of duration `sd` seconds, given a sampling frequency of `Fs` samples per second.

`strips(x, sd, Fs, scale)` scales the vertical axes.

If `x` is a matrix, `strips(x, n)`, `strips(x, sd, Fs)`, and `strips(x, sd, Fs, scale)` plot the different columns of `x` on the same strip plot.

`strips` ignores the imaginary part of `x` if it is complex.

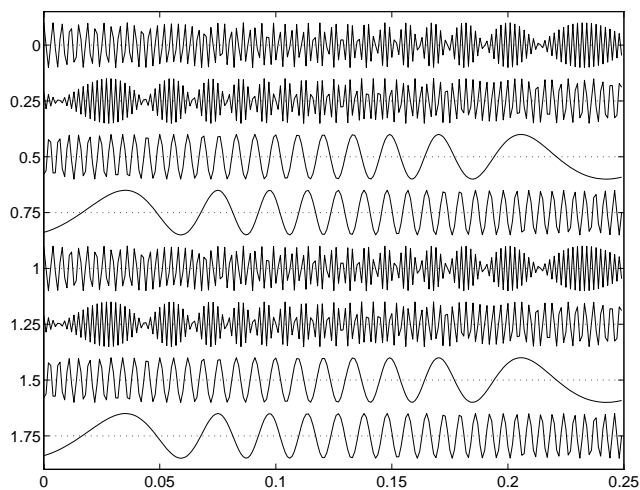
Example

Plot two seconds of a frequency modulated sinusoid in 0.25 second strips:

```

Fs = 1000;                                % sampling frequency
t = 0: 1/Fs: 2;                            % time vector
x = vco(sin(2*pi*t), [10 490], Fs); % FM waveform
strips(x, 0.25, Fs)

```

**See Also**

`plot`

Linear two-dimensional plot (see the online *MATLAB Function Reference*).

`stem`

Plot discrete sequence data (see the online *MATLAB Function Reference*).

tf2latc

Purpose	Transfer function to lattice filter conversion.	
Syntax	<pre>[k, v] = tf2latc(num, den) k = tf2latc(1, den) [k, v] = tf2latc(1, den) k = tf2latc(num)</pre>	
Description	<p>[k, v] = tf2latc(num, den) finds the lattice parameters k and the ladder parameters v for an IIR (ARMA) lattice-ladder filter, normalized by den(1). Note that an error will be generated if any poles of the transfer function lie on the unit circle.</p> <p>k = tf2latc(1, den) finds the lattice parameters k for an IIR all-pole (AR) lattice filter.</p> <p>[k, v] = tf2latc(1, den) returns a scalar ladder coefficient v.</p> <p>k = tf2latc(num) finds the lattice parameters k for an FIR (MA) lattice filter, normalized by num(1).</p> <p>The function latc2tf is the inverse of tf2latc.</p>	
See Also	latc2tf	Lattice filter to transfer function conversion.
	latcfilt	Lattice and lattice-ladder filter implementation.
	poly2rc	Reflection coefficients from polynomial coefficients.
	rc2poly	Polynomial coefficients from reflection coefficients.

Purpose Transfer function to state-space conversion.

Syntax `[A, B, C, D] = tf2ss(num, den)`

Description `tf2ss` converts a transfer function representation of a given system to an equivalent state-space representation.

`[A, B, C, D] = tf2ss(num, den)` finds a state-space representation:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

given a system in transfer function form:

$$H(s) = \frac{\text{num}(s)}{\text{den}(s)} = C(sI - A)^{-1}B + D$$

from a single input. Input vector `den` contains the denominator coefficients in descending powers of s . Array `num` contains the numerator coefficients with as many rows as there are outputs y . `tf2ss` returns the A, B, C, and D matrices in controller canonical form.

`tf2ss` also works for discrete systems, but you must pad the numerator with trailing zeros to make it the same length as the denominator.

The function `ss2tf` is the inverse of `tf2ss`.

Example Consider the system

$$H(s) = \frac{\begin{bmatrix} 2s + 3 \\ s^2 + 2s + 1 \end{bmatrix}}{s^2 + 0.4s + 1}$$

To convert this system to state-space:

```
num = [ 0 2 3; 1 2 1];
den = [ 1 0.4 1];
[A, B, C, D] = tf2ss(num, den)
```

```
A =
    -0.4000    -1.0000
     1.0000         0
```

```
B =
     1
     0
```

```
C =
     2.0000     3.0000
     1.6000         0
```

```
D =
     0
     1
```

There is disagreement in the literature on naming conventions for the canonical forms. It is easy, however, to generate similarity transformations that convert to other forms. For example:

```
T = flipr(eye(n));
A = T\A*T;
```

Algorithm

tf2ss writes the output in controller canonical form by inspection.

See Also

ss2tf	State-space to transfer function conversion.
ss2zp	State-space to zero-pole-gain conversion.
tf2zp	Transfer function to zero-pole-gain conversion.
zp2ss	Zero-pole-gain to state-space conversion.
zp2tf	Zero-pole-gain to transfer function conversion.

Purpose	Transfer function to zero-pole-gain conversion.
Syntax	<code>[z, p, k] = tf2zp(num, den)</code>
Description	tf2zp finds the zeros, poles, and gains of a system in polynomial transfer function form.

`[z, p, k] = tf2zp(num, den)` finds the single-input, multi-output (SIMO) factored transfer function form:

$$H(s) = \frac{Z(s)}{p(s)} = k \frac{(s - Z(1))(s - Z(2)) \dots (s - Z(m))}{(s - p(1))(s - p(2)) \dots (s - p(n))}$$

given a SIMO system in polynomial transfer function form:

$$\frac{num(s)}{den(s)} = \frac{num(1)s^{nn-1} + \dots + num(nn-1)s + num(nn)}{den(1)s^{nd-1} + \dots + den(nd-1)s + den(nd)}$$

Vector den specifies the coefficients of the denominator in descending powers of s . Matrix num indicates the numerator coefficients with as many rows as there are outputs. The zero locations are returned in the columns of matrix z, with as many columns as there are rows in num. The pole locations are returned in column vector p and the gains for each numerator transfer function in vector k.

tf2zp also works for discrete systems. The function zp2tf is the inverse of tf2zp.

Example Find the zeros, poles, and gains of the system

```
H(s) = (2s + 3) / (s^2 + 0.4s + 1)

num = [ 2 3];
den = [ 1 0.4 1];
[z, p, k] = tf2zp(num, den)

z =

    -1.5000

p =

    -0.2000 + 0.9798i
    -0.2000 - 0.9798i

k =

     2
```

Algorithm The system is converted to state-space using `tf2ss` and then to zeros, poles, and gains using `ss2zp`.

See Also	<code>ss2tf</code>	State-space to transfer function conversion.
	<code>ss2zp</code>	State-space to zero-pole-gain conversion.
	<code>tf2ss</code>	Transfer function to state-space conversion.
	<code>zp2ss</code>	Zero-pole-gain to state-space conversion.
	<code>zp2tf</code>	Zero-pole-gain to transfer function conversion.

Purpose	Transfer function estimate from input and output.
Syntax	<pre> Txy = tfe(x, y) Txy = tfe(x, y, nfft) [Txy, f] = tfe(x, y, nfft, Fs) Txy = tfe(x, y, nfft, Fs, window) Txy = tfe(x, y, nfft, Fs, window, overlap) Txy = tfe(x, y, ..., 'flag') tfe(x, y) </pre>
Description	<p><code>Txy = tfe(x, y)</code> finds a transfer function estimate <code>Txy</code> given input signal vector <code>x</code> and output signal vector <code>y</code>. The <i>transfer function</i> is the quotient of the cross spectrum of <code>x</code> and <code>y</code> and the power spectrum of <code>x</code>:</p> $T_{xy}(f) = \frac{P_{xy}(f)}{P_{xx}(f)}$ <p>The relationship between the input <code>x</code> and output <code>y</code> is modeled by the linear, time-invariant transfer function <code>Txy</code>.</p> <p>Vectors <code>x</code> and <code>y</code> must be the same length. <code>Txy = tfe(x, y)</code> uses the following default values:</p> <ul style="list-style-type: none"> • <code>nfft = min(256, (length(x)))</code> • <code>Fs = 2</code> • <code>window = hanning(nfft)</code> • <code>overlap = 0</code> <p><code>nfft</code> specifies the FFT length that <code>tfe</code> uses. This value determines the frequencies at which the power spectrum is estimated. <code>Fs</code> is a scalar that specifies the sampling frequency. <code>window</code> specifies a windowing function and the number of samples <code>tfe</code> uses in its sectioning of the <code>x</code> and <code>y</code> vectors. <code>overlap</code> is the number of samples by which the sections overlap. Any arguments that omitted from the end of the parameter list use the default values shown above.</p> <p>If <code>x</code> is real, <code>tfe</code> estimates the transfer function at positive frequencies only; in this case, the output <code>Txy</code> is a column vector of length <code>nfft/2+1</code> for <code>nfft</code> even and <code>(nfft+1)/2</code> for <code>n</code> odd. If <code>x</code> or <code>y</code> is complex, <code>tfe</code> estimates the transfer function for both positive and negative frequencies and <code>Txy</code> has length <code>nfft</code>.</p>

`Txy = tfe(x, y, nfft)` uses the specified FFT length `nfft` in estimating the transfer function. Specify `nfft` as a power of 2 for fastest execution.

`[Txy, f] = tfe(x, y, nfft, Fs)` returns a vector `f` of frequencies at which `tfe` estimates the transfer function. `Fs` is the sampling frequency. `f` is the same size as `Txy`, so `plot(f, Txy)` plots the transfer function estimate versus properly scaled frequency. `Fs` has no effect on the output `Txy`; it is a frequency scaling multiplier.

`Txy = tfe(x, y, nfft, Fs, window)` specifies a windowing function and the number of samples per section of the `x` vector. If you supply a scalar for `window`, `Txy` uses a Hanning window of that length. The length of the window must be less than or equal to `nfft`; `tfe` zero pads the sections if the length of the window exceeds `nfft`.

`Txy = tfe(x, y, nfft, Fs, window, overlap)` overlaps the sections of `x` by `overlap` samples.

You can use the empty matrix `[]` to specify the default value for any input argument except `x` or `y`. For example,

```
Txy = tfe(x, y, [], [], kaiser(128, 5))
```

uses 256 as the value for `nfft` and 2 as the value for `Fs`.

`Txy = tfe(x, y, ..., 'dflag')` specifies a detrend option, where `dflag` is

- `linear`, to remove the best straight-line fit from the prewindowed sections of `x` and `y`
- `mean`, to remove the mean from the prewindowed sections of `x` and `y`
- `none`, for no detrending (default)

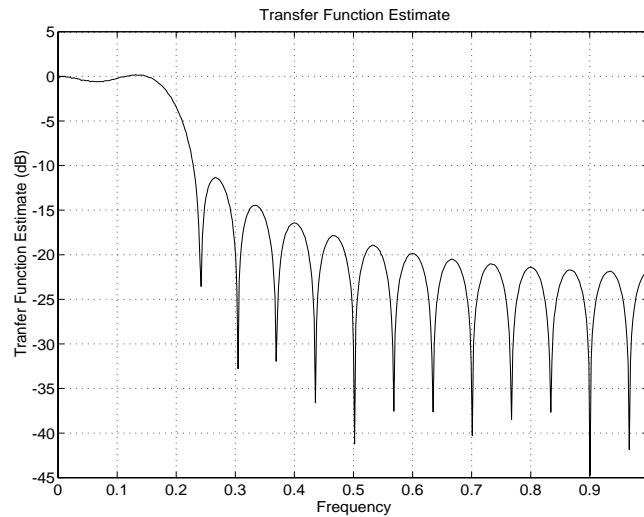
The `dflag` parameter must appear last in the list of input arguments. `tfe` recognizes a `dflag` string no matter how many intermediate arguments are omitted.

`tfe` with no output arguments plots the magnitude of the transfer function estimate in decibels versus frequency in the current figure window.

Example

Compute and plot the transfer function estimate between two colored noise sequences x and y :

```
h = fir1(30, 0.2, boxcar(31));
x = randn(16384, 1);
y = filter(h, 1, x);
tfe(x, y, 1024, [], [], 512)
```

**Algorithm**

`tfe` uses a four-step algorithm:

- 1 It multiplies the detrended sections by window.
- 2 It takes the length `nfft` FFT of each section.
- 3 It averages the squares of the spectra of the x sections to form P_{xx} and averages the products of the spectra of the x and y sections to form P_{xy} .
- 4 It calculates T_{xy} :

$$T_{xy} = P_{xy} / P_{xx}$$

Diagnostics An appropriate diagnostic message is displayed when incorrect arguments are used:

Requires window's length to be no greater than the FFT length.
Requires NOVERLAP to be strictly less than the window length.
Requires positive integer values for NFFT and NOVERLAP.
Requires vector (either row or column) input.
Requires inputs X and Y to have the same length.

See Also

etfe	Compute empirical transfer function estimate and periodogram (see <i>System Identification Toolbox User's Guide</i>).
cohere	Estimate magnitude squared coherence function between two signals.
csd	Estimate the cross spectral density (CSD) of two signals.
psd	Estimate the power spectral density (PSD) of a signal using Welch's method.
spa	Perform spectral analysis for input-output data (see <i>System Identification Toolbox User's Guide</i>).

Purpose Triangular window.

Syntax `w = triang(n)`

Description `triang(n)` returns an n -point triangular window in the column vector w . The coefficients of a triangular window are

For n odd:

$$w[k] = \begin{cases} \frac{2k}{n+1}, & 1 \leq k \leq \frac{n+1}{2} \\ \frac{2(n-k+1)}{n+1}, & \frac{n+1}{2} \leq k \leq n \end{cases}$$

For n even:

$$w[k] = \begin{cases} \frac{2k-1}{n}, & 1 \leq k \leq \frac{n}{2} \\ \frac{2(n-k)+1}{n}, & \frac{n}{2}+1 \leq k \leq n \end{cases}$$

The triangular window is very similar to a Bartlett window. The Bartlett window always ends with zeros at samples 1 and n , while the triangular window is nonzero at those points. For n odd, the center $n-2$ points of `triang(n-2)` are equivalent to `bartlett(n)`.

See Also	<code>bartlett</code>	Bartlett window.
	<code>blackman</code>	Blackman window.
	<code>boxcar</code>	Rectangular window.
	<code>chebwin</code>	Chebyshev window.
	<code>hamming</code>	Hamming window.
	<code>hanning</code>	Hanning window.
	<code>kaiser</code>	Kaiser window.

References [1] Oppenheim, A.V., and R.W. Schaffer, *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

tripuls

Purpose Sampled aperiodic triangle generator.

Syntax `y = tripuls(T)`
 `y = tripuls(T, w)`
 `y = tripuls(T, w, s)`

Description `y = tripuls(T)` returns a continuous, aperiodic, symmetric, unity-height triangular pulse at the times indicated in array T, centered about T=0 and with a default width of 1.

`y = tripuls(T, w)` generates a triangle of width w.

`y = tripuls(T, w, s)` generates a triangle with skew s, where $-1 < s < 1$. When s is 0, a symmetric triangle is generated.

See Also	<code>chirp</code>	Swept-frequency cosine generator.
	<code>cos</code>	Cosine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
	<code>diric</code>	Dirichlet or periodic sinc function.
	<code>gauspuls</code>	Gaussian-modulated sinusoidal pulse generator.
	<code>pulstran</code>	Pulse train generator.
	<code>rectpuls</code>	Sampled aperiodic rectangle generator.
	<code>sawtooth</code>	Sawtooth or triangle wave generator.
	<code>sin</code>	Sine of vector/matrix elements (see the online <i>MATLAB Function Reference</i>).
	<code>sinc</code>	Sinc or $\sin(\pi t)/\pi t$ function.
	<code>square</code>	Square wave generator.

Purpose	Unwrap phase angles.	
Syntax	<code>p = unwrap(p)</code>	
Description	<p><code>p = unwrap(p)</code> corrects the phase angles in vector <code>p</code> by adding multiples of $\pm 2\pi$, where needed, to smooth the transitions across branch cuts. When <code>p</code> is a matrix, <code>unwrap</code> corrects the phase angles down each column. The phase must be in radians.</p> <p>This function is part of the standard MATLAB environment.</p>	
Limitations	<code>unwrap</code> tries to detect branch cut crossings, but it can be fooled by sparse, rapidly changing phase values.	
See Also	<code>abs</code>	Absolute value (magnitude).
	<code>angle</code>	Phase angle.

upfirdn

Purpose Upsample, apply an FIR filter, and downsample.

Syntax

```
yout = upfirdn(xin, h)
yout = upfirdn(xin, h, p)
yout = upfirdn(xin, h, p, q)
```

Description upfirdn performs a cascade of three operations:

- 1 Upsampling by p (zero inserting)
- 2 FIR filtering with the impulse response given in h
- 3 Downsampling by q (throwing away samples)

upfirdn has been implemented as a MEX-file for maximum speed, so only the outputs actually needed are computed. The FIR filter is usually a lowpass filter, which you must design using another function such as `remez` or `fir1`.

NOTE The function `resample` performs an FIR design using `fir1`, followed by rate changing implemented with `upfirdn`.

`yout = upfirdn(xin, h)` returns the output signal `yout`. If `yout` is a row or column vector, then it represents one signal; if `yout` is an array, then each column is a separate output. `xin` is the input signal. If `xin` is a row or column vector, then it represents one signal; if `xin` is an array, then each column is filtered. `h` is the impulse response of the FIR filter. If `h` is a row or column vector, then it represents one filter; if `h` is an array, then each column is a separate impulse response.

`yout = upfirdn(xin, h, p)` specifies the upsampling factor `p`. `p` is an integer with a default of 1.

`yout = upfirdn(xin, h, p, q)` specifies the downsampling factor `q`. `q` is an integer with a default of 1.

NOTE Since `upfirdn` performs convolution and rate changing, the `yout` signals have a different length than `xin`. The length of $y[n]$ is approximately p/q times the length of $x[n]$.

Remarks

Usually the inputs `xin` and the filter `h` are vectors, in which case only one output signal is produced. However, when these arguments are arrays, each column is treated as a separate signal or filter. Valid combinations are

- 1 `xin` is a vector and `h` is a vector.

There is one filter and one signal, so the function convolves `xin` with `h`. The output signal `yout` is a row vector if `xin` is a row; otherwise, it is a column vector.

- 2 `xin` is an array and `h` is a vector.

There is one filter and many signals, so the function convolves `h` with each column of `xin`. The resulting `yout` will be an array with the same number of columns as `xin`.

- 3 `xin` is a vector and `h` is an array.

There are many filters and one signal, so the function convolves each column of `h` with `xin`. The resulting `yout` will be an array with the same number of columns as `h`.

- 4 `xin` is an array and `h` is an array, both with the same number of columns.

There are many filters and many signals, so the function convolves corresponding columns of `xin` and `h`. The resulting `yout` is an array with the same number of columns as `xin` and `h`.

Examples

If both `p` and `q` are equal to 1 (that is, there is no rate changing), the result is ordinary convolution of two signals (equivalent to `conv`):

```
yy = upfirdn(xx, hh);
```

This example implements a seven-channel filter bank by convolving seven different filters with one input signal, then downsamples by five:

```
% Assume that hh is an L-by-7 array of filters.
yy = upfirdn(xx, hh, 1, 5);
```

Implement a rate change from 44.1 kHz (CD sampling rate) to 48 kHz (DAT rate), a ratio of 160/147. This requires a lowpass filter with cutoff frequency at $\omega_c = 2\pi/160$:

```
% Design lowpass filter with cutoff at 1/160th of Fs.  
hh = fir1(300, 2/160); % need a very long lowpass filter  
yy = upfirdn(xx, hh, 160, 147);
```

In this example, the filter design and resampling are separate steps. Note that `resample` would do both steps as one.

Algorithm

`upfirdn` uses a polyphase interpolation structure. The number of multiply-add operations in the polyphase structure is approximately $(L_h L_x - p L_x)/q$ where L_h and L_x are the lengths of $h[n]$ and $x[n]$, respectively.

A more accurate flops count is computed in the program, but the actual count is still approximate. For long signals $x[n]$, the formula is quite often exact.

Diagnostics

There must be one output argument and at least two input arguments. If either of these conditions are violated, `upfirdn` gives the appropriate error message:

```
UPFIRDN needs at least two input arguments.  
UPFIRDN should have exactly one output argument.
```

If the arrays are sparse, `upfirdn` gives the error message

```
H must be full numeric matrix.
```

When the input signals are in the columns of a matrix and there are multiple filters also in the columns of a matrix, the number of signals and filters must be the same. If they are not, `upfirdn` gives the error message

```
X and H must have the same number of columns, if more than one.
```

The arguments `p` and `q` must be integers. If they are not, `upfirdn` gives the error message

```
P and/or Q must be greater than zero
```

If the arguments `p` and `q` are not relatively prime, `upfirdn` gives the warning message

```
WARNING (upfirdn) p & q have common factor
```


See Also

conv	Convolution and polynomial multiplication.
decimate	Decrease the sampling rate for a sequence (decimation).
filter	Filter data with a recursive (IIR) or nonrecursive (FIR) filter.
interp	Increase sampling rate by an integer factor (interpolation).
intfilt	Interpolation FIR filter design.
resample	Change sampling rate by any factor.

References

- [1] Crochiere, R.E., and L.R. Rabiner. *Multi-Rate Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1983. Pgs. 88-91.
- [2] Crochiere, R.E. "A General Program to Perform Sampling Rate Conversion of Data by Rational Ratios." In *Programs for Digital Signal Processing*. IEEE Press. New York: John Wiley & Sons, 1979. Pgs. 8.2-1 to 8.2-7.

VCO

Purpose Voltage controlled oscillator.

Syntax
 $y = \text{vco}(x, F_c, F_s)$
 $y = \text{vco}(x, [F_{\min} \ F_{\max}], F_s)$

Description $y = \text{vco}(x, F_c, F_s)$ creates a signal that oscillates at a frequency determined by the real input vector or array x with sampling frequency F_s . F_c is the carrier or reference frequency; when x is 0, y is an F_c Hz cosine with amplitude 1 sampled at F_s Hz. x ranges from -1 to 1, where -1 corresponds to a 0 frequency output, 0 to F_c , and 1 to $2 \cdot F_c$. y is the same size as x .

$y = \text{vco}(x, [F_{\min} \ F_{\max}], F_s)$ scales the frequency modulation range so that -1 and 1 values of x yield oscillations of F_{\min} Hz and F_{\max} Hz respectively. For best results, F_{\min} and F_{\max} should be in the range 0 to $F_s/2$.

By default, F_s is 1 and F_c is $F_s/4$.

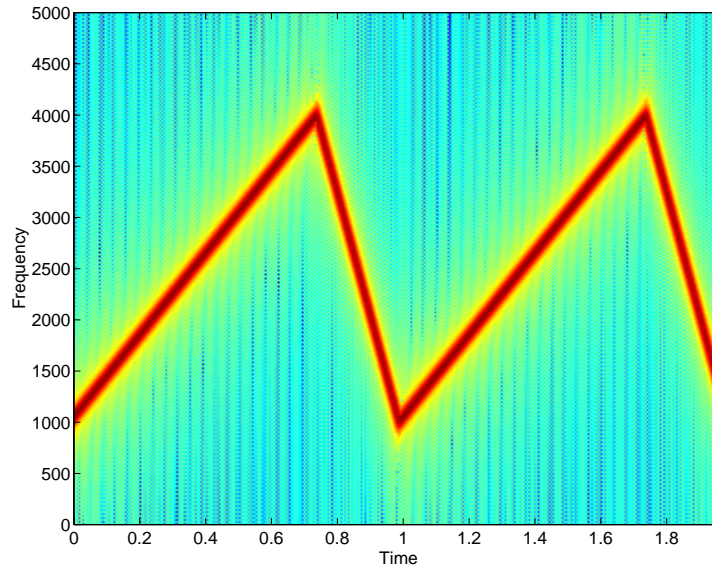
If x is a matrix, vco produces a matrix whose columns oscillate according to the columns of x .

Example Generate two seconds of a signal sampled at 10,000 samples/second whose instantaneous frequency is a triangle function of time:

```
Fs = 10000;  
t = 0: 1/Fs: 2;  
x = vco(sawtooth(2*pi*t, 0.75), [.1 0.4]*Fs, Fs);
```

Plot the spectrogram of the generated signal.

```
specgram(x, 512, Fs, kaiser(256, 5), 220)
```



Algorithm

vco performs FM modulation using the `modulate` function.

Diagnostics

If any values of `x` lie outside $[-1, 1]$, `vco` gives the following error message:

X outside of range $[-1, 1]$.

See Also

`demod`

Demodulation for communications simulation.

`modulate`

Modulation for communications simulation.

Purpose Cross-correlation function estimate.

Syntax

```
c = xcorr(x, y)
c = xcorr(x)
c = xcorr(x, y, 'option')
c = xcorr(x, y, maxlags)
c = xcorr(x, y, maxlags, 'option')
[c, lags] = xcorr(x, y)
[c, lags] = xcorr(x, y, maxlags)
[c, lags] = xcorr(x, y, maxlags, 'option')
```

Description xcorr estimates the cross-correlation sequence of random processes. Autocorrelation is handled as a special case.

The true cross-correlation sequence is

$$\gamma_{xy}(m) = E\{x_n y_{n+m}^*\}$$

where x_n and y_n are stationary random processes, $-\infty < n < \infty$, and $E\{\}$ is the expected value operator. xcorr must estimate the sequence because, in practice, access is available to only a finite segment of the infinite-length random process.

`c = xcorr(x, y)` returns the cross-correlation sequence in a length $2N-1$ vector, where x and y are length N vectors.

`c = xcorr(x)` is the autocorrelation sequence for the vector x . Where x is an N -by- P matrix, `c = xcorr(X)` returns a matrix with $2N-1$ rows whose P^2 columns contain the cross-correlation sequences for all combinations of the columns of X .

By default, xcorr computes raw correlations with no normalization. For a length N vector:

$$c_{xy}(m) = \begin{cases} \sum_{n=0}^{N-|m|-1} x_{n+1} y_{n+m+1}^* & m \geq 0 \\ c_{yx}^*(-m) & m < 0 \end{cases}$$

The output vector c has elements given by $c(m) = c_{xy}(m-N)$, $m=1,\dots,2N-1$.

The correlation function requires normalization to estimate the function properly.

$c = \text{xcorr}(x, y, 'option')$ specifies a scaling option, where '*option*' is

- **biased**, for biased estimates of the cross-correlation function:

$$c_{xy,biased}(m) = \frac{1}{N} c_{xy}(m)$$

- **unbiased**, for unbiased estimates of the cross-correlation function:

$$c_{xy,unbiased}(m) = \frac{1}{N-|m|} c_{xy}(m)$$

- **coeff**, to normalize the sequence so the autocorrelations at zero lag are identically 1.0
- **none**, to use the raw, unscaled cross-correlations (default)

See reference [1] for more information on the properties of biased and unbiased correlation estimates.

$[c, lags] = \text{xcorr}(x, y)$ where x and y are length N vectors, returns the cross-correlation sequence in a length $2N-1$ vector and the output lags in the vector $[-N+1:N-1]$. That is, the maximum lag is $N-1$.

$[c, lags] = \text{xcorr}(x, y, maxlags)$ where x and y are length N vectors, returns the cross-correlation sequence in a length $2*maxlags+1$ vector c . $lags$ is a vector of the lag indices where c was estimated, that is, $[-maxlags:maxlags]$.

$[c, lags] = \text{xcorr}(x, maxlags)$ returns the autocorrelation sequence over the lag range $[-maxlags:maxlags]$.

$[c, lags] = \text{xcorr}(X, maxlags)$ where x is an M -by- P matrix, is a matrix with $2*maxlags+1$ rows whose P^2 columns contain the cross-correlation sequences for all combinations of the columns of X .

```
[c, lags] = xcorr(x, maxlags, 'option') and
```

`[c, lags] = xcorr(x, y, maxlags, 'option')` specifies both a maximum number of lags and a scaling option.

In all cases, `xcorr` gives an output such that the zeroth lag of the correlation vector is in the middle of the sequence, at element or row `maxlags+1` or at `N`.

Examples

The second output `lags` is useful when plotting. For example, the estimated autocorrelation of zero-mean Gaussian white noise $c_{ww}(m)$ can be displayed for $-10 \leq m \leq 10$ using

```
ww = randn(1000, 1);  
[c_ww, lags] = xcorr(ww, 10, 'coeff');  
stem(lags, c_ww)
```

Swapping the `x` and `y` input arguments reverses (and conjugates) the output correlation sequence. For row vectors, the resulting sequences are reversed left to right; for column vectors, up and down. The following example illustrates this property (`mat2str` is used for a compact display of complex numbers):

```
x = [1, 2i, 3]; y = [4, 5, 6];  
[c1, lags] = xcorr(x, y);  
c1 = mat2str(c1, 2), lags  
  
c1 =  
[ 12+i *8. 9e-016 15+i *8 22+i *10 5+i *12 6-i *8. 9e-016]  
  
lags =  
-2 -1 0 1 2  
  
c2 = conj(fliplr(xcorr(y, x)));  
c2 = mat2str(c2, 2)  
  
c2 =  
[ 12+i *8. 9e-016 15+i *8 22+i *10 5+i *12 6-i *8. 9e-016]
```

For the case where input argument `x` is a matrix, the output columns are arranged so that extracting a row and rearranging it into a square array

produces the cross-correlation matrix corresponding to the lag of the chosen row. For example, the cross-correlation at zero lag can be retrieved by

```
randn('seed', 0)
X = randn(2, 2);
[M, P] = size(X);
c = xcorr(X);
c0 = zeros(P); c0(:) = c(M,:) % Extract zero-lag row

c0 =

    1.7500    0.3079
    0.3079    0.1293
```

You can calculate the matrix of correlation coefficients that the MATLAB function `corrcoef` generates by substituting

```
c = xcov(X, 'coef')
```

in the last example. The function `xcov` subtracts the mean and then calls `xcorr`.

Use `fftshift` to move the second half of the sequence starting at the zeroth lag to the front of the sequence. `fftshift` swaps the first and second halves of a sequence.

Algorithm

For more information on estimating covariance and correlation functions, see [1] and [2].

Diagnostics

There must be at least one vector input argument; otherwise, `xcorr` gives the following error message:

```
1st arg must be a vector or matrix.
```

The string '*option*' must be the last argument; otherwise, `xcorr` gives the following error message:

```
Argument list not in correct order.
```

If the second argument was entered as a scalar, it is taken to be `maxlag` and no succeeding input can be a scalar. When the second argument is a vector, the first must also be a signal vector. The third argument, when present, must be

a scalar or a string. If they are not, xcorr gives the appropriate error message(s):

```
3rd arg is maxlag, 2nd arg cannot be scalar.  
When b is a vector, a must be a vector.  
Maxlag must be a scalar.
```

Normally the lengths of the vector inputs should be the same; if they are not, then the only allowable scaling option is ' none' . If it is not, xcorr gives the following error message:

```
OPTION must be ' none' for different length vectors A and B.
```

See Also

conv	Convolution and polynomial multiplication.
corrcoef	Correlation coefficient matrix.
cov	Covariance matrix.
xcorr2	Two-dimensional cross-correlation.
xcov	Cross-covariance function estimate (equal to mean-removed cross-correlation).

References

[1] Bendat, J.S., and A.G. Piersol. *Random Data: Analysis and Measurement Procedures*. New York: John Wiley & Sons, 1971. Pg. 332.

[2] Oppenheim, A.V., and R.W. Schafer. *Digital Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1975. Pgs. 63-67, 746-747, 839-842.

Purpose Two-dimensional cross-correlation.

Syntax `C = xcorr2(A)`
`C = xcorr2(A, B)`

Description `C = xcorr2(A, B)` returns the cross-correlation of matrices A and B with no scaling. `xcorr2` is the two-dimensional version of `xcorr`. It has its maximum value when the two matrices are aligned so that they are shaped as similarly as possible.

`xcorr2(A)` is the autocorrelation matrix of input matrix A. It is identical to `xcorr2(A, A)`.

See Also

<code>conv2</code>	Two-dimensional convolution.
<code>filter2</code>	Two-dimensional digital filtering.
<code>xcorr</code>	Cross-correlation function estimate.

Purpose Cross-covariance function estimate (equal to mean-removed cross-correlation).

Syntax

```
v = xcov(x, y)
v = xcov(x)
v = xcov(x, 'option')
[c, lags] = xcov(x, y, maxlags)
[c, lags] = xcov(x, maxlags)
[c, lags] = xcov(x, y, maxlags, 'option')
```

Description `xcov` estimates the cross-covariance sequence of random processes. Autocovariance is handled as a special case.

The true cross-covariance sequence is the mean-removed cross-correlation sequence

$$\phi_{xy}(m) = E\{(x_n - m_x)(y_{n+m} - m_y)^*\}$$

where m_x and m_y are the mean values of the two stationary random processes, and $E\{\}$ is the expected value operator. `xcov` estimates the sequence because, in practice, access is available to only a finite segment of the infinite-length random process.

`v = xcov(x, y)` returns the cross-covariance sequence in a length $2N-1$ vector, where `x` and `y` are length N vectors.

`v = xcov(x)` is the autocovariance sequence for the vector `x`. Where `x` is an N -by- P array, `v = xcov(X)` returns an array with $2N-1$ rows whose P^2 columns contain the cross-covariance sequences for all combinations of the columns of `X`.

By default, `xcov` computes raw covariances with no normalization. For a length N vector:

$$c_{xy}(m) = \begin{cases} \sum_{n=0}^{N-|m|-1} \left(x(n) - \frac{1}{N} \sum_{i=0}^{N-1} x_i \right) \left(y_{n+m}^* - \frac{1}{N} \sum_{i=0}^{N-1} y_i^* \right) & m \geq 0 \\ c_{yx}^*(-m) & m < 0 \end{cases}$$

The output vector c has elements given by $c(m) = c_{xy}(m-N)$, $m=1,\dots,2N-1$.

The covariance function requires normalization to estimate the function properly.

$v = \text{xcov}(x, 'option')$ specifies a scaling option, where *option* is

- *biased*, for biased estimates of the cross-covariance function
- *unbiased*, for unbiased estimates of the cross-covariance function
- *coeff*, to normalize the sequence so the auto-covariances at zero lag are identically 1.0
- *none*, to use the raw, unscaled cross-covariances (default)

See [1] for more information on the properties of biased and unbiased correlation and covariance estimates.

$[c, \text{lags}] = \text{xcov}(x, y, \text{maxlags})$ where x and y are length m vectors, returns the cross-covariance sequence in a length $2*\text{maxlags}+1$ vector c . lags is a vector of the lag indices where c was estimated, that is, $[-\text{maxlags}: \text{maxlags}]$.

$[c, \text{lags}] = \text{xcov}(x, \text{maxlags})$ is the autocovariance sequence over the range of lags $[-\text{maxlags}: \text{maxlags}]$.

$[c, \text{lags}] = \text{xcov}(x, \text{maxlags})$ where x is an m -by- p array, returns array c with $2*\text{maxlags}+1$ rows whose P^2 columns contain the cross-covariance sequences for all combinations of the columns of X .

$[c, \text{lags}] = \text{xcov}(x, y, \text{maxlags}, 'option')$ specifies a scaling option, where *option* is the last input argument.

In all cases, `xcov` gives an output such that the zeroth lag of the covariance vector is in the middle of the sequence, at element or row $\text{maxlags}+1$ or at m .

Examples

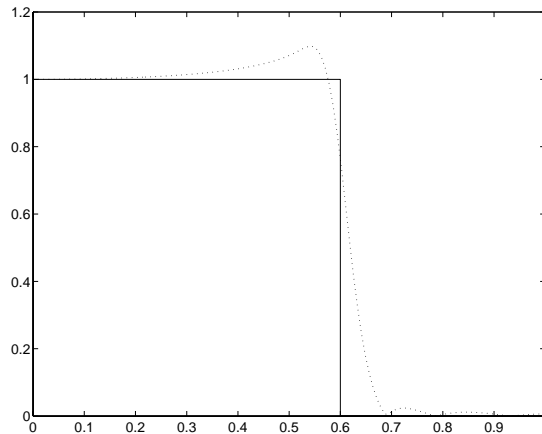
The second output lags is useful when plotting. For example, the estimated autocovariance of uniform white noise $c_{ww}(m)$ can be displayed for $-10 \leq m \leq 10$ using

```
ww = randn(1000, 1); % generate uniform noise with mean = 1/2
[cov_ww, lags] = xcov(ww, 10, 'coeff');
stem(lags, cov_ww)
```

Algorithm	xcov computes the mean of its inputs, subtracts the mean, and then calls xcorr. For more information on estimating covariance and correlation functions, see [1] and [2].	
Diagnostics	xcov does not check for any errors other than the correct number of input arguments. Instead, it relies on the error checking in xcorr, which it calls.	
See Also	conv	Convolution and polynomial multiplication.
	corrcoef	Correlation coefficient matrix.
	cov	Covariance matrix.
	xcorr	Cross-correlation function estimate.
	xcorr2	Two-dimensional cross-correlation.
References	<p>[1] Bendat, J.S., and A.G. Piersol. <i>Random Data: Analysis and Measurement Procedures</i>. New York: John Wiley & Sons, 1971. Pg. 332.</p> <p>[2] Oppenheim, A.V., and R.W. Schaffer. <i>Digital Signal Processing</i>. Englewood Cliffs, NJ: Prentice Hall, 1975. Pgs. 63-67, 746-747, 839-842.</p>	

Purpose	Recursive digital filter design.
Syntax	<code>[b, a] = yulewalk(n, f, m)</code>
Description	<p><code>yulewalk</code> designs recursive IIR digital filters using a least-squares fit to a specified frequency response.</p> <p><code>[b, a] = yulewalk(n, f, m)</code> returns row vectors <code>b</code> and <code>a</code> containing the $n+1$ coefficients of the order n IIR filter whose frequency-magnitude characteristics approximately match those given in vectors <code>f</code> and <code>m</code>:</p> <ul style="list-style-type: none"> • <code>f</code> is a vector of frequency points, specified in the range between 0 and 1, where 1 corresponds to half the sample frequency (the Nyquist frequency). The first point of <code>f</code> must be 0 and the last point 1, with all intermediate points in increasing order. Duplicate frequency points are allowed, corresponding to steps in the frequency response. • <code>m</code> is a vector containing the desired magnitude response at the points specified in <code>f</code>. • <code>f</code> and <code>m</code> must be the same length. • <code>plot(f, m)</code> displays the filter shape. <p>The output filter coefficients are ordered in descending powers of z,</p> $\frac{B(z)}{A(z)} = \frac{b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n}}{a(1) + a(2)z^{-1} + \dots + a(n+1)z^{-n}}$ <p>When specifying the frequency response, avoid excessively sharp transitions from passband to stopband. You may need to experiment with the slope of the transition region to get the best filter design.</p>
Example	<p>Design an 8th-order lowpass filter and overplot the desired frequency response with the actual frequency response:</p> <pre>f = [0 0.6 0.6 1]; m = [1 1 0 0];</pre>

```
[b, a] = yulewalk(8, f, m);  
[h, w] = freqz(b, a, 128);  
plot(f, m, w/pi, abs(h), ' - - ')
```



Algorithm

`yulewalk` performs a least-squares fit in the time domain. It computes the denominator coefficients using modified Yule-Walker equations, with correlation coefficients computed by inverse Fourier transformation of the specified frequency response. To compute the numerator, `yulewalk` takes the following steps:

- 1 Computes a numerator polynomial corresponding to an additive decomposition of the power frequency response.
- 2 Evaluates the complete frequency response corresponding to the numerator and denominator polynomials.
- 3 Uses a spectral factorization technique to obtain the impulse response of the filter.
- 4 Obtains the numerator polynomial by a least-squares fit to this impulse response.

See Also

butter	Butterworth analog and digital filter design.
cheby1	Chebyshev type I filter design (passband ripple).
cheby2	Chebyshev type II filter design (stopband ripple).
ellip	Elliptic (Cauer) filter design.
fir2	Window-based finite impulse response filter design—arbitrary response.
firls	Least square linear-phase FIR filter design.
maxflat	Generalized digital Butterworth filter design.
remez	Parks-McClellan optimal FIR filter design.

References

- [1] Friedlander, B., and B. Porat. "The Modified Yule-Walker Method of ARMA Spectral Estimation." *IEEE Transactions on Aerospace Electronic Systems*. AES-20, No. 2 (March 1984). Pgs. 158-173.

Purpose Zero-pole-gain to second-order section conversion.

Syntax
`sos = zp2sos(z, p, k)`
`sos = zp2sos(z, p, k, 'order')`

Description `zp2sos` converts a zero-pole-gain representation of a given system to an equivalent second-order section representation.

`sos = zp2sos(z, p, k)` finds a matrix `sos` in second-order section form equivalent to the zero-pole-gain system represented by input arguments `z`, `p`, and `k`. Vectors `z` and `p` contain the zeros and poles of the system $H(z)$, not necessarily in any order:

$$H(z) = k \frac{(z - z(1))(z - z(2)) \dots (z - z(N))}{(p - p(1))(p - p(2)) \dots (p - p(M))}$$

`k` is a scalar gain. The zeros and poles must be real or come in complex conjugate pairs. `sos` is an L -by-6 matrix:

$$sos = \begin{bmatrix} b_{01} & b_{11} & b_{21} & a_{01} & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & a_{02} & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0L} & b_{1L} & b_{2L} & a_{0L} & a_{1L} & a_{2L} \end{bmatrix}$$

whose rows contain the numerator and denominator coefficients b_{ik} and a_{ik} of the second-order sections of $H(z)$:

$$H(z) = \prod_{k=1}^L H_k(z) = \prod_{k=1}^L \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{a_{0k} + a_{1k}z^{-1} + a_{2k}z^{-2}}$$

The number of rows L of matrix `sos` is the maximum of the ceiling of $N/2$ and the ceiling of $M/2$, where N and M are the lengths of `z` and `p`, respectively.

`sos = zp2sos(z, p, k, 'order')` specifies the order of the rows in `sos`, where *order* is

- down, to order the sections so the first row of `sos` contains the poles closest to the unit circle
- up, to order the sections so the first row of `sos` contains the poles farthest from the unit circle (default)

Example

Find a second-order section form of a Butterworth lowpass filter:

```
[z, p, k] = butter(5, 0.2);
sos = zp2sos(z, p, k);
```

Algorithm

`zp2sos` uses a four-step algorithm to determine the second-order section representation for an input zero-pole-gain system:

- 1 It groups the zeros and poles into complex conjugate pairs using the `cplxpair` function.
- 2 It forms the second-order section by matching the pole and zero pairs according to the following rules:
 - a Match the poles closest to the unit circle with the zeros closest to those poles.
 - b Match the poles next closest to the unit circle with the zeros closest to those poles.
 - c Continue until all of the poles and zeros are matched.

`zp2sos` groups real poles into sections with the real poles closest to them in absolute value. The same rule holds for real zeros.

- 3 It orders the sections according to the proximity of the pole pairs to the unit circle. `zp2sos` normally orders the sections with poles closest to the unit circle last in the cascade. You can tell `zp2sos` to order the sections in the reverse order by specifying the down flag.

Putting “high Q” sections at the beginning of the cascade, by specifying the down flag, reduces the response sensitivity of the filter to quantization noise near those poles. Putting “high Q” sections at the end of the cascade (the default) prevents reduction in signal power level early in the cascade.

`zp2sos` orders all zero sections according to the minimum of $|z_i|$ and $|z_i^{-1}|$

where z_i (for $i = 1, 2$) are the zeros in the section. References [1] and [2] provide detailed discussions of section ordering.

- 4 `zp2sos` scales the sections so the maximum of the magnitude of the transfer function of the first N sections in cascade is less than 1:

$$\max_{|\omega| \leq \pi} \left| \prod_{i=1}^N H_i(e^{j\omega}) \right| < 1, \quad N = 1, \dots, L-1$$

subject to the constraint that the overall gain, k , stays the same:

$$\prod_{k=1}^L \frac{b_{0k}}{a_{0k}} = k$$

This scaling is an attempt to minimize overflow in some standard fixed point implementations of filtering.

See Also	<code>cplxpair</code>	Group complex numbers into complex conjugate pairs.
	<code>sos2ss</code>	Second-order section to state-space conversion.
	<code>sos2tf</code>	Second-order section to transfer function conversion.
	<code>sos2zp</code>	Second-order section to zero-pole-gain conversion.
	<code>ss2sos</code>	State-space to second-order section conversion.

References

[1] Oppenheim, A.V., and R.W. Schaffer. *Discrete-Time Signal Processing*. Englewood Cliffs, NJ: Prentice Hall, 1989. Pgs. 363-370.

[2] Jackson, L.B. *Digital Filters and Signal Processing*. Third Ed. Boston: Kluwer Academic Publishers, 1989. Pgs. 319-324.

Purpose	Zero-pole-gain to state-space conversion.	
Syntax	$[A, B, C, D] = \text{zp2ss}(Z, p, k)$	
Description	<p>zp2ss converts a zero-pole-gain representation of a given system to an equivalent state-space representation. It forms state-space models from the zeros, poles, and gains of systems in transfer function form.</p> <p>$[A, B, C, D] = \text{zp2ss}(Z, p, k)$ finds a single input, multiple output, state-space representation</p> $\dot{x} = Ax + Bu$ $y = Cx + Du$ <p>given a system in factored transfer function form</p> $H(s) = \frac{z(s)}{p(s)} = k \frac{(s - z(1))(s - z(2)) \dots (s - z(n))}{(s - p(1))(s - p(2)) \dots (s - p(n))}$ <p>Column vector p specifies the pole locations, and array Z the zero locations with as many columns as there are outputs. The gains for each numerator transfer function are in vector k. The A, B, C, and D matrices are returned in controller canonical form.</p> <p>Inf values may be used as place holders in Z if some columns have fewer zeros than others.</p>	
Algorithm	zp2ss, for single-input systems, groups complex pairs together into two-by-two blocks down the diagonal of the A matrix. This requires the zeros and poles to be real or come in complex conjugate pairs.	
See Also	ss2tf	State-space to transfer function conversion.
	ss2zp	State-space to zero-pole-gain conversion.
	tf2ss	Transfer function to state-space conversion.
	tf2zp	Transfer function to zero-pole-gain conversion.
	zp2tf	Zero-pole-gain to transfer function conversion.

Purpose	Zero-pole-gain to transfer function conversion.	
Syntax	<code>[num, den] = zp2tf(Z, p, k)</code>	
Description	<p><code>zp2tf</code> forms transfer function polynomials from the zeros, poles, and gains of a system in factored form.</p> <p><code>[num, den] = zp2tf(z, p, k)</code> finds a rational transfer function:</p> $\frac{num(s)}{den(s)} = \frac{num(1)s^{nn-1} + \dots + num(nn-1)s + num(nn)}{den(1)s^{nd-1} + \dots + den(nd-1)s + den(nd)}$ <p>given a system in factored transfer function form:</p> $H(s) = \frac{Z(s)}{p(s)} = k \frac{(s - Z(1))(s - Z(2)) \dots (s - Z(m))}{(s - p(1))(s - p(2)) \dots (s - p(n))}$ <p>Column vector <code>p</code> specifies the pole locations, and array <code>Z</code> the zero locations, with as many columns as there are outputs. The gains for each numerator transfer function are in vector <code>k</code>. The zeros and poles must be real or come in complex conjugate pairs. The polynomial coefficients are returned in vectors: the denominator coefficients in row vector <code>den</code> and the numerator coefficients in matrix <code>num</code>, with as many rows as there are columns of <code>z</code>.</p> <p><code>Inf</code> values can be used as place holders in <code>Z</code> if some columns have fewer zeros than others.</p>	
Algorithm	The system is converted to transfer function form using <code>poly</code> with <code>p</code> and the columns of <code>Z</code> .	
See Also	<code>ss2tf</code>	State-space to transfer function conversion.
	<code>ss2zp</code>	State-space to zero-pole-gain conversion.
	<code>tf2ss</code>	Transfer function to state-space conversion.
	<code>tf2zp</code>	Transfer function to zero-pole-gain conversion.
	<code>zp2ss</code>	Zero-pole-gain to state-space conversion.

Purpose Zero-pole plot.

Syntax `zplane(z, p)`
`zplane(b, a)`
`[hz, hp, ht] = zplane(z, p)`

Description This function displays the poles and zeros of discrete-time systems.

`zplane(z, p)` plots the zeros specified in column vector `z` and the poles specified in column vector `p` in the current figure window. The symbol 'o' represents a zero and the symbol 'x' represents a pole. The plot includes the unit circle for reference. If `z` and `p` are arrays, `zplane` plots the poles and zeros in the columns of `z` and `p`, respectively, in different colors.

You can override the automatic scaling of `zplane` using

```
axis([xmi n xmax ymi n ymax])
```

or

```
set(gca, 'ylim', [ymi n ymax])
```

or

```
set(gca, 'xlim', [xmi n xmax])
```

after calling `zplane`. This is useful in the case where one or a few of the zeros or poles have such a large magnitude that the others are grouped around the origin and are thus hard to distinguish.

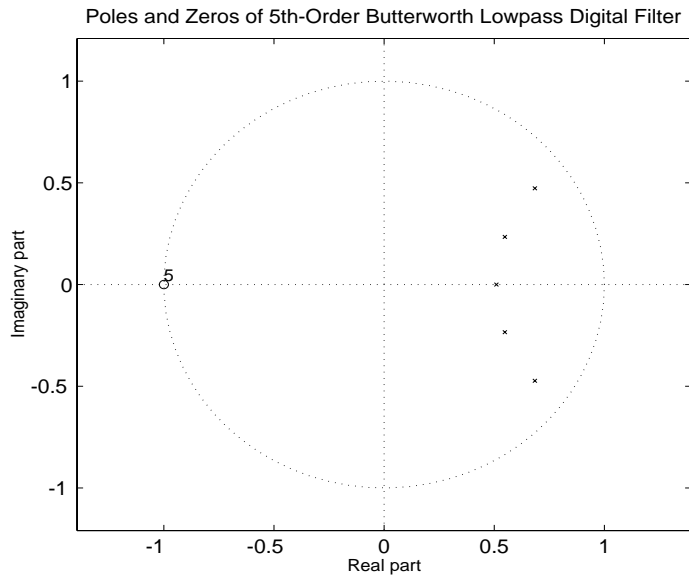
`zplane(b, a)` where `b` and `a` are row vectors, first uses `roots` to find the zeros and poles of the transfer function represented by numerator coefficients `b` and denominator coefficients `a`.

`[hz, hp, ht] = zplane(z, p)` returns vectors of handles to the zero lines, `hz`, and the pole lines, `hp`. `ht` is a vector of handles to the axes/unit circle line and to text objects, which are present when there are multiple zeros or poles. If there are no zeros or no poles, `hz` or `hp` is set to the empty matrix `[]`.

Examples

Plot the poles and zeros of a 5th-order Butterworth lowpass digital filter with cutoff frequency of 0.2:

```
[z, p, k] = butter(5, 0.2);  
zplane(z, p)
```



To generate the same plot with a transfer function representation of the filter:

```
[b, a] = butter(5, 0.2);    % transfer function  
zplane(b, a)
```

See Also

`freqz`

Frequency response of digital filters.

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